Continuous Workout Mortgages

Adelaide Business School Research Seminar

Robert J. Shiller — Yale University, USA
Rafał M. Wojakowski* — **Lancaster University, UK**
M. Shahid Ebrahim — Bangor University, UK
Mark B. Shackleton — Lancaster University, UK

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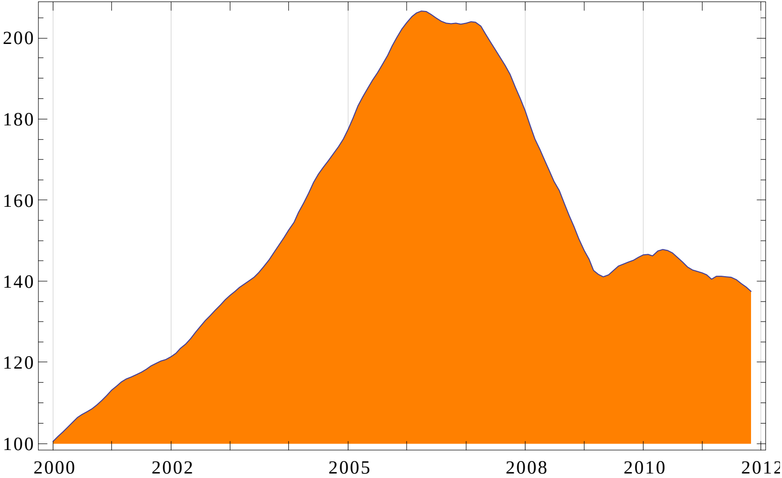
Home loans for everybody: short-term "solution" $\Longrightarrow$ not sustainable in long term

- In the US 25% of mortgages are "underwater" (Economist, October 2011):
  - Balance $>$ House value $\Longrightarrow$ cannot refinance (at historically low interest rates) $\Longrightarrow$ Foreclosure
- The new "Obama's Foreclosure Plan:" (end January 2012)
  - Lender lowers interest or principal, government provides subsidies (to the lender);
  - Criticized for helping banks at taxpayers' expense.

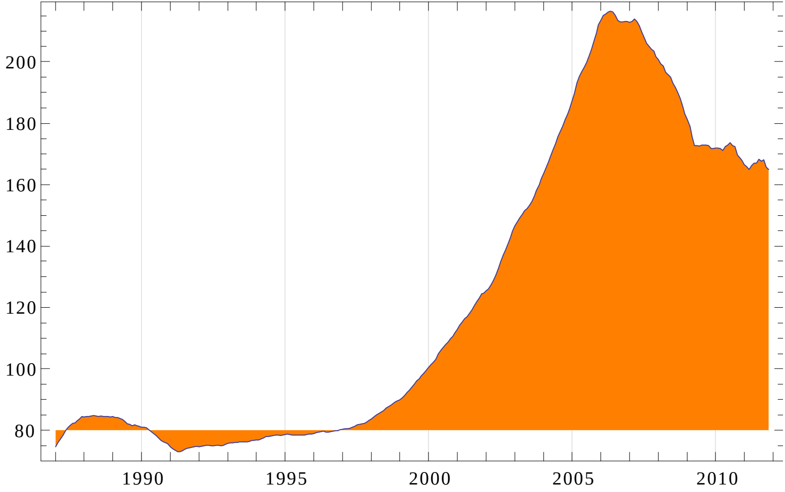
We incorporate long-term $\Longrightarrow$ Robust to economic downturn $\Longrightarrow$ Sustainable

Case-Shiller house price index

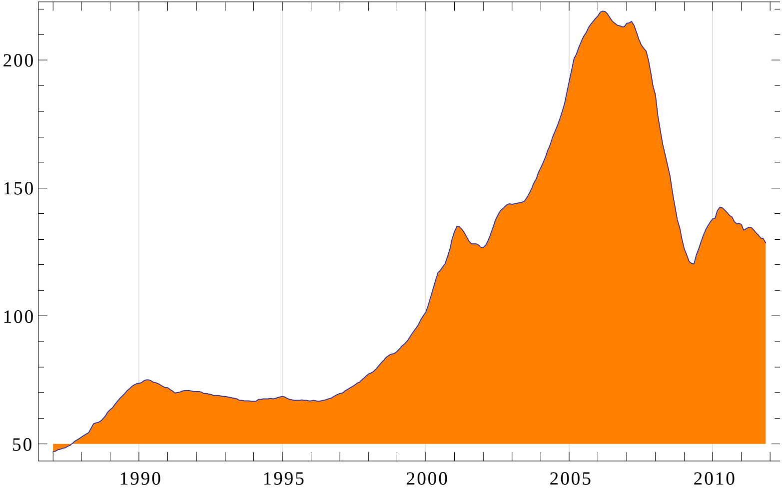
- Case-Shiller house price index is below the 2009 low.



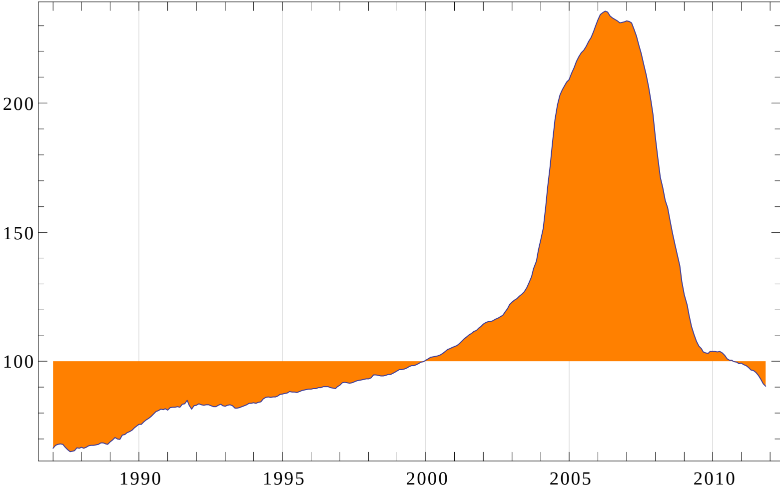
S&P Case-Shiller home price index, 20–city composite (12 yrs → Nov 2011)



S&P Case-Shiller home price index, New York metro area (25 yrs → Nov 2011)

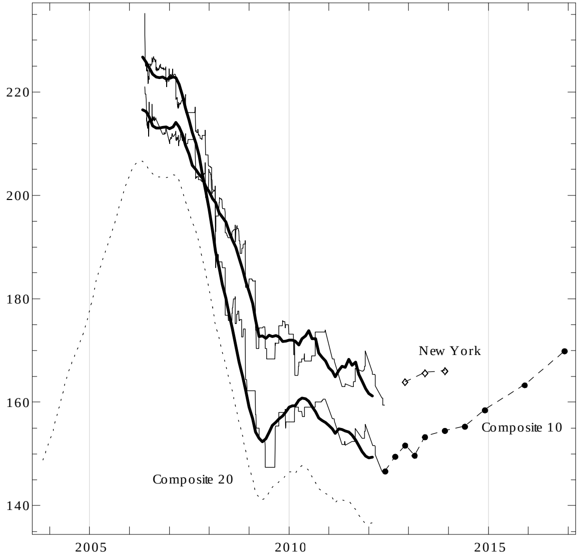


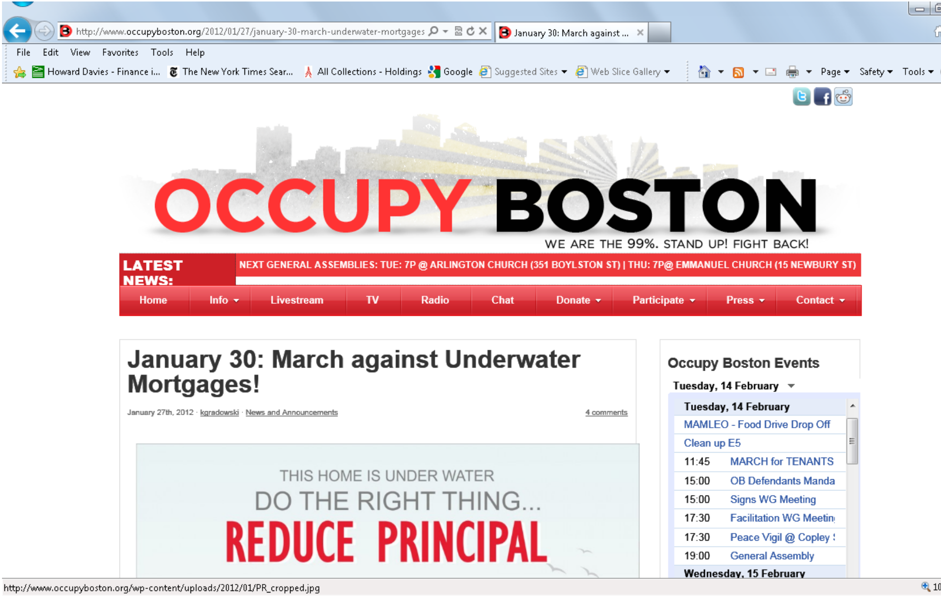
S&P Case-Shiller home price index, San Francisco (25 yrs → Nov 2011)



S&P Case-Shiller home price index, Las Vegas (25 yrs → Nov 2011)

Case-Shiller Index Futures (CME)

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January 30: March against Underwater Mortgages!
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THIS HOME IS UNDER WATER
DO THE RIGHT THING...
REDUCE PRINCIPAL
Occupy Boston Events
Tuesday, 14 February
Tuesday, 14 February
MAMLEO - Food Drive Drop Off
Clean up E5
11:45 MARCH for TENANTS
15:00 OB Defendants Manda
15:00 Signs WG Meeting
17:30 Facilitation WG Meetin
17:30 Peace Vigil @ Copley
19:00 General Assembly
Wednesday, 15 February
http://www.occupyboston.org/wp-content/uploads/2012/01/PR_cropped.jpg


THIS HOME IS UNDER WATER
DO THE RIGHT THING...
REDUCE PRINCIPAL
CLVU
CLVU
CLVU
S.S. City Life Vida Urbana
Fannie Mae
Freddie Mac
US Bank
Bank of America

"If workouts are part of the original mortgage contract, then they can be done continually, systematically, and automatically, eliminating the delays, irregularities and uncertainties that are seen with the workouts we have observed in the subprime crisis. A great deal of human suffering at a time of economic contraction would be eliminated."

Robert Shiller (2009), Article 4, p. 11

- Modernizing finance institutions and loan contracts in response to the crisis is needed to promote confidence.
  - My **Participating Mortgages (PMs)** paper (JBF (2011)) offers an *ex-post* solution to foreclosure problem.
  - **Continuous Workout Mortgages (CWMs)** paper (JEBO (2012)) offers an *ex-ante* solution to foreclosure problem.

Participating vs Continuous Workout mortgages

PMs

- A mortgage that *reduces the monthly interest payment*.
  - Cost to homeowner: Lender takes portion of appreciation or income.

CWMs

- A mortgage where the *principal (and monthly payment) goes down when house prices go down*.
  - A two-in-one product: loan + insurance against real estate market decline.

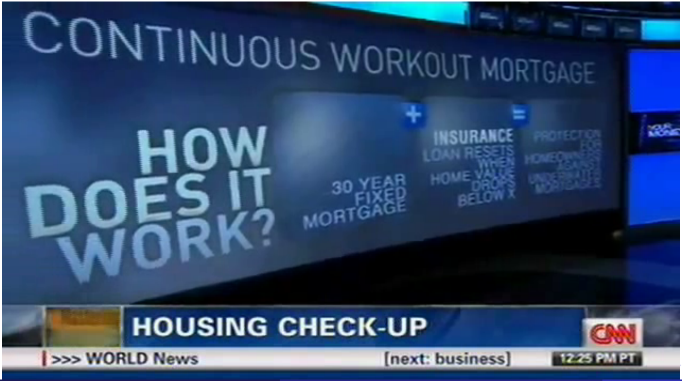
Continuous Workouts

Home price changes $\Longrightarrow$ Mortgage balance adjusts $\Longrightarrow$ Monthly payment adjusts automatically

1. Eliminates incentive to default on monthly payments;
2. Helps prevent foreclosures;
3. Eliminates costly workout procedures;
4. Preserves uninterrupted stream of monthly payments to the lender.

CWM @ CNN Money, April 2012

click



Motivation

- Continuous Workout Mortgages (CWMs) have never been considered prior to the current crisis.
- Shiller in his book *"Subprime Solution"* (2008) and paper (2009) studies CWMs as an extension of the Price-Level Adjusted Mortgages (PLAMs):
    - PLAM: adjusts to a *consumer price index;*
    - CWM: adjusts to a *house price index.*
- Shiller (2010, JPM): financial regulation should promote innovation to make markets more resilient and less prone to crisis.
- In a recent paper Ambrose and Buttimer (2012, REE) numerically investigate properties of Adjustable Balance Mortgages which bear many similarities to CWMs.
- Methodology to value CWM cash flows is available, see e.g. Carr, Lipton and Madan (2000) and Shackleton & Wojakowski (2007).

Purpose of our work

Goals:

1. Simple analytical model to see how CWM mechanism works;
2. Design variants of CWMs and price them via *closed form solutions*:
   - Interest only CWMs
   - Repayment CWMs
3. Study impact of prepayments

Interest-only Continuous Workout Mortgage: Assumptions

Interest-only:

- Contract rate i, 100% LTV, riskless rate r
- Property acquired at $t = 0$, initial cost H_0, subsequent values $H_t : t > 0$ *not observable*
- Instead, consider h_t, **initial home value weighted by the local house price index:**
  - If the index goes up, h_t goes up
  - If the index goes down, h_t goes down
- e.g. H_0 weighted by the Case-Shiller index (note that $h_0 = H_0$)

Interest-only Continuous Workout Mortgage: Payoffs

- **At maturity** $t = T$ the CWM protects against falls in the home value

$$\text{Repayment at maturity } = \min\{H_0, h_T\} = H_0 - \underbrace{(H_0 - h_T)^+}_{\text{Put}}$$

- More importantly, **before maturity** $t < T$, the CWM also protects

$$\text{Intermediate balance} = \min\{H_0, h_t\} = H_0 - \boxed{\underbrace{(H_0 - h_t)^+}_{\textbf{Floorlet}}}$$

Interest-only Continuous Workout Mortgage: Valuation

Valuation of the CWM:

- **Not trivial** because it is necessary to *sum up* (integrate over time) all the intermediate *expected interest payments*

$$\overline{V} = V - \frac{i}{r} \boxed{\underbrace{\int_0^T E\left[e^{-rt}\left(rH_0 - rh_t\right)^+\right] dt}_{\substack{P\left(rH_0, rH_0, T, r, \delta, \sigma\right) \\ \textbf{Floor on flow } rh_t}}} - \underbrace{E\left[e^{-rT}\left(H_0 - h_T\right)^+\right]}_{\substack{p\left(H_0, H_0, T, r, \delta, \sigma\right) \\ \text{Put on value } h_T}}$$

where $V = i\frac{H_0}{r}\left(1 - e^{-rT}\right) + e^{-rT} H_0$

Proposition

An interest-only Continuous Workout Mortgage (CWM) with protective put and floor has **equilibrium contract rate** *i equal to*

$$i = r\left[\frac{H_0\left(1-e^{-rT}\right)+p\left(H_0,H_0,T,r,\delta,\sigma\right)}{H_0\left(1-e^{-rT}\right)-P\left(rH_0,rH_0,T,r,\delta,\sigma\right)}\right]$$

where $P\left(rH_0,rH_0,T,r,\delta,\sigma\right)$ is a **floor on flow** *rH with value given by a* **closed form formula** *(Shackleton & Wojakowski, JEDC (2007)).*

Other parameters:

- Service flow rate δ
- Volatility σ of the house price index

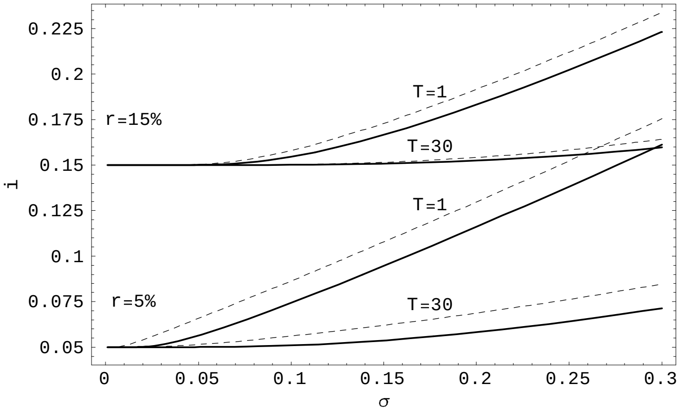


Figure: **Interest-only Continuous Workout Mortgage.** Contract rate i as a function of risk σ. Tenure is $T = 30$ years or $T = 1$ years to maturity, riskless rates are $r = 5\%$ and $r = 15\%$ and service flow rate is either $\delta = 1\%$ (thick lines) or 4% (dashed lines).

Interest-only Continuous Workout Mortgage: "Mechanics"

Market incorporates insurance offered by the CWM into an ex ante increase of the contract rate i

House prices may fall	$\Longrightarrow$	Protective floor P and put p may appreciate	$\Longrightarrow$	$i > r$

Risk

- *Contract rates i increase as risk σ increases*

Interest rates

- *When interest rates go up, the spread $i - r$ decreases*

Time to maturity

- *When time to maturity is short, the contract rate (price of insurance) increases*

Service flow

- *Guarantee more expensive for higher service flows*

Fully amortizing repayment mortgages:

- **Major invention** introduced during years of Great Depression
  - Constant repayment flow rate R, balance decreases to zero at maturity $Q_T = 0$
  - Amount owed to the lender

$$Q_t = \int_t^T R\ e^{-r(s-t)} ds = \frac{R}{r}\left(1 - e^{-r(T-t)}\right) \tag{1}$$

is equal to the present value of remaining payments

- **Mortgage brokers** around the world use this formula

$$\boxed{R = \frac{rQ_0}{1 - e^{-rT}}} \tag{2}$$

  - Computes payment R as a function of initial balance Q_0, rate r and term T.

Repayment Continuous Workout Mortgage: Assumptions

Repayment flow R **scaled down** *proportionally* when house price index decreases

$$\begin{array}{ll} R(h_t) = \rho & \text{if } h_t \text{ above } H_0 \\ R(h_t) < \rho & \text{if } h_t \text{ below } H_0 \end{array}$$

where h_t is the **initial home value weighted by the local house price index**

- $R(h_t)$ is capped from above by ρ
  - **"Maximal repayment flow parameter"** ρ must be *higher* $\rho > R$ than for a standard repayment mortgage
- We generalize this to $R_\alpha(h_t)$ to obtain a **Partial Workout**
  - **Extra parameter** "workout proportion" $\alpha \in [0, 1]$
  - Full protection: $\alpha = 1$, no protection: $\alpha = 0$

Repayment Partial CWM

Generalization: Partial workout

- Mortgage payment scaled down to

$$R_{\alpha}\left(h_t\right) = \rho \min\left\{1, 1 - \alpha\left(1 - \frac{h_t}{h_0}\right)\right\} = \rho\left[1 - \alpha\left(1 - \frac{h_t}{h_0}\right)^{+}\right]$$

  where $0 \leq \alpha \leq 1$
- Full protection: $\alpha = 1$, no protection: $\alpha = 0$

Repayment CWM: Full vs Partial Workout

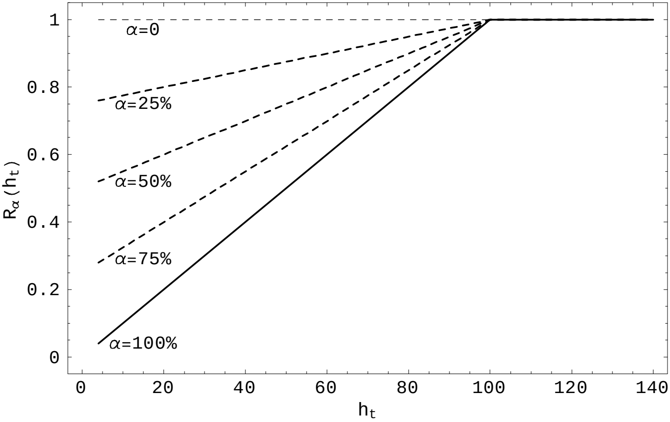


Figure: **Repayment Continuous Workout Mortgage (RCWM)**. Reduction of mortgage payment $R_\alpha(h_t)$ as a function of current adjusted house price level h_t and different "workout proportions" α. Full workout is $\alpha = 100\%$ (thick solid line), no workout (standard repayment mortgage, thin dashed line) is $\alpha = 0$.

Repayment CWM

Proposition

For a given "workout proportion" $\alpha \in [0,1]$, the mortgage repayment flow is

$$\rho = \frac{rQ_0}{1 - e^{-rT} - \frac{\alpha}{H_0} P\left(rH_0, rH_0, T, r, \delta, \sigma\right)},$$

per annum, where Q_0 is the initial value of the loan and $P\left(rH_0, rH_0, T, r, \delta, \sigma\right)$ is a **floor on flow** *rH with value given by a* **closed form formula** *(Shackleton & Wojakowski, JEDC (2007)).*

- For $\alpha = 0$ (no workout) reverts to the standard fixed rate repayment mortgage
- **For a 30-year repayment mortgage the continuous workout premium is not very large making RCWMs very attractive**

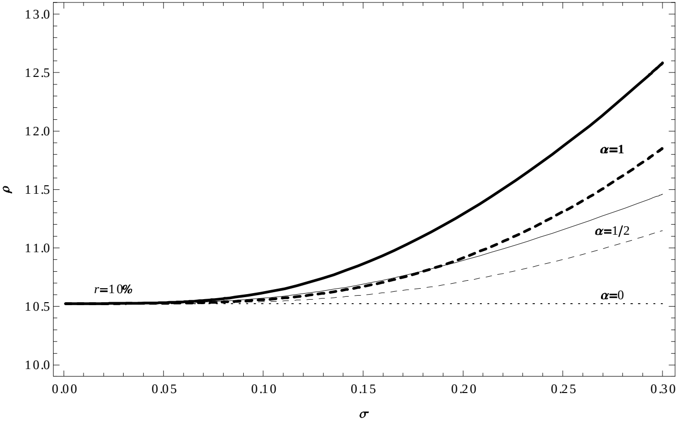


Figure: **Repayment Continuous Workout Mortgage (RCWM).** Mortgage payment ρ as a function of risk σ. Full workout ($\alpha = 1$) is represented by thick lines, standard repayment level (no workout, $\alpha = 0$) are the thin flat lines. Tenure is $T = 30$ years to maturity, riskless rates are $r = 5\%$ and $r = 15\%$ and service flow rate is either $\delta = 1\%$ (thick lines) or 4% (thick dashed lines).

Repayment CWM with Protective Threshold

Threshold Repayment Continuous Workout Mortgage (TRCWM):

- Idea: Freely set a "protection threshold" $\overline{K}$
  - Higher or lower than the initial house price index level $\overline{H}_0$
  - For example. if $\overline{K} < \overline{H}_0$ the continuous workout kicks-in *only after* house values fall below $\overline{K}$

We obtain **closed form expressions** for:

- Mortgage balance Q_t at time $t > 0$ of a TRCWM
- Mortgage payment flow ρ of a TRCWM

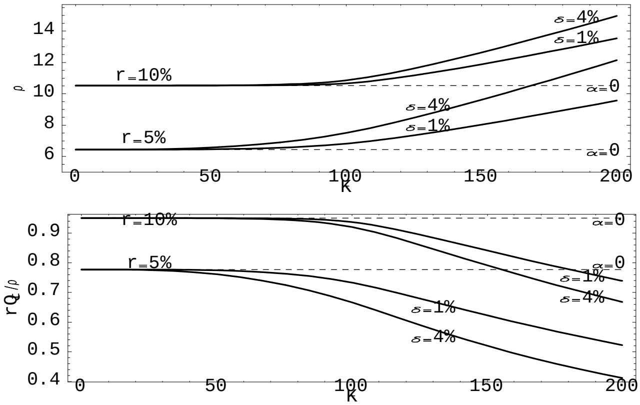


Figure: **Threshold Repayment Continuous Workout Mortgage (TRCWM)** Mortgage balance rQ_t/ρ, as a function of “workout threshold” $\overline{K}$. Interest rate $r = 5\%$ and 10%, service flow rate $\delta = 1\%$ and 4%, volatility $\sigma = 15\%$, term $T = 30$ years to repayment, initial house price and index $\overline{H}_0 = 100$.

Prepayments and CWMs

- Empirical evidence confirms that **borrowers do not prepay optimally**, i.e. they do not maximize the loss to the lender.
- Modelling: Public Securities Association **(PSA) benchmark** (see e.g. Fabozzi (2005)).
  - Consistent with **hazard rate** (prepayment intensity) approach: Random prepayment time τ_p is a Poisson process with intensity λ, *independent* of the house prices processes.
- **Prepayment penalty**: Financial institutions typically charge a **fixed fraction α of the principal outstanding** if refinancing occurs before some lock-in date T^*

Valuation of CWMs when there are prepayments

- We obtain a **closed-form CWM valuation condition** which we solve numerically for implicit variables
  - Based on the following mortgage cash flows valuation condition

$$M_0 = E\left[\underbrace{\int_0^T e^{-\lambda t} e^{-rt} c_t\, dt}_{payments} + \underbrace{\int_0^T \lambda e^{-\lambda t} e^{-rt} S_t\, dt}_{prepayment} + \underbrace{e^{-\lambda T} e^{-rT} S_T}_{repayment}\right],$$

    where $e^{-\lambda t}$, $e^{-\lambda T}$ are "survival" probabilities (beyond t, T) and $\lambda e^{-\lambda t}$ is prepayment probability to occur within $(t, t + dt]$
    - $c_t = i \min \{h_0, h_t\}$ are contractual payments received until prepayment or maturity, whichever comes first
    - $S_t = \min \{h_0, h_t\} + \Pi_t$ is the prepayment lump sum
    - $S_T = \min \{h_0, h_t\}$ is the repayment lump sum (if prepayment did not occur before maturity)

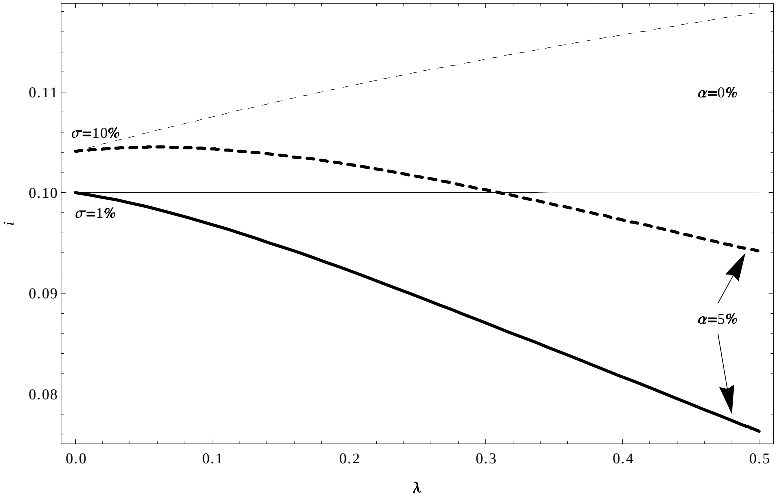


Figure: **Continuous Workout Mortgage contract rate** i as a function of the prepayment rate λ.

Numerical Example

1. Contract rate i will be higher for higher volatilities σ
2. The higher the prepayment rate λ, the higher the contract rate i
   - However, when prepayment penalty ($\alpha = 5\%$) is added, the contract rate i can become a *decreasing* function of the prepayment rate λ
   - Also, contract rate can become *lower* than the standard rate (when prepayment penalty finances workout insurance)
3. Contract rate i will not be very much different from the standard rate ($r = 10\%$) unless volatility σ, prepayment rate λ or penalty α are high

Theorem

We obtain the following identity, which links a weighted sum of two annuities (left hand side) with a weighted sum of two floors and one put (right hand side)

$$\begin{aligned}(i-r)\times A\left(1,r+\lambda,T\right) & \\ +\lambda\alpha\times A\left(1,r+\lambda,T^{*}\right) &= (i+\lambda)\times P\left(1,1,T,r+\lambda,\delta+\lambda,\sigma\right) \\ &\quad +\lambda\alpha\times P\left(1,1,T^{*},r+\lambda,\delta+\lambda,\sigma\right) \\ &\quad +p\left(1,1,T,r+\lambda,\delta+\lambda,\sigma\right)\ ,\end{aligned} \tag{3}$$

where we used the annuity function

$$A\left(x,\rho,\tau\right)=\frac{x}{\rho}\left(1-e^{-\rho T}\right)\ .$$

The right hand side of (3) takes into account the benefits to the lender per unit value of housing. Similarly, the left hand side takes into account the costs. The benefits (costs) to the lender (homeowner) are:

1. The insurance premium paid by the homeowner;
2. The expected early prepayment penalty paid by the homeowner.

The costs (benefits) to the lender (homeowner) are:

1. The expected continuous workout paid to the homeowner;
2. The expected decrease of prepayment penalty when there is workout early;[1]
3. The expected terminal workout paid to the homeowner.

Not surprisingly, identity (3) does not depend on the level of housing H_0. Moreover, it can be solved *explicitly* for the mortgage rate i.

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[1] Early workout will lower the outstanding balance Q_t, to which the prepayment penalty is proportional here.

Corollary

When $M_0 = H_0$ and in absence of prepayment penalties ($\alpha = 0$) the CWM valuation equation simplifies to

$$\underbrace{\frac{H_0\,(i-r)}{r+\lambda}\left(1-e^{-(r+\lambda)T}\right)}_{\textit{Insurance premium (annuity)}} = \underbrace{P\left(\left(i+\lambda\right)H_0, \left(i+\lambda\right)H_0, T, r+\lambda, \delta+\lambda, \sigma\right)}_{\textbf{Continuous Workout}} + \underbrace{p\left(H_0, H_0, T, r+\lambda, \delta+\lambda, \sigma\right)}_{\textit{Terminal Workout}} \tag{4}$$

i.e. in absence of arbitrage the extra income generated by charging the customer the premium $i - r > 0$ must compensate for the cost of continuous workout insurance expressed by the floor P (intermediate, continuous workouts) and the put p (automatic workout at maturity).

Concluding Remarks

Economy fragile to plain vanilla mortgage instruments:

- CWMs reduce default risk, reduce moral hazard

Our contributions:

- **Simple explicit modelling** of several variants of CWMs:
  - Interest-only and repayment contracts
  - Negotiable parameters: "workout proportion" α and "protection threshold" $\overline{K}$
- **Closed form formulae** for key parameters of CWMs:
  - *Repayment*: for **mortgage balances** *and* **payments**
  - *Interest-only*: for **mortgage interest**
- **Impact of prepayment risk** investigated

Thank you!

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Paper + Presentation
Dynamic Link Libraries (DLLs) for *Mathematica* and *Excel*

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Appendix: Floor and Put formulae

Floors P on flow s with strike flow level k for finite horizon T can be computed using the following closed-form formula (see Shackleton and Wojakowski (2007)):

$$P(s_0, k, T, r, \delta, \sigma) = As_0^a \left(\mathbf{1}_{s_0<k} - N(-d_a)\right) - \frac{s_0}{\delta}\left(\mathbf{1}_{s_0<k} - e^{-\delta T} N(-d_1)\right) \tag{5}$$

$$+ \frac{k}{r}\left(\mathbf{1}_{s_0<k} - e^{-rT} N(-d_0)\right) - Bs_0^b \left(\mathbf{1}_{s_0<k} - N(-d_b)\right),$$

where

$$A = \frac{k^{1-a}}{a-b}\left(\frac{b}{r} - \frac{b-1}{\delta}\right), \tag{6}$$

$$B = \frac{k^{1-b}}{a-b}\left(\frac{a}{r} - \frac{a-1}{\delta}\right),$$

and

$$a, b = \frac{1}{2} - \frac{r-\delta}{\sigma^2} \pm \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}, \tag{7}$$

whereas the cumulative normal integrals $N(\cdot)$ are labelled with parameters d_β

$$d_\beta = \frac{\ln s_0 - \ln k + \left(r - \delta + \left(\beta - \frac{1}{2}\right)\sigma^2\right) T}{\sigma\sqrt{T}} \tag{8}$$

(different to the standard textbook notation) for elasticity β which takes one of four values $\beta \in \{a, b, 0, 1\}$.

Standard Black-Scholes (1973) put on S with strike value of K can be computed using

$$p(S_0, K, T, r, \delta, \sigma) = Ke^{-rT} N(-d_0) - S_0 e^{-\delta T} N(-d_1) \tag{9}$$

where d_0 and d_1 can be computed using formula (8) in which *values* S_0 and K can (formally) be used in place of *flows* s_0 and k.

Both floor (5) and put (9) formulae assume that the underlying flow s or asset S follows the stochastic differential equation

$$\frac{ds_t}{s_t} = \frac{dS_t}{S_t} = (r - \delta)\, dt + \sigma dZ_t \tag{10}$$

with initial values s_0 and S_0, respectively. Clearly, (10) describes a geometric Brownian motion under risk-neutral measure where Z_t is the standard Brownian motion, σ is the volatility, r is the riskless rate and δ is the service flow. We assume that (10) describes the dynamics of the repayment flow s. Similarly, (10) also defines the dynamics of the value S of the real estate property.