Subsidy for Education and Fertility: long-run evolution of the economy with heterogeneous households and idiosyncratic shocks to the human capital

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Introduction

- Analyse the effect of the government support for education and fertility on the long-run development of the economy
- Perform our analysis in the environment with heterogeneous households who experience idiosyncratic shocks during human capital formation
- Utilise the model of overlapping generations introduced by *de la* Croix and Doepke (AER, 2003) to formalise behaviour of households and producer
  - This model features endogenous choices of households for education provision and childbearing decisions which forms the human capital and population of future generations
- Expand original model economy to incorporate the government sector and uncertainty for the human capital formation

Literature

Analysis for the deterministic environment that uses OLG framework:

- Subsidy for education:
  - Long-run increase in number of children and abilities of adults (Fanti and Gori, 2011; and Chen, 2015)
  - Long-run welfare improvement (Chen, 2015)
- Subsidy for children (child allowance):
  - Long-run decrease in level of human capital, and long-run increase in fertility (Chen, 2015)
  - Long-run decrease in level of human capital, however, no effect on fertility (Fanti and Gori, 2011)
  - Long-run decrease in fertility when parents spend small amount of time on raising children
  - Long-run increase in fertility when parents spend large amount of time on raising children (Momota, 2000)

Literature (cont.)

Analysis for the stochastic environment:

- Kogan and Walker (2007), real option theory:
  - An environment with higher risks (in the labour market) creates the incentives for higher education attainment which increases human capital.
  - Additionally, with increase in the labour income tax, individuals decide to stay in education longer.
- Akyol and Athreya (2004), dynamic heterogeneous agent model:
  - Subsidy for tertiary education increases participation in the higher education and produces welfare improvement when there is uncertainty for returns on the education investment

Summary for the results

Subsidy for education:

- At the average level, it improves education attainment, which, however, diminishes fertility
- It increases the average level of human capital, but population decreases in its size
- It improves utility at individual and average levels, but it is found to be ineffective for stimulus of education provision by lower ability groups who still optimally provide their children with zero education
- It decreases level of inequality in distribution of human capital across the population

Summary for the results (cont.)

Subsidy for fertility:

- At the average and aggregate levels, it leads to the outcomes which are opposite from the case of subsidy for education
- At individual level, it only improves utility of lower ability households who drastically increase their choices for fertility and decrease their education provision
- Higher ability households simultaneously increase their education and fertility choices, however

Model economy – Households

- Three generations of households: young, adult and elderly
- Households are heterogeneous in the level of human capital
- Young: receive education optimally chosen by their parents. Amount of education received is one of forms the human capital
- Additionally, we consider that young households receive idiosyncratic shocks during human capital accumulation
- Adult households make all economically relevant decisions
- Adults: maximise personal utility, participate in the labour market, earn labour income, consume, save, pay taxes, receive subsidies, make decision for level of education to provide for their children and number of children to have
- Adult households, however, experience the trade-offs in their decisions for the number of children to have and level of education to provide their children with
- Elderly: use savings made during adulthood, consume, pay taxes

Model economy – Households (cont.)

For $i \in [1, I]$ and $t \in [1, T]$

$$\max_{<c_t^i, s_t^i, e_t^i, n_t^i>} E_t[u_t^i] = E_t\left[\ln c_t^i + \beta \ln d_{t+1}^i + \gamma \ln n_t^i h_{t+1}^i\right]$$

$$(1+\tau_t^c)c_t^i + s_t^i + e_t^i n_t^i w_t \bar{h}_t = (1-\tau_t^l) w_t h_t^i (1-\phi n_t^i) + n_t^i w_t \bar{h}_t (e_t^i sub_t^e + \bar{e}_t sub_t^n)$$

$$E_t[(1+\tau_{t+1}^c) d_{t+1}^i] = E_t\left[\frac{1}{(1+\rho)}\left(1 + r_{t+1}(1-\tau_{t+1}^k)\right) s_t^i\right]$$

$$E_t[h_{t+1}^i] = E_t\left[\frac{1}{(1+\rho)} B(\theta + e_t^i)^\eta (h_t^i)^\pi (\bar{h}_t)^\kappa \times \exp^\varepsilon\right]$$

Model Economy – Producer

- Production sector: single firm maximises its profit by optimally choosing its factor inputs – physical capital stock and effective labour force

$$\max_{<K_t, L_t>} \Pi_t = Y_t - w_t L_t - (r_t + \delta) K_t$$

$$Y_t = A K_t^{\alpha} L_t^{1-\alpha}$$

- Both factor inputs are supplied by households

$$L_t = \frac{\sum_{i=1}^{I} p_t^i \left[h_t^i (1 - \phi n_t^i) - n_t^i e_t^i \bar{h}_t \right]}{\sum_{i=1}^{I} p_t^i}$$

$$K_t = \frac{1}{(1+\rho)} \left(\frac{\sum_{i=1}^{I} p_{t-1}^i s_{t-1}^i}{\sum_{i=1}^{I} p_t^i} + (1 - \delta) K_{t-1} \right)$$

Model Economy – Government

- We introduce the government in form of policy maker which provides the households with exogenously chosen subsidy rates for education and fertility
- We assume that government budget constraint is always balanced

$$\sum_{i=1}^{I} p_t^i \tau_t^c c_t^i + \sum_{i=1}^{I} p_{t-1}^i \tau_t^c d_t^i + \sum_{i=1}^{I} p_t^i r_t \tau_t^k s_t^i + \sum_{i=1}^{I} p_t^i \tau_t^l w_t h_t^i (1 - \phi n_t^i) =$$

$$= \sum_{i=1}^{I} p_t^i n_t^i w_t \bar{h}_t (e_t^i sub_t^e + \bar{e}_t sub_t^n)$$

- Government balances its budget through the endogenous choice for the tax rates, given the optimal decisions of the households and the firm, and the subsidy program in place

Policy Experiments

- We consider that government enters into the economy and provides subsidy rate for education and fertility of ten percent
- We examine the case of these two subsidies separately, however, to avoid the overlap in fertility and education responses given the presence of parental ‘quality-quantity‘ trade-off
- For each of the tax option we perform its own policy experiment to analyse the type of distortion that given tax option has and consider possible implications for human capital development, population growth, welfare and inequality in distribution of human capital across the population

Subsidy for Education

Subsidy for education – Initial response

As government enters the economy:

- Adult households from every ability group (with exception for three groups with lowest level of human capital) increase education provision for their children
- Fertility decisions for these households, however, diminish
- Due to increase in required education provision level, there is an increase in number of teachers required, which decreases the size of the effective labour force
- Due to initial decrease in consumption of adult and elderly households as a result of introduction of the taxes and changes in the wage and interest rates, utility at individual and average levels diminishes
- However, as a result of increase in education provision
  - children more easily observe negative ability shocks
  - population share o future higher ability households begins to increase; while the population share of future lower ability households begins to decrease

Subsidy for education – Education, Fertility and Population share – Final steady-state

	e^i		n^i		p^i/P	
	τ^c,τ^k	τ^l	τ^c,τ^k	τ^l	τ^c,τ^k	τ^l
$i=1$	—	—	—	0.60%	-62.81%	-60.45%
$i=2$	—	—	—	0.60%	-60.79%	-58.60%
$i=3$	—	—	—	0.60%	-57.23%	-54.90%
$i=4$	10.76%	10.10%	-1.09%	-0.43%	-50.50%	-48.29%
$i=5$	5.03%	4.73%	-0.79%	-0.14%	-41.35%	-39.37%
$i=6$	3.50%	3.29%	-0.59%	0.05%	-31.68%	-30.02%
$i=7$	2.81%	2.64%	-0.45%	0.18%	-22.09%	-20.81%
$i=8$	2.41%	2.27%	-0.35%	0.27%	-11.76%	-10.95%
$i=9$	2.17%	2.03%	-0.27%	0.35%	0.68%	0.80%
$i=10$	2.00%	1.88%	-0.22%	0.40%	16.18%	15.34%
$i=11$	1.88%	1.76%	-0.17%	0.44%	34.33%	32.28%
$i=12$	1.79%	1.68%	-0.14%	0.47%	53.15%	49.72%
$i=13$	1.72%	1.62%	-0.11%	0.50%	74.10%	69.96%
$i=14$	1.67%	1.57%	-0.09%	0.52%	113.29%	104.77%
$i=15$	1.63%	1.53%	-0.07%	0.54%	212.99%	195.46%
mean	16.25%	15.27%	-2.61%	-1.88%		

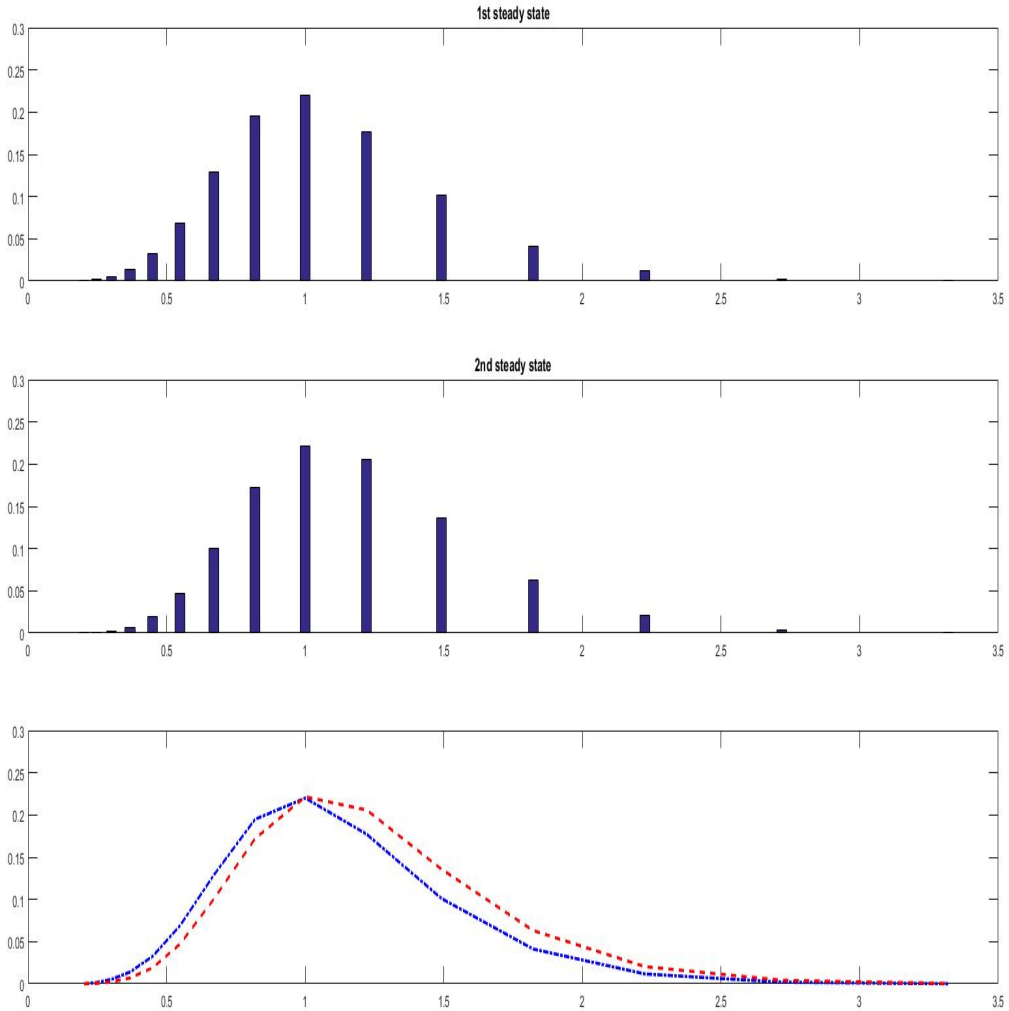
Subsidy for education – Final steady-state

	τ^c	τ^l	τ^k
τ^c	0.0052	0	0
τ^l	0	0.0060	0
τ^k	0	0	0.0102
$\bar{h}$	9.50%	8.93%	9.50%
P	-63.97%	-51.86%	-63.97%
L	6.61%	6.77%	6.61%
K	10.96%	9.04%	10.96%
Y	8.04%	7.52%	8.04%
w	1.34%	0.70%	1.34%
r	-3.57%	-1.89%	-3.57%
$c^i \ \forall i$	0.82%	0.10%	1.34%
$\bar{c}$	10.39%	9.04%	10.96%
$s^i \ \forall i$	1.34%	0.10%	1.34%
$\bar{s}$	10.96%	9.04%	10.96%
$d^i \ \forall i$	-1.83%	-1.29%	-2.06%
$\bar{d}$	7.49%	7.52%	7.24%

Subsidy for education – Utility and Welfare – Final steady-state

u^i	τ^c	τ^l	τ^k
$i = 1$	0.09%	-0.06%	0.24%
$i = 2$	0.10%	-0.07%	0.27%
$i = 3$	0.11%	-0.08%	0.30%
$i = 4$	0.03%	-0.18%	0.25%
$i = 5$	0.07%	-0.18%	0.32%
$i = 6$	0.11%	-0.19%	0.40%
$i = 7$	0.15%	-0.21%	0.51%
$i = 8$	0.22%	-0.26%	0.69%
$i = 9$	0.33%	-0.35%	1.01%
$i = 10$	0.62%	-0.61%	1.84%
$i = 11$	3.19%	-2.91%	9.23%
$i = 12$	1.10%	-0.95%	3.14%
$i = 13$	0.48%	-0.40%	1.35%
$i = 14$	0.31%	-0.25%	0.86%
$i = 15$	0.23%	-0.18%	0.63%
mean	18.39%	16.77%	18.98%

Subsidy for education – Distribution of the population across ability groups



	steady state		
	initial	τ^c, τ^k	τ^l
Gini coefficient	0.2018	0.1968↓	0.1971↓
Coefficient of variation	0.3724	0.3637↓	0.3640↓
Mean absolute deviation	0.2818	0.2864↑	0.2866↑
Kuznets ratio, $\leqslant 40\%$	0.1230	0.0764↓	0.0786↓
Kuznets ratio $\geqslant 60\%$	0.3332	0.4299↑	0.4241↑
The range	3.1190	2.8474↓	2.8622↓

Subsidy for Fertility

Subsidy for fertility – Initial response

As government enters the economy:

- Adult households from every ability group increase their fertility choices
- Due to inter-dependency of education and fertility decisions, all household ability groups (with exception for ones who provide their children with zero education) diminish the education provision for their children
- Due to increase in number of children born, there is an increase in number of teachers required and decrease in time that adult households can contribute for labour market participation. This decreases the size of the effective labour force
- There is increase in utility at individual and average level, due to sizeable increase in fertility decisions
- However, as a result of decrease in education provision, the human capital of children decreases and population share of lower ability households increases for the next generations

Subsidy for fertility – Education, Fertility and Population share – Final steady-state

	e^i		n^i		p^i/P	
	τ^c,τ^k	τ^l	τ^c,τ^k	τ^l	τ^c,τ^k	τ^l
$i=1$	—	—	40.42%	40.81%	396.02%	408.22%
$i=2$	—	—	30.84%	31.30%	411.58%	422.10%
$i=3$	—	—	23.91%	24.41%	337.56%	345.65%
$i=4$	-71.14%	-70.78%	29.39%	29.90%	202.68%	207.89%
$i=5$	-22.83%	-22.74%	19.55%	20.09%	84.41%	87.30%
$i=6$	-9.94%	-9.93%	13.87%	14.43%	24.13%	25.75%
$i=7$	-4.06%	-4.08%	10.24%	10.81%	-0.61%	0.24%
$i=8$	-0.74%	-0.78%	7.76%	8.34%	-11.31%	-11.10%
$i=9$	1.36%	1.30%	5.99%	6.57%	-15.46%	-15.82%
$i=10$	2.78%	2.71%	4.68%	5.26%	-15.50%	-16.36%
$i=11$	3.79%	3.71%	3.70%	4.28%	-12.25%	-13.60%
$i=12$	4.53%	4.45%	2.94%	3.52%	-5.84%	-7.80%
$i=13$	5.08%	5.00%	2.35%	2.94%	4.65%	1.80%
$i=14$	5.51%	5.42%	1.89%	2.48%	25.96%	21.40%
$i=15$	5.84%	5.75%	1.53%	2.11%	85.14%	75.77%
mean	-8.80%	-9.57%	14.98%	15.82%		

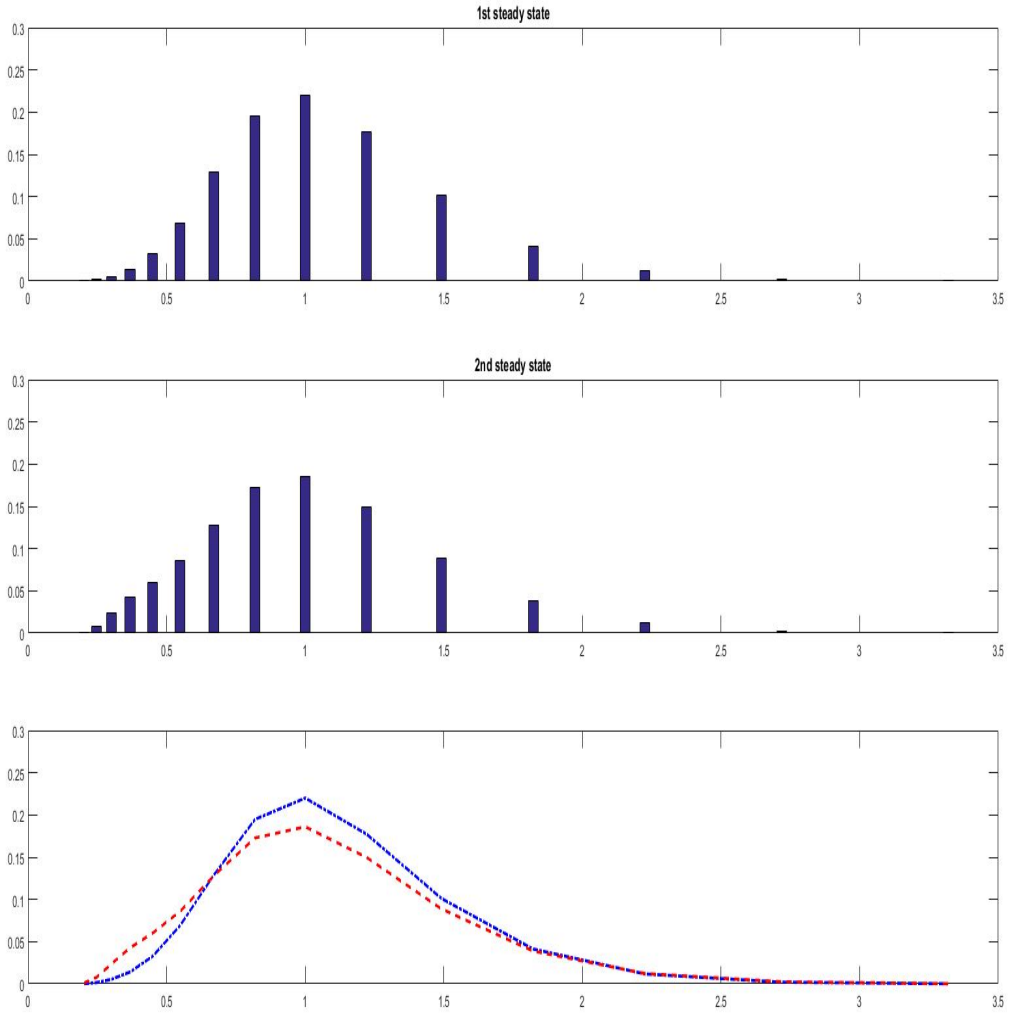
Subsidy for fertility – Final steady-state

	τ^c	τ^l	τ^k
τ^c	0.0047	0	0
τ^l	0	0.0058	0
τ^k	0	0	0.0084
$\bar{h}$	-6.68%	-7.14%	-6.68%
P	192.87	255.78	192.87
L	1.58%	1.73%	1.58%
K	-10.55%	-12.06%	-10.55%
Y	-2.63%	-3.09%	-2.63%
w	-4.15%	-4.74%	-4.15%
r	12.01%	13.83%	12.01%
$c^i \ \forall i$	-4.60%	-5.29%	-4.15%
$\bar{c}$	-10.97%	-12.06%	-10.55%
$s^i \ \forall i$	-4.15%	-5.29%	-4.15%
$\bar{s}$	-10.55%	-12.06%	-10.55%
$d^i \ \forall i$	3.84%	4.36%	3.66%
$\bar{d}$	-3.09%	-3.09%	-3.26%

Subsidy for fertility – Utility and Welfare – Final steady-state

u^i	τ^c	τ^l	τ^k
$i = 1$	0.86%	0.68%	1.00%
$i = 2$	0.47%	0.28%	0.63%
$i = 3$	0.11%	-0.10%	0.29%
$i = 4$	0.51%	0.26%	0.71%
$i = 5$	-0.21%	-0.49%	0.02%
$i = 6$	-0.83%	-1.15%	-0.55%
$i = 7$	-1.49%	-1.89%	-1.15%
$i = 8$	-2.38%	-2.89%	-1.93%
$i = 9$	-3.88%	-4.61%	-3.24%
$i = 10$	-7.58%	-8.89%	-6.42%
$i = 11$	-39.89%	-46.35%	-34.15%
$i = 12$	-14.03%	-16.21%	-12.10%
$i = 13$	-6.20%	-7.12%	-5.37%
$i = 14$	-4.04%	-4.63%	-3.51%
$i = 15$	-3.02%	-3.45%	-2.63%
mean	-21.23%	-22.84%	-20.66%

Subsidy for fertility – Distribution of the population across ability groups



	steady state		
	initial	τ^c,τ^k	τ^l
Gini coefficient	0.2018	0.2389↑	0.2391↑
Coefficient of variation	0.3724	0.4376↑	0.4383↑
Mean absolute deviation	0.2818	0.3442↑	0.3449↑
Kuznets ratio, $\leqslant 40\%$	0.1230	0.2209↑	0.2242↑
Kuznets ratio $\geqslant 60\%$	0.3332	0.2924↓	0.2881↓
The range	3.1190	3.3402↑	3.3580↑

Conclusion

- We analysed the long-run effects of government support for education and fertility in the environment with heterogeneous households and idiosyncratic shocks to the human capital
- As a foundation for our analysis, we utilised the model of overlapping generations of *de la* Croix and Doepke (AER, 2003)
- We find that largest welfare improvement takes place when government provides subsidy for education and levies tax on capital income.
- Largest increase in population size takes place when government provides subsidy for fertility and levies the tax on labour income
- Finally, the smallest level of inequality in distribution of the human capital is reached when government uses tax on consumption or capital income to finance the subsidy for education

Appendix

Model economy – Households – Deterministic case – Analytical solution

$$e_t^i = \frac{\eta\phi(1-\tau_t^l)x_t^i - \theta(1-sub_t^e) - \eta\bar{e}_t sub_t^n}{(1-sub_t^e)(1-\eta)}$$

$$c_t^i = \frac{(1-\tau_t^l)}{(1+\tau_t^c)}\frac{1}{(1+\beta+\gamma)}w_t h_t^i$$

$$s_t^i = (1-\tau_t^l)\frac{\beta}{(1+\beta+\gamma)}w_t h_t^i$$

$$d_{t+1}^i = \frac{1}{(1+\rho)}\frac{(1-\tau_t^l)}{(1+\tau_{t+1}^c)}\frac{\beta}{(1+\beta+\gamma)}\left(1+r_{t+1}(1-\tau_{t+1}^k)\right)w_t h_t^i$$

$$n_t^i = \frac{\gamma}{(1+\beta+\gamma)}x_t^i\left[(1-\tau_t^l)\phi x_t^i + e_t^i(1-sub_t^e) - \bar{e}_t sub_t^n\right]^{-1}$$

where $x_t^i = h_t^i/\bar{h}_t$

Computational algorithm

1. Discretise the population into 15 ability groups with each group being defined by its marginal level of human capital. Before uncertainty is introduced, we consider a uniform distribution of human capital across the population
2. Discretise the random component of the human capital formation function, which is log-normally distributed, into 5 nodes using Binomial probability to calculate probability of occurrence of each node
3. Introduce idiosyncratic shocks into the human capital accumulation process, and obtain the stable distribution for the original steady state without government presence ($t = 1$)
4. Introduce the government policy and calculate the transition path to the second steady state ($t \in [2, T]$)
5. Reach the second steady state with government presence, and obtain the stable distribution ($t = T$)

Parametrization

- Closely follow original calibration of de la Croix and Doepke (2003)
- (Note: one period of analysis = one generation = 30 years)
- We, however, normalise values of A, B and γ to obtain $w_1 = 1$, $\bar{h}_1 = 1$ and $\bar{n}_1 = 1$
- We assume that standard deviation of idiosyncratic shocks to the human capital σ follows standard deviation of lifetime labour income, which is 20% (Bosworth et al, 2000; and Millimet et al, 2003)

Extrapolation procedure

<table>
<tr><td colspan="2">if $0 \leq h^i_{t,j} < h^1$</td></tr>
<tr><td>if $(h^2 - h^1) > (h^1 - h^i_{t,j})$
$p^1_t = \frac{(h^2 - h^1)}{(h^2 - h^i_{t,j})} p^i_{t-1} b^i_j + p^1_t$
$p^2_t = \frac{(h^1 - h^i_{t,j})}{(h^2 - h^i_{t,j})} p^i_{t-1} b^i_j + p^2_t$</td><td>if $(h^1 - h^i_{t,j}) > (h^2 - h^1)$
$p^1_t = \frac{(h^1 - h^i_{t,j}))}{(h^2 - h^i_{t,j})} p^i_{t-1} b^i_j + p^1_t$
$p^2_t = \frac{(h^2 - h^1}{(h^2 - h^i_{t,j})} p^i_{t-1} b^i_j + p^2_t$</td></tr>
<tr><td colspan="2">if $h^L \leq h^i_{t,j} \leq h^H$</td></tr>
<tr><td colspan="2">$p^L_t = \frac{(h^H - h^i_{t,j})}{(h^H - h^L)} p^i_{t-1} b^i_j + p^L_t$
$p^H_t = \frac{(h^i_{t,j} - h^L)}{(h^H - h^L)} p^i_{t-1} b^i_j + p^H_t$</td></tr>
<tr><td colspan="2">if $h^i_{t,j} > h^{15}$</td></tr>
<tr><td>if $(h^{15} - h^{14}) > (h^i_{t,j} - h^{15})$
$p^{14}_t = \frac{(h^i_{t,j} - h^{15})}{(h^i_{t,j} - h^{14})} p^i_{t-1} b^i_j + p^{14}_t$
$p^{15}_t = \frac{(h^{15} - h^{14})}{(h^i_{t,j} - h^{14})} p^i_{t-1} b^i_j + p^{15}_t$</td><td>if $(h^i_{t,j} - h^{15}) > (h^{15} - h^{14})$
$p^{14}_t = \frac{(h^{15} - h^{14})}{(h^i_{t,j} - h^{14})} p^i_{t-1} b^i_j + p^{14}_t$
$p^{15}_t = \frac{(h^i_{t,j} - h^{15})}{(h^i_{t,j} - h^{14})} p^i_{t-1} b^i_j + p^{15}_t$</td></tr>
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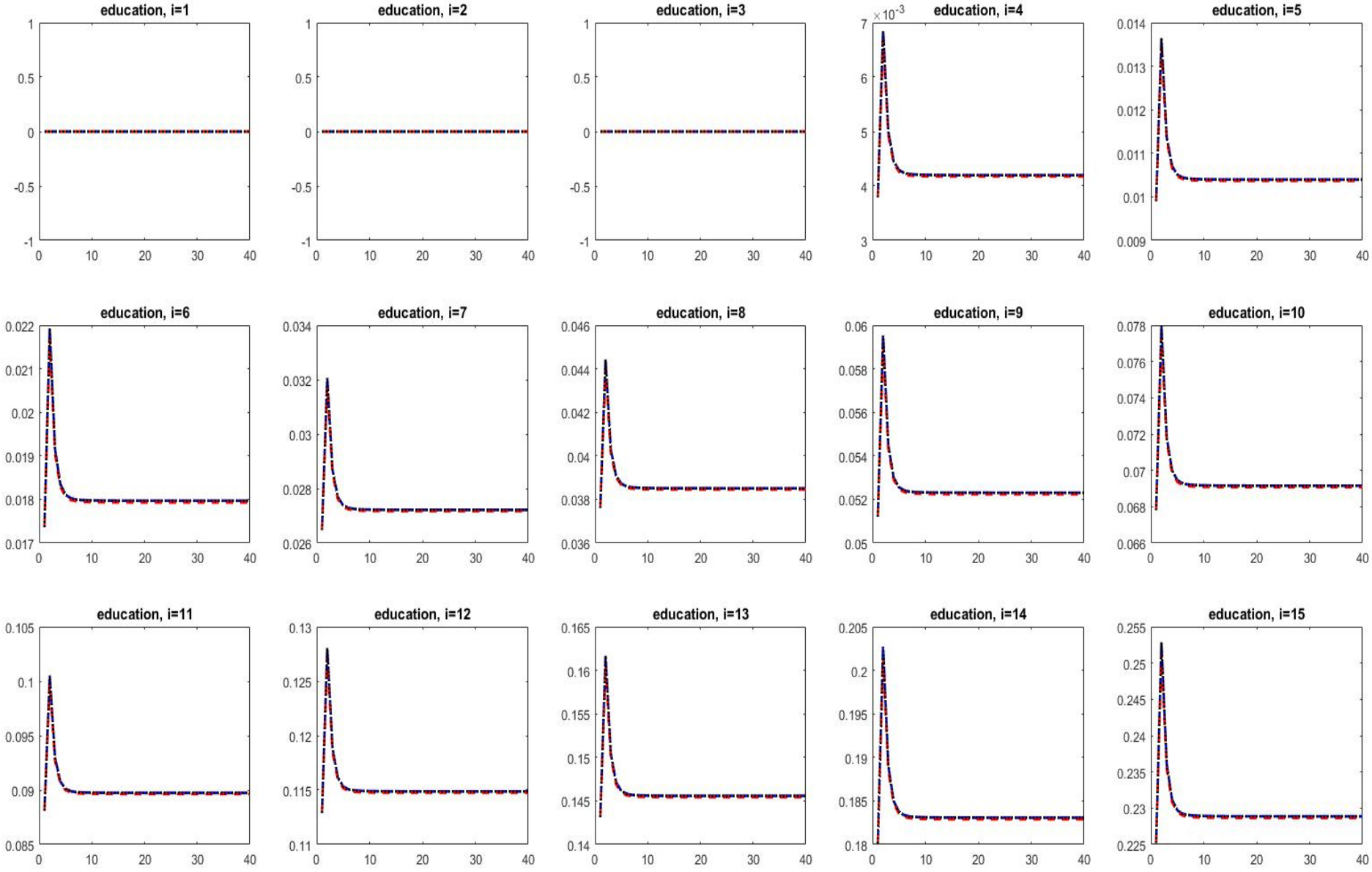
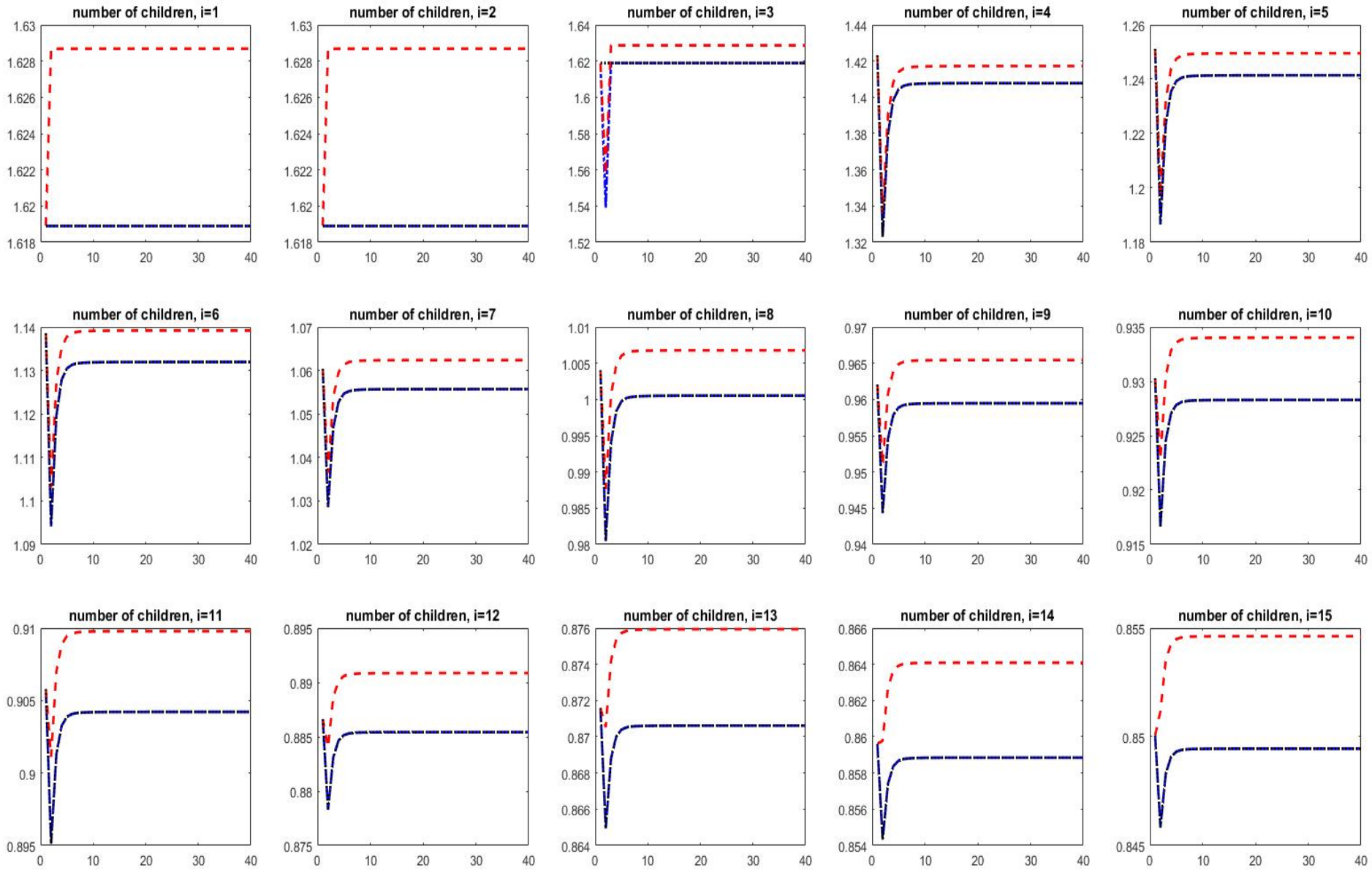
Subsidy for education – Transition to the final steady state

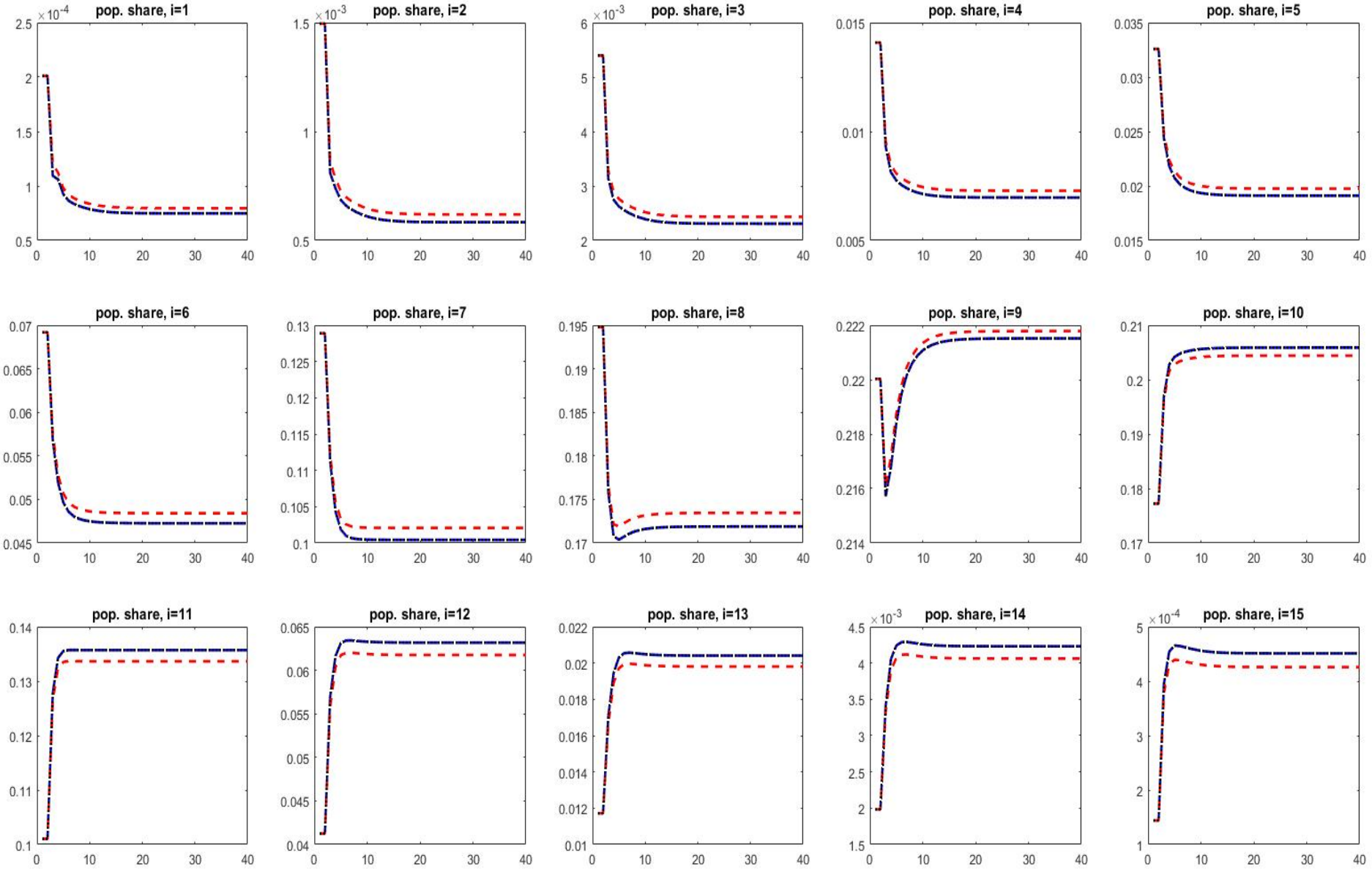
Subsidy for education – education



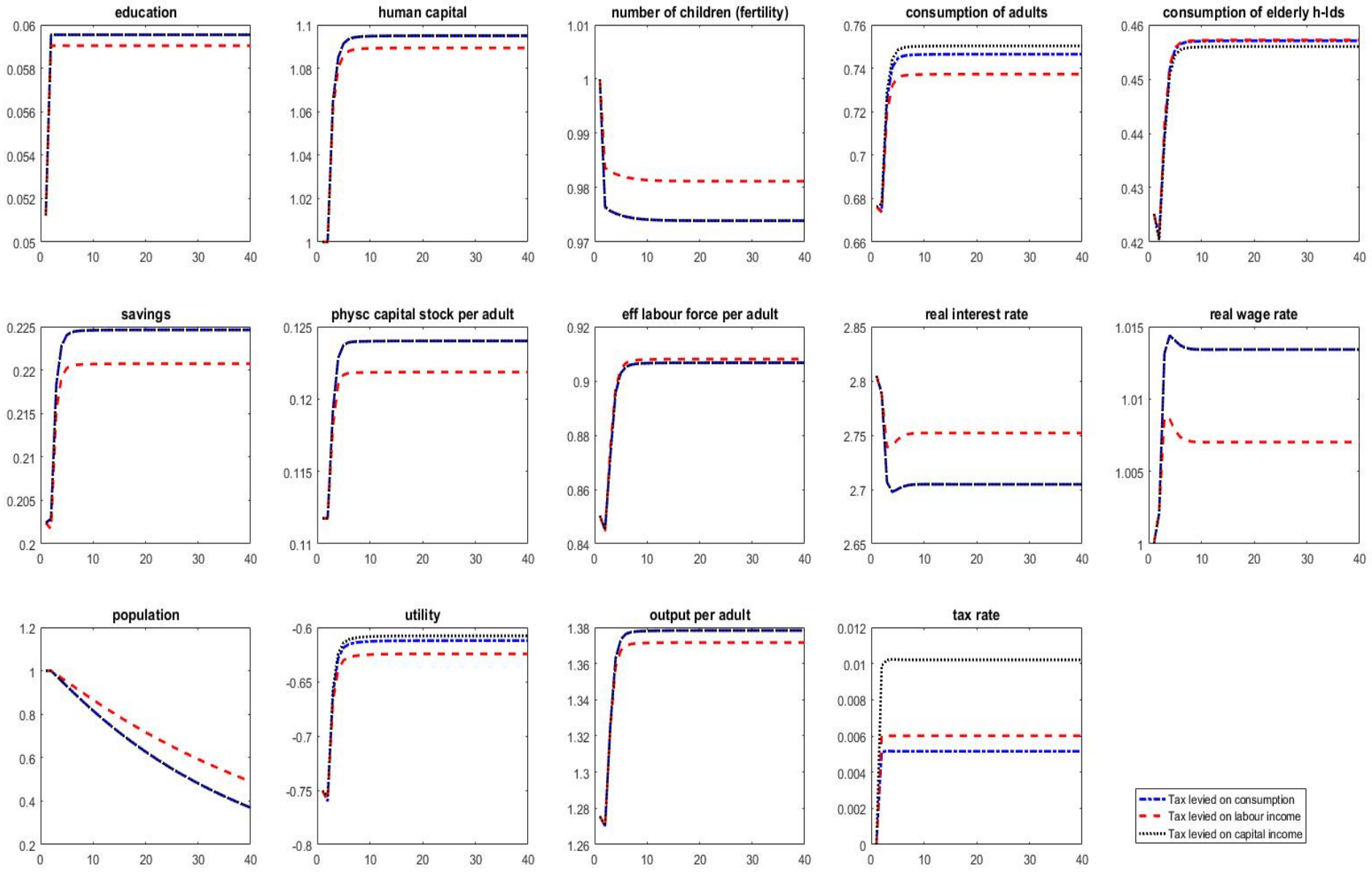
Subsidy for education – fertility



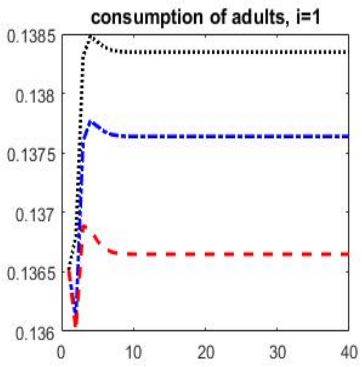
Subsidy for education – population share

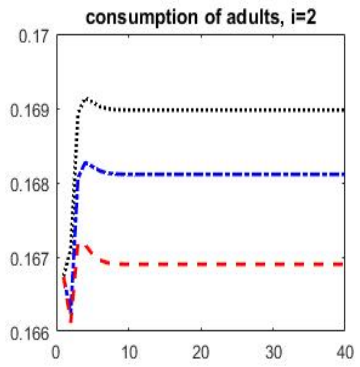


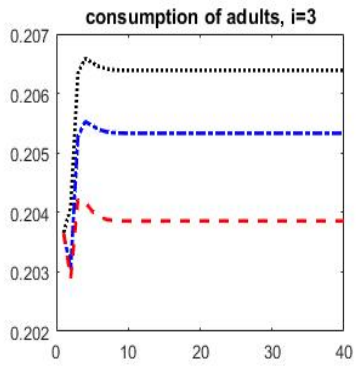
Subsidy for education

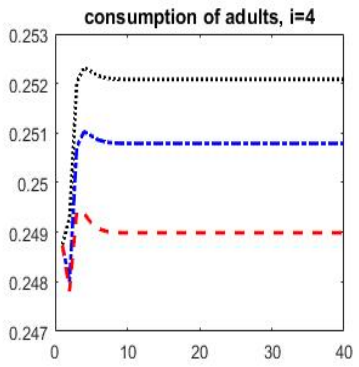


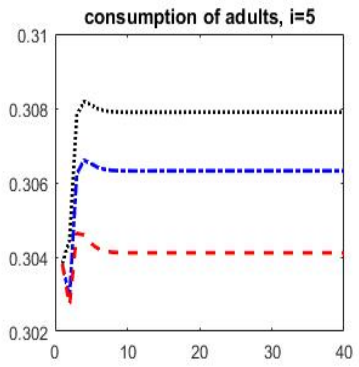
Subsidy for education – consumption of adult households

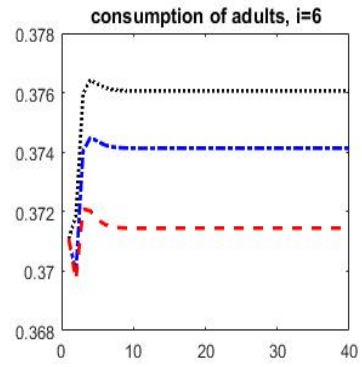


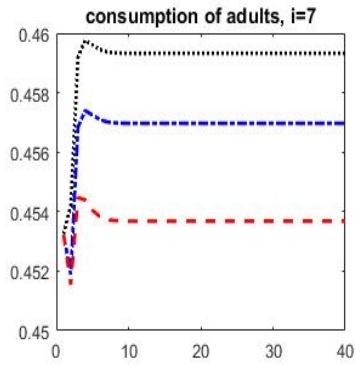


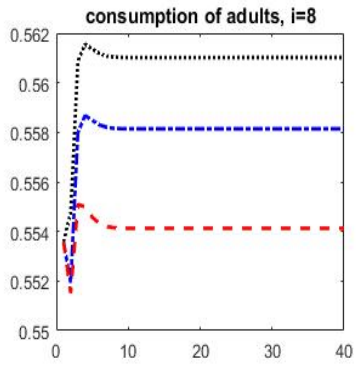


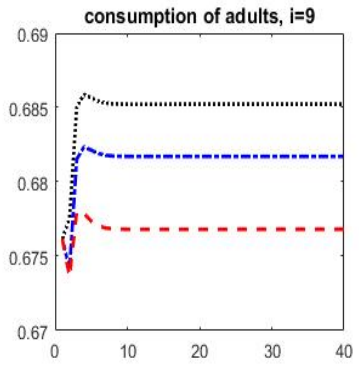


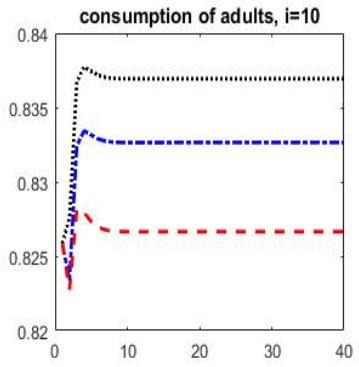


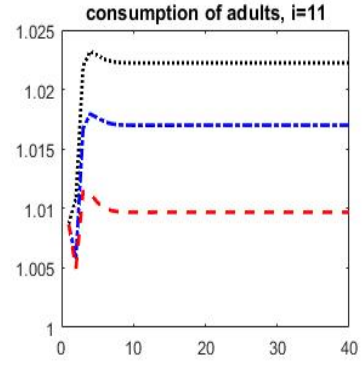


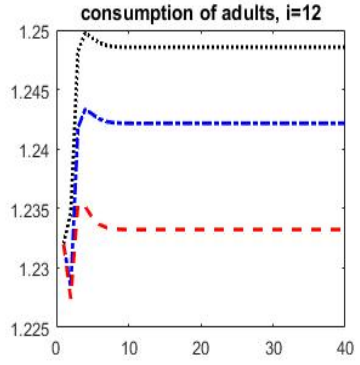


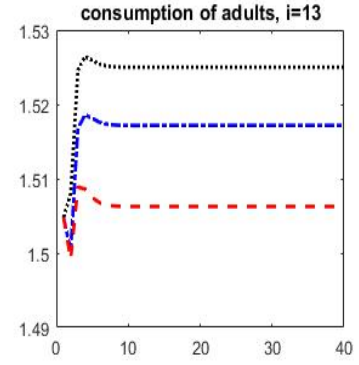


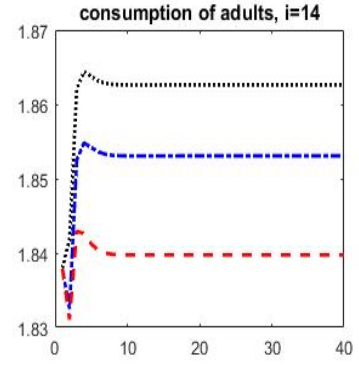


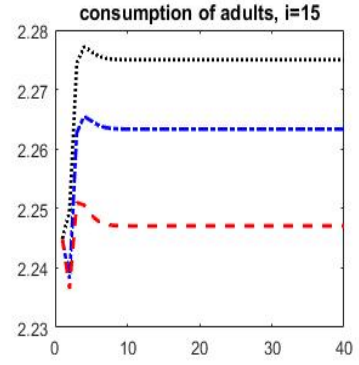




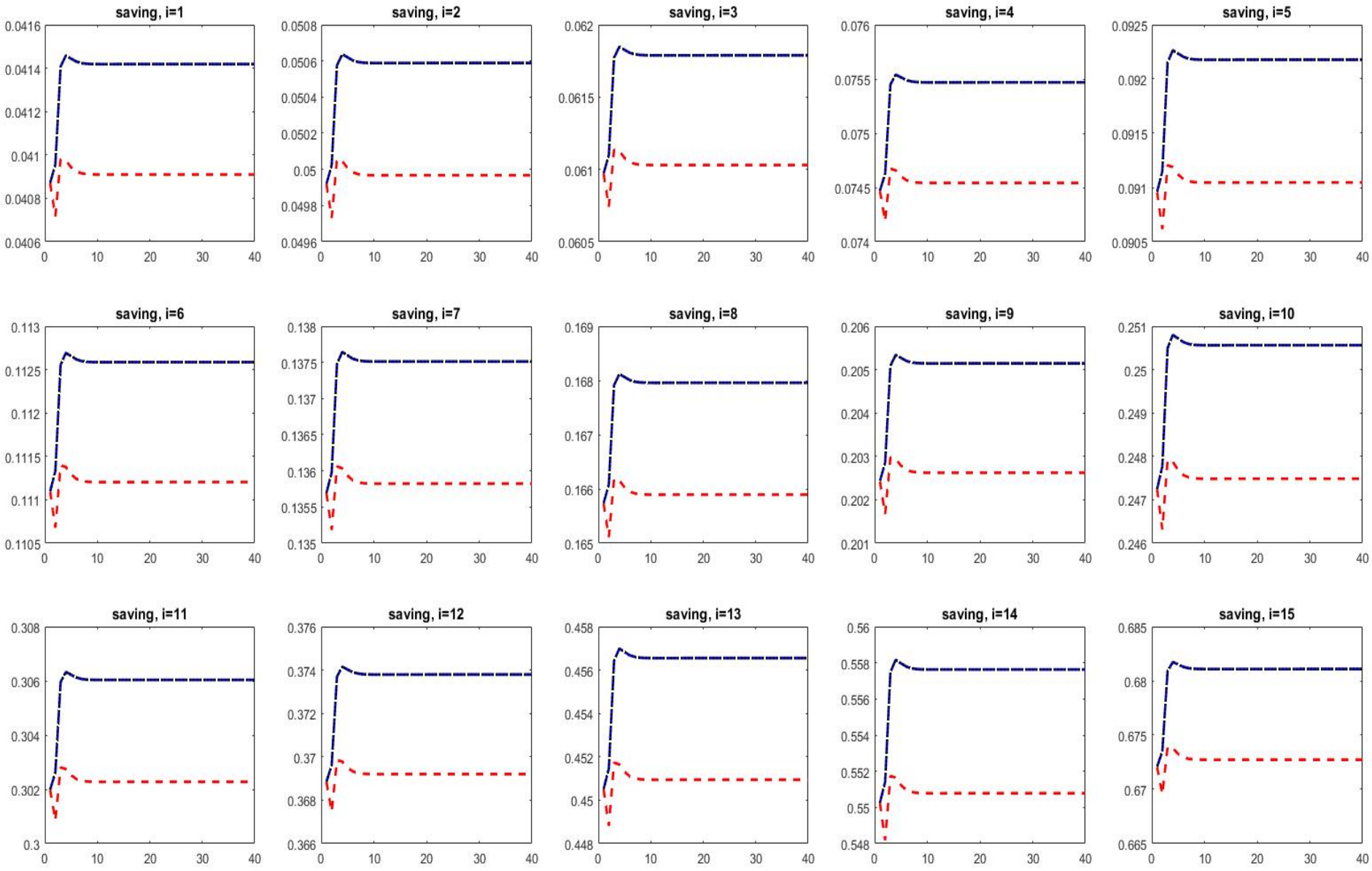




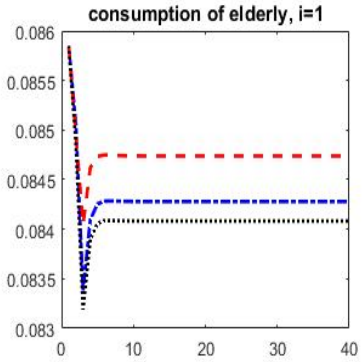


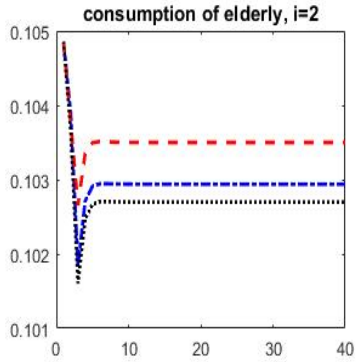


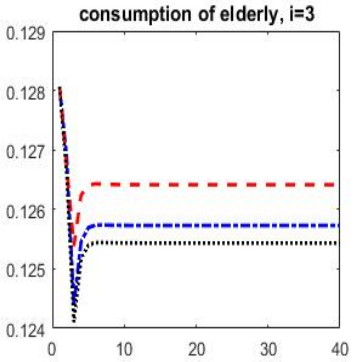
Subsidy for education – saving of adult households

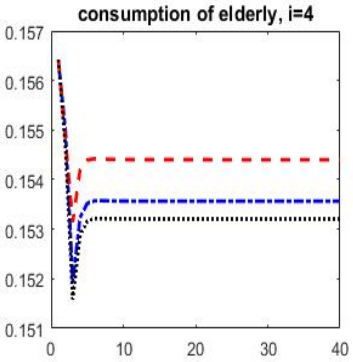


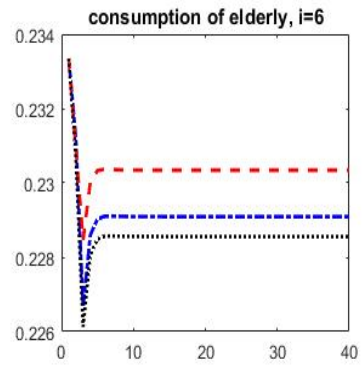
Subsidy for education – consumption of elderly households

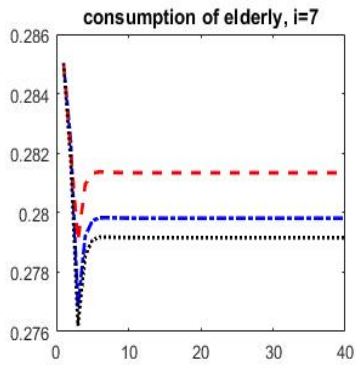


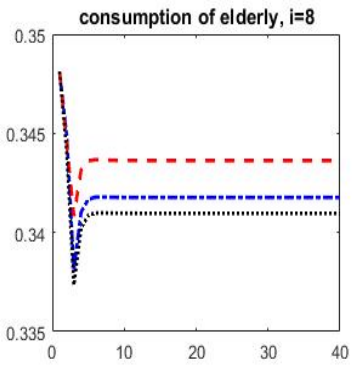


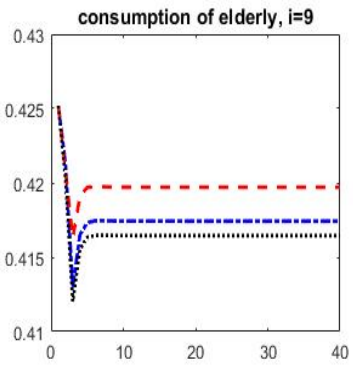


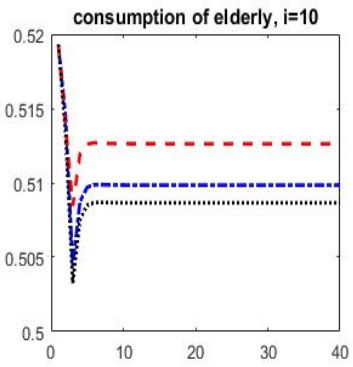


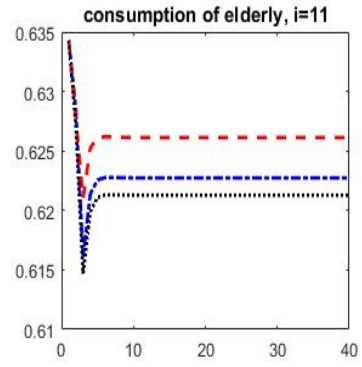


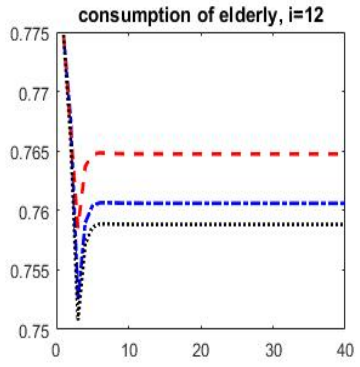


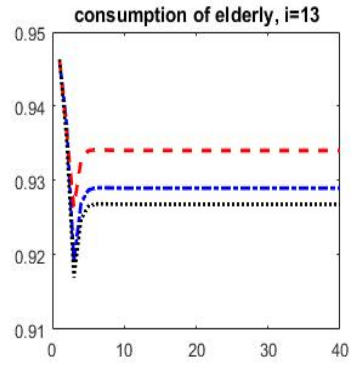


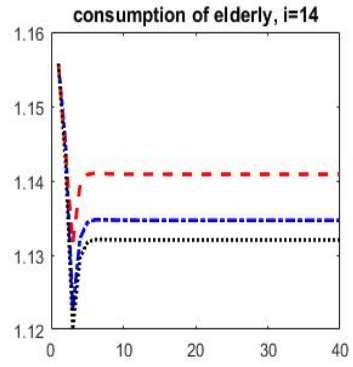


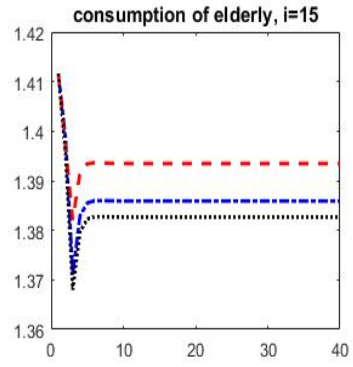




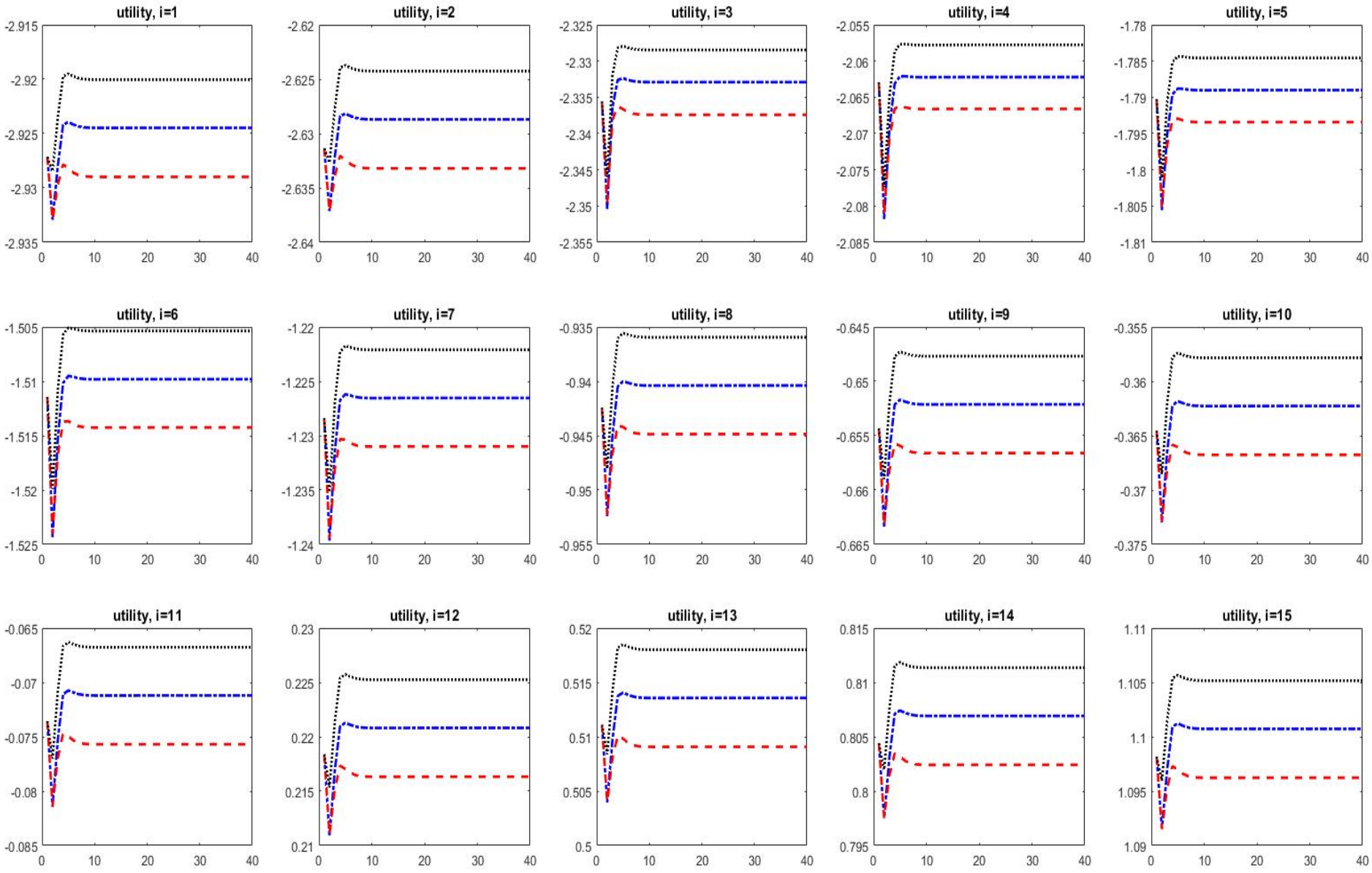






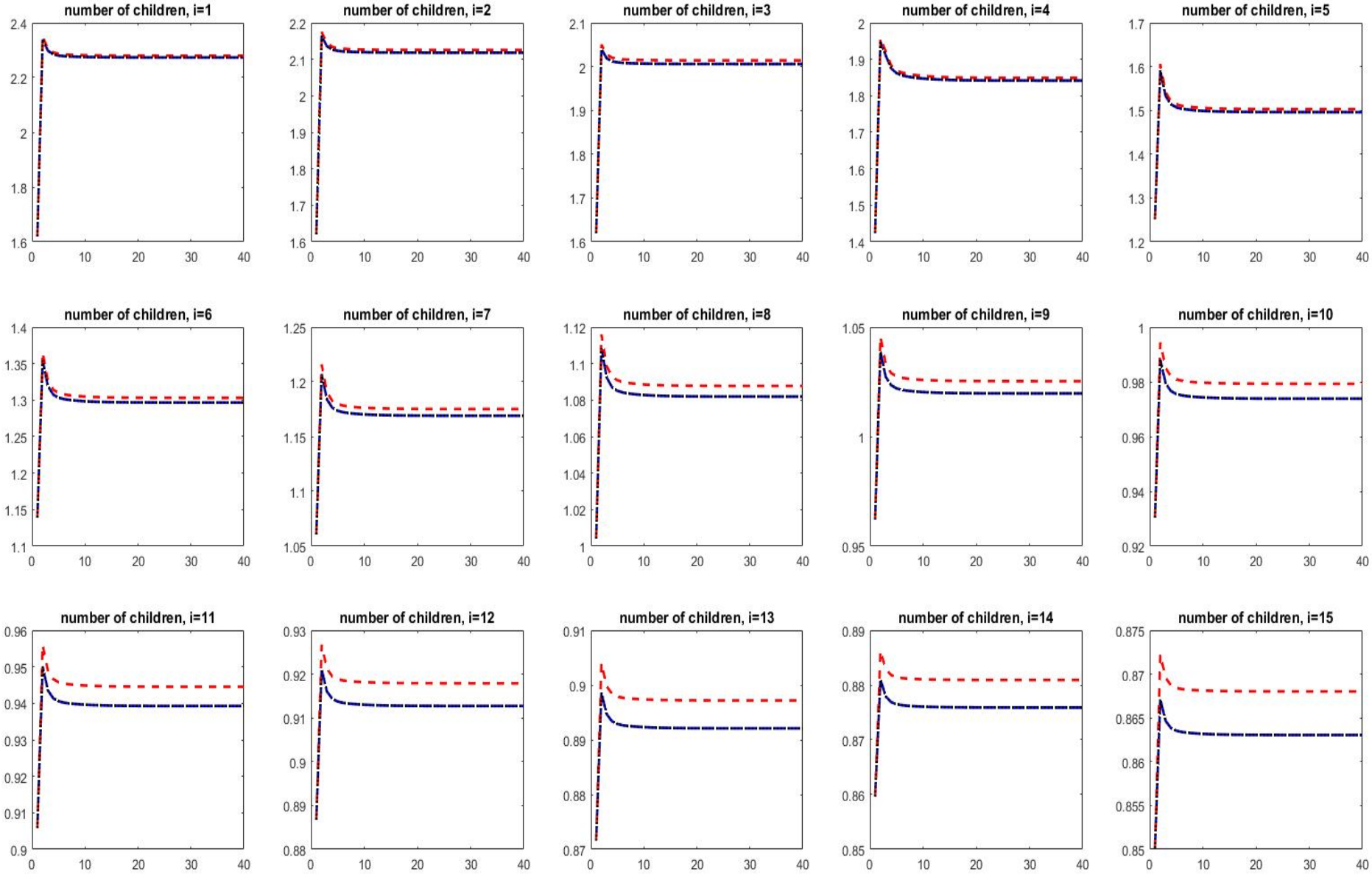


Subsidy for education – utility

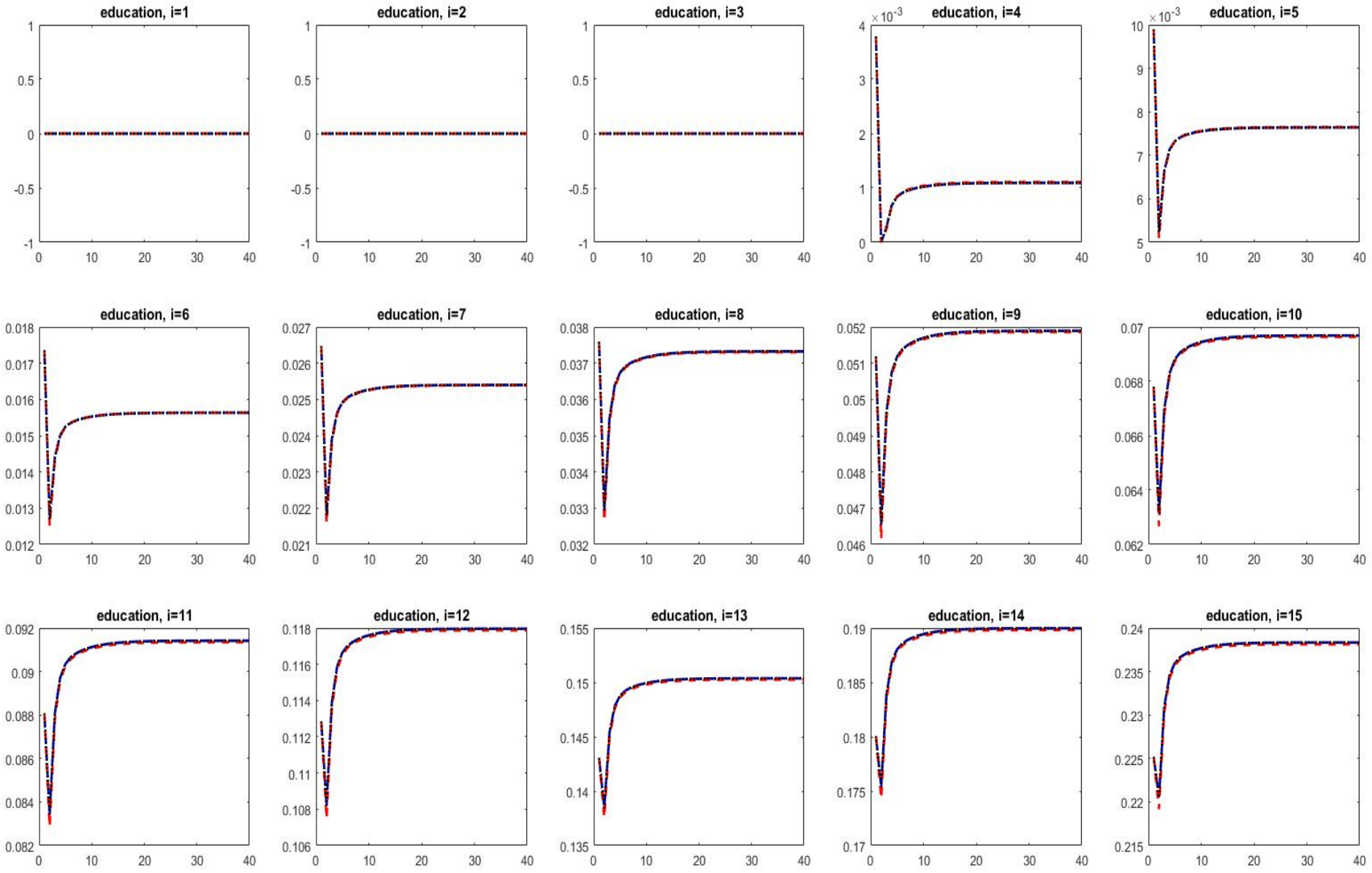


Subsidy for education – Transition to the final steady state

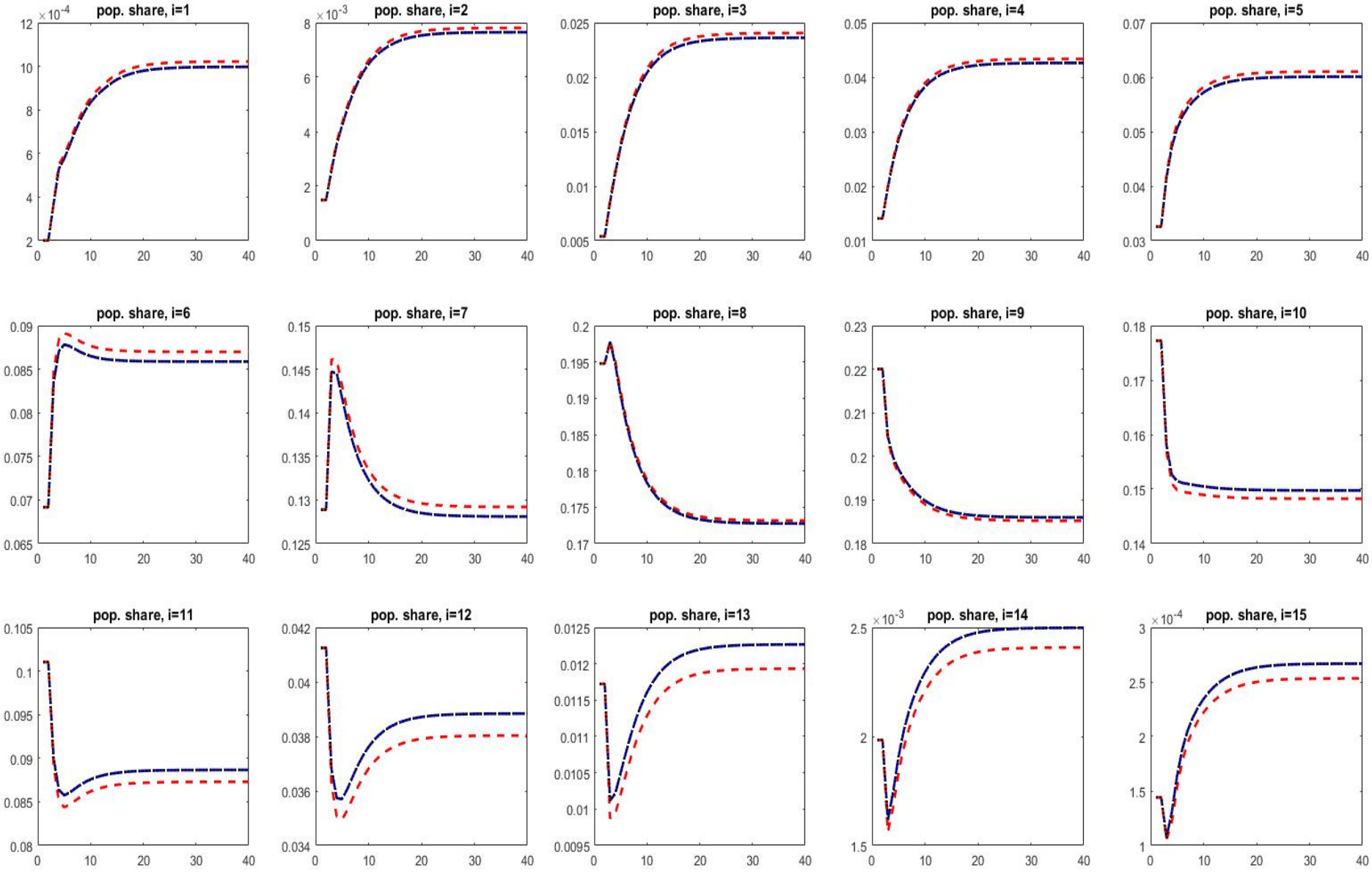
Subsidy for fertility – fertility



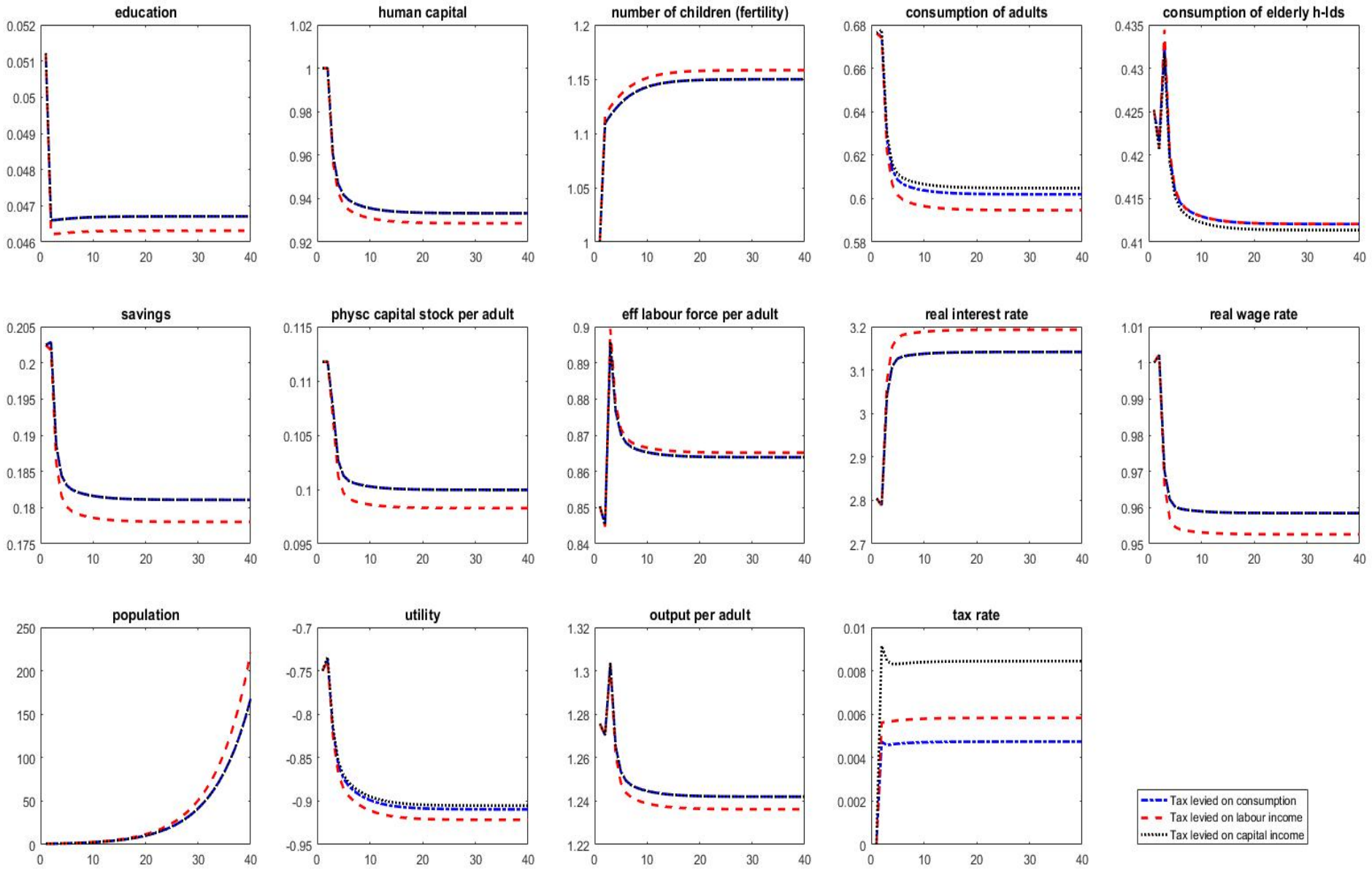
Subsidy for fertility – education



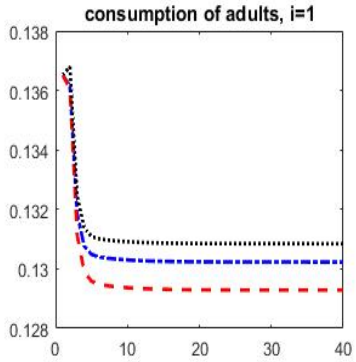
Subsidy for fertility – population share

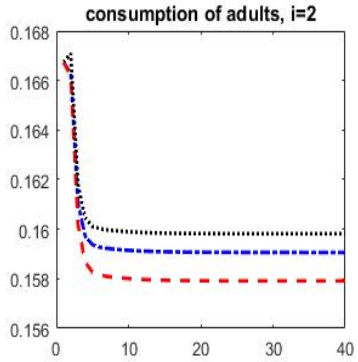


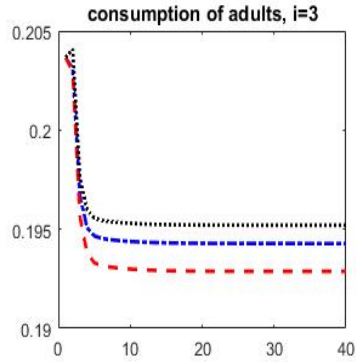
Subsidy for children

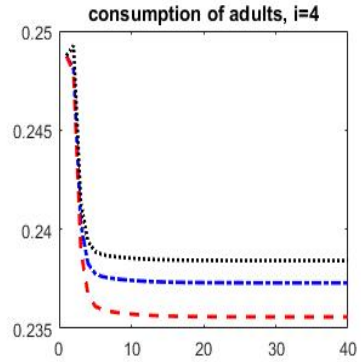


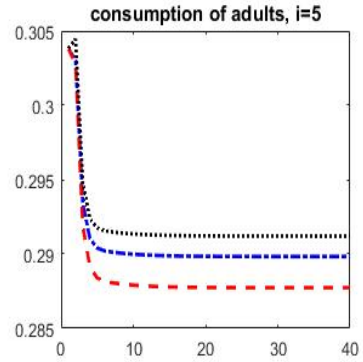
Subsidy for fertility – consumption of adult households

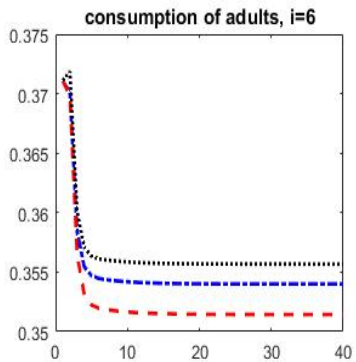


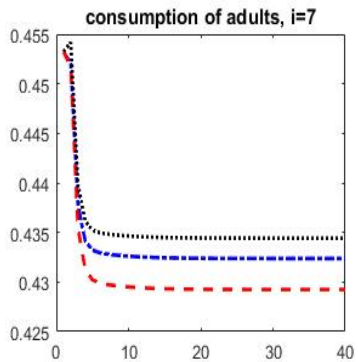


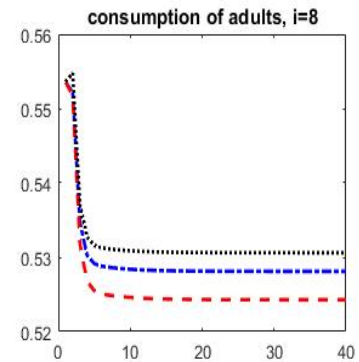


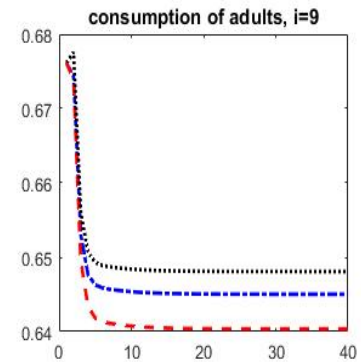


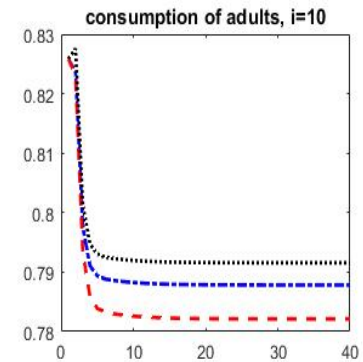


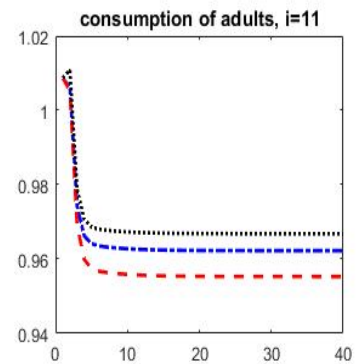


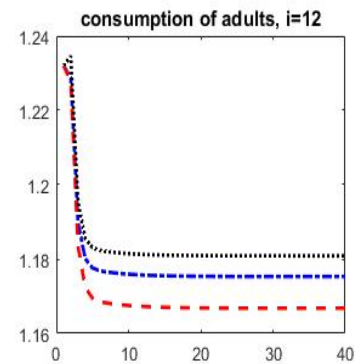


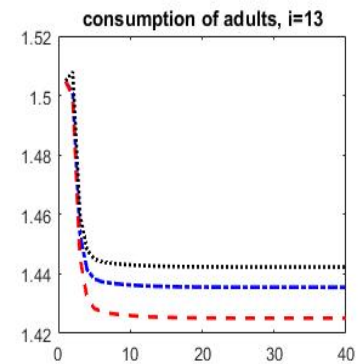


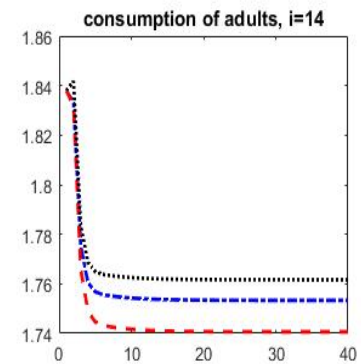


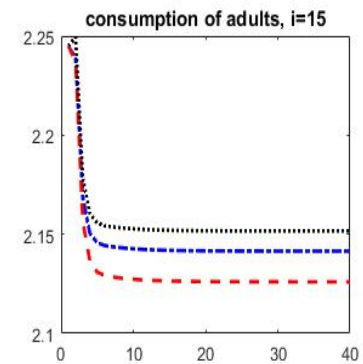




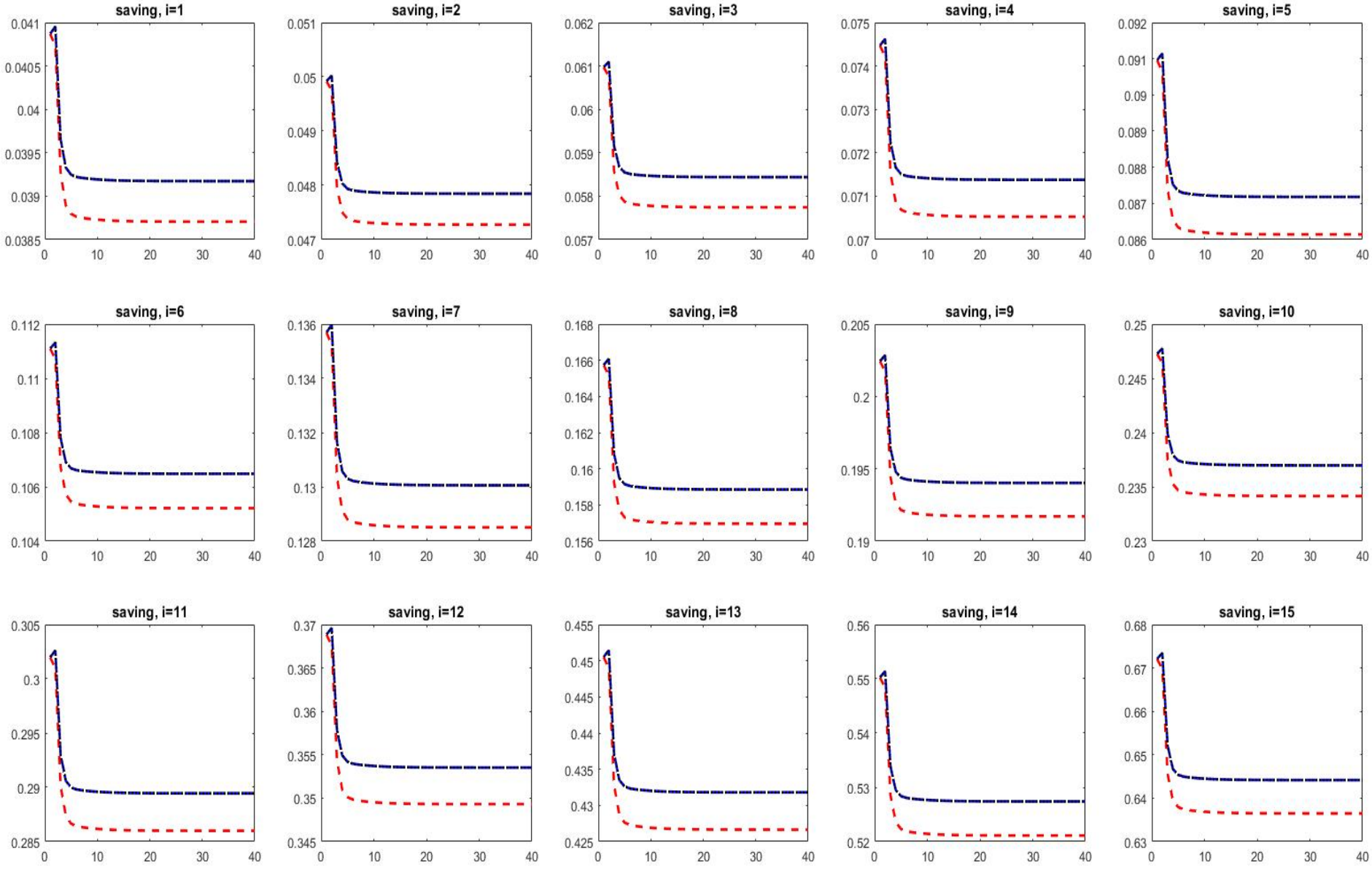




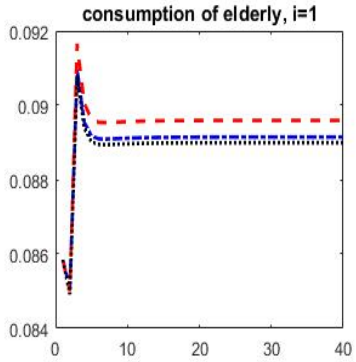


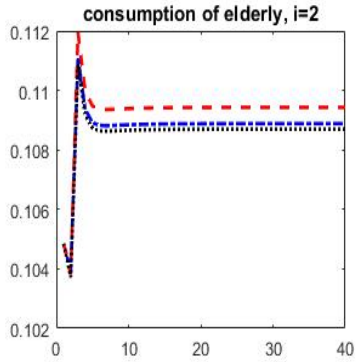


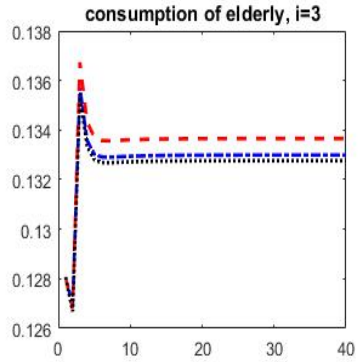
Subsidy for fertility – saving of adult households

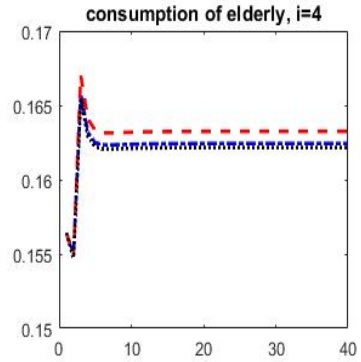


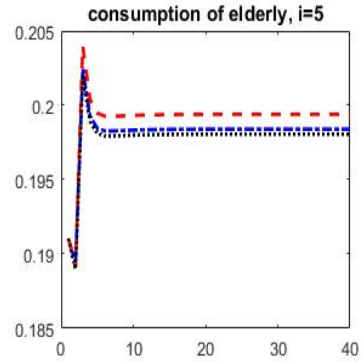
Subsidy for fertility – consumption of elderly households

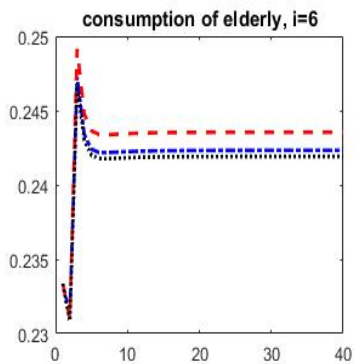


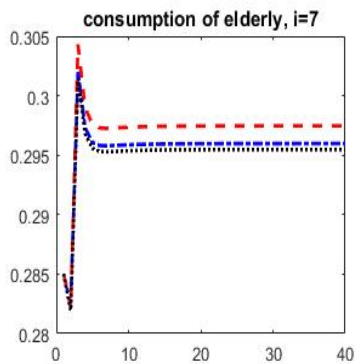


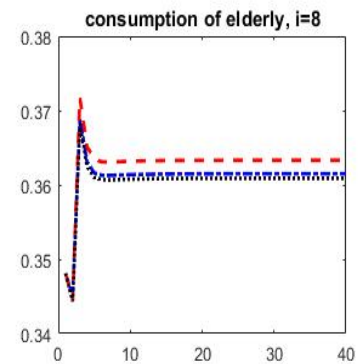


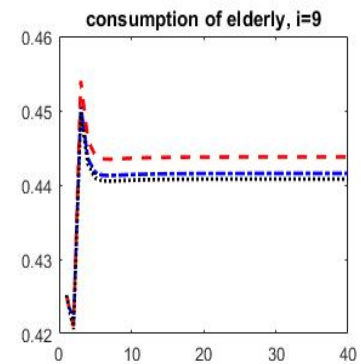


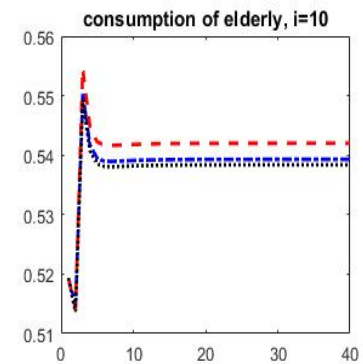


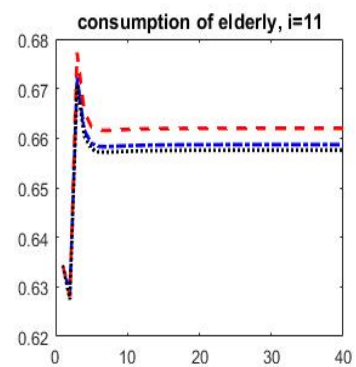


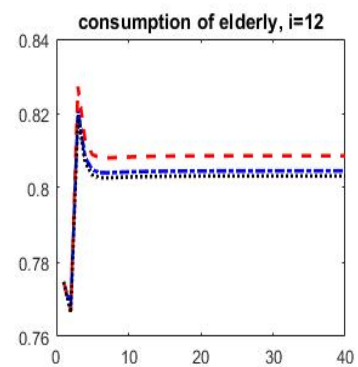


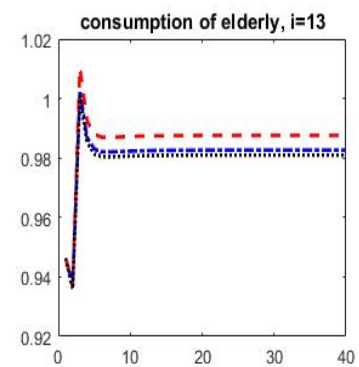


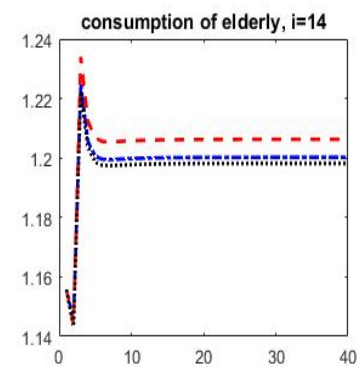


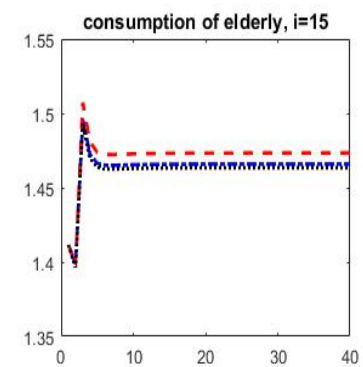












Subsidy for fertility – utility