Thermal Inflation with a Waterfall Field Mass Coupled to a Light Auxiliary Scalar Field

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Outline

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1. Thermal Inflation
2. Our New Model
3. "End of Inflation" Mechanism
   - Generating ζ
   - Constraining the Free Parameters
   - Results

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- Solves the moduli problem

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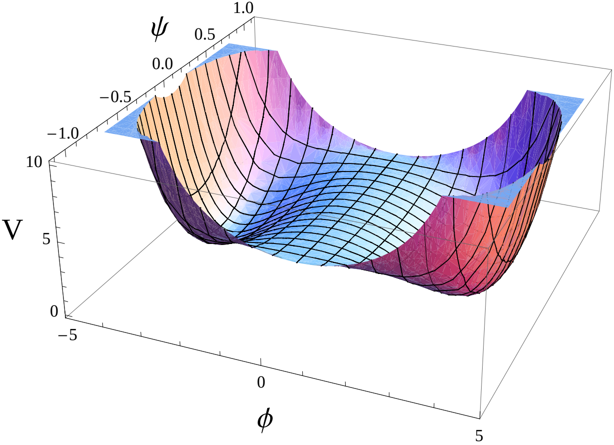
$$V(\phi, T, \psi) = V_0 + (g^2 T^2 - \frac{1}{2} m^2)\phi^2 + \lambda \frac{\phi^{2n+4}}{M_P^{2n}} + \frac{1}{2} m_\psi^2 \psi^2$$

where

$$m^2 \equiv m_0^2 - 2h^2 \frac{\psi^{2\alpha}}{M_P^{2\alpha-2}}$$

$$\alpha, n \geq 1$$



Arbitrary Units

We don't get domain walls, as we can interpret ϕ as being the real part of a complex field, whose potential contains only 1 continuous VEV.

T_1 and T_2

From

$$\rho_r = \frac{\pi^2}{30} g_* T^4$$

we obtain

$$T_1 \sim V_0^{\frac{1}{4}}$$

$$T_2 = \frac{m}{\sqrt{2}g}$$

Other Quantities

$$\langle\phi\rangle = \left(\frac{mM_P^n}{\sqrt{(2n+4)\lambda}}\right)^{\frac{1}{n+1}}$$

$$V_0 \sim \left(\frac{m_0^{2n+4}M_P^{2n}}{\lambda}\right)^{\frac{1}{n+1}} \qquad (V = 0 \text{ at the VEV})$$

$$H_{TI} \sim \left(\frac{m_0^{n+2}}{\sqrt{\lambda}M_P}\right)^{\frac{1}{n+1}}$$

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δN Formalism

$$\zeta = \delta N_{TI} = \frac{dN_{TI}}{dm} \delta m + \frac{1}{2!} \frac{d^2 N_{TI}}{dm^2} \delta m^2 + \frac{1}{3!} \frac{d^3 N_{TI}}{dm^3} \delta m^3 + \ldots$$

Therefore we obtain, to 3rd order

$$\zeta = \delta N_{TI} = -\frac{\delta m}{m} + \frac{1}{2}\frac{\delta m^2}{m^2} - \frac{1}{3}\frac{\delta m^3}{m^3}$$

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Substituting m and δm into this gives us, to 1st order,

$$\mathcal{P}_\zeta^{\frac{1}{2}} = \frac{\alpha h^2 H_* \psi^{2\alpha-1}}{\pi m^2 M_P^{2\alpha-2}}$$

where we have used

$$\delta\psi = \frac{H_*}{2\pi}$$

as the most probable value of $\delta\psi$.

Constraining the Free Parameters

Primordial Inflation Scale

Constraining the Free Parameters

Primordial Inflation Scale

We want the scale of primordial inflation to be

$$V^{\frac{1}{4}} \lesssim 10^{14}\ \text{GeV}$$

Therefore, from

$$3M_P^2 H_*^2 = V$$

we require

$$H_* \lesssim 10^{10}\ \text{GeV}$$

Constraining the Free Parameters

Inflationary Dynamics

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Inflationary Dynamics

In order that the ψ field does not affect the inflationary dynamics during thermal inflation, we require, from our definition of m,

$$m_0^2 \gg h^2 \frac{\psi_*^{2\alpha}}{M_P^{2\alpha-2}}$$

Therefore we have

$$m \approx m_0$$

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  Make the field equation for ψ look like that of a free field, i.e. make the coupling term to ϕ in the field equation sub-dominant.

  From the field equation

$$\ddot{\psi} + 3H\dot{\psi} + \frac{\partial V}{\partial \psi} = 0$$

  we obtain

$$\ddot{\psi} + 3H\dot{\psi} + m_\psi^2 \psi + 2\alpha h^2 \frac{\psi^{2\alpha-1}}{M_P^{2\alpha-2}} \phi^2 = 0$$

Therefore, in order that the field equation looks like that of a free field, we require

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- Substituting the observed spectrum value

$$\mathcal{P}_\zeta^{\frac{1}{2}} = 4.9\times 10^{-5}$$

into our equation for $\mathcal{P}_\zeta^{\frac{1}{2}}$ gives us the constraint

$$\psi_* \sim \left(\frac{10^{-4} m_0^2 M_P^{2\alpha-2}}{h^2 H_*}\right)^{\frac{1}{2\alpha-1}}$$

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We have

$$m^2_{\psi,eff} = m^2_\psi + (4\alpha^2 - 2\alpha)h^2\phi^2 \left(\frac{\psi}{M_P}\right)^{2\alpha-2}$$

Therefore, we require

$$m_\psi \ll H_*$$

and

$$h\phi_* \left(\frac{\psi_*}{M_P}\right)^{\alpha-1} \ll H_*$$

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This gives us the constraint

$$m_\psi \ll H_{TI}$$

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In order to keep $m_{\psi,eff}$ light, we therefore require

$$hT_1 \left(\frac{\psi_*}{M_P} \right)^{\alpha - 1} \ll H_{TI}$$

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$$\Gamma_{Therm} > H$$

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Thermalization rate between ϕ and thermal bath is

$$\Gamma_{Therm} \sim g^4 T$$

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$$\phi_{BD} \sim \left(\frac{M_P^n H_*^2}{\sqrt{\lambda}}\right)^{\frac{1}{n+2}}$$

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$$\phi_{BD} \sim \left(\frac{M_P^n H_*^2}{\sqrt{\lambda}} \right)^{\frac{1}{n+2}}$$

- ϕ light, in which a SUGRA correction to the potential is appreciable, in which ϕ rolls down to $\phi = 0$.

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Time for Transition from Thermal Inflation to Field Oscillation

[1] **D.H. Lyth**, *JCAP* **1205** (2012) 022

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Time for Transition from Thermal Inflation to Field Oscillation

In order for "end of inflation" mechanism to produce observed ζ, the transition from thermal inflation to field oscillation has to be sufficiently fast[1].

We require

$$H_{TI} \ll 10^{-5} m_0$$

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As ψ has acquired perturbations, we require it to not dominant energy density of universe after end of thermal inflation, at which time $m_{\psi,eff}$ is increased significantly.

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Therefore we require

$$m_{\psi,osc}\psi_* \ll M_P \Gamma$$

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However, we require $H_* \lesssim 10^{10}$ GeV!

Modulated Decay Rate

[2] **G. Dvali, A. Gruzinov and M. Zaldarriaga**, *Phys.Rev.* **D69** (2004) 023505

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The decay rate of ϕ is

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The primordial curvature perturbation that is produced by a varying decay rate[2] going from field oscillation after thermal inflation to waterfall field decay is given to 1st order by

$$\zeta_{osc\to dec} = \delta N_{osc\to dec} = -\frac{1}{6}\frac{\delta\Gamma}{\Gamma}$$

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- *Paper to appear soon*

Thank you

Any questions?

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