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Quantum Turbulence in ^{4}He, Oscillating Grids, and where do we go next?

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Experimental approaches to the study of quantum turbulence (QT) in superfluid ^{4}He in the low temperature limit, where the normal fluid density is effectively zero, are considered. A succinct general introduction covers liquid ^{4}He, superfluidity, critical velocities for the onset of dissipation, quantized vortex lines, and QT. The QT can be created mechanically by the oscillation of wires or grids above characteristic critical velocities. The interesting dynamics of the oscillating grid are discussed. It exhibits an enhanced effective mass due to backflow, as expected from classical hydrodynamics. It is found that the critical velocity attributable to the onset of QT production rises with increasing temperature. Oscillating objects like grids or wires create QT that is not well-characterised in terms of length scale, and the QT is not spatially homogeneous. The QT can be detected by the trapping of negative ions on vortex cores. Although the corresponding capture cross-section has not yet been measured, it is evidently very small, so that the technique cannot be expected to be a very sensitive one. In the future it is hoped to create well-characterised, homogeneous QT by means of a drawn grid. Improved sensitivity in the detection of QT is being sought through calorimetric techniques that monitor the temperature rise of the liquid caused by the decay of the vortex lines.

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1. INTRODUCTION

The liquid phases of helium have a quantum character that endows them with many remarkable properties[1]. In particular, the combination of weak Van der Waals interatomic forces and large zero-point energy results in both liquid ^{4}He and liquid ^{3}He having extremely low densities (respectively about 3× and 4× less than for hard spheres in contact), such that the atoms are well separated. Correspondingly, neither isotope solidifies under its saturated vapour pressure but apparently remains liquid right down to absolute zero. On account of their low densities, the liquids are in many ways gas-like so that their quantum statistics play an important role in determining their physical properties. It is a role that becomes crucially important at low temperatures where liquid ^{4}He and liquid ^{3}He behave respectively as degenerate, imperfect, Bose-Einstein and Fermi-Dirac gases. One of the most remarkable of their properties is superfluidity.

For both isotopes, the inviscid superfluid component (see Sec. 1.1. below) is of particular interest for studies of turbulence, and experiments[2–5] are now starting to probe the low temperature regime of pure *quantum turbulence* (QT) where the dissipative normal fluid component is essentially absent. Fundamental questions can be asked about QT under these conditions. One can enquire, for example, how classical turbulence is modified where flow is severely restricted by the quantization of angular velocity that governs the motion of the superfluid. One can also ask whether and, if so, how the turbulence decays in the absence of viscosity.

In this paper we review recent experiments on pure QT in liquid ^{4}He and consider some of those that are now being planned. In particular, we consider the experimental problems that must be faced in the creation and detection of QT at very low temperatures. For those who are unfamiliar with quantum fluids, we start by reviewing in Sec. 1.1. the salient properties of superfluid ^{4}He. In Sec. 1.2. we introduce the concept of QT, and we mention some early experiments relevant to QT in Sec. 1.3. In Sec. 2. we report results of on-going experiments on the production of QT with an oscillating grid, discussing the resonance curves (Sec. 2.1.), the hydrodynamic effective mass (Sec. 2.2.) and the temperature dependence of the critical velocity for QT creation. We draw conclusions and consider future prospects in Sec. 3., both in relation to the grid resonances (Sec. 3.1.) and to possible new techniques for creation and detection of QT (Sec. 3.2.).

As we will see, QT is not only of great intrinsic interest in its own right, but its study could lead to a better understanding of classical turbulence.

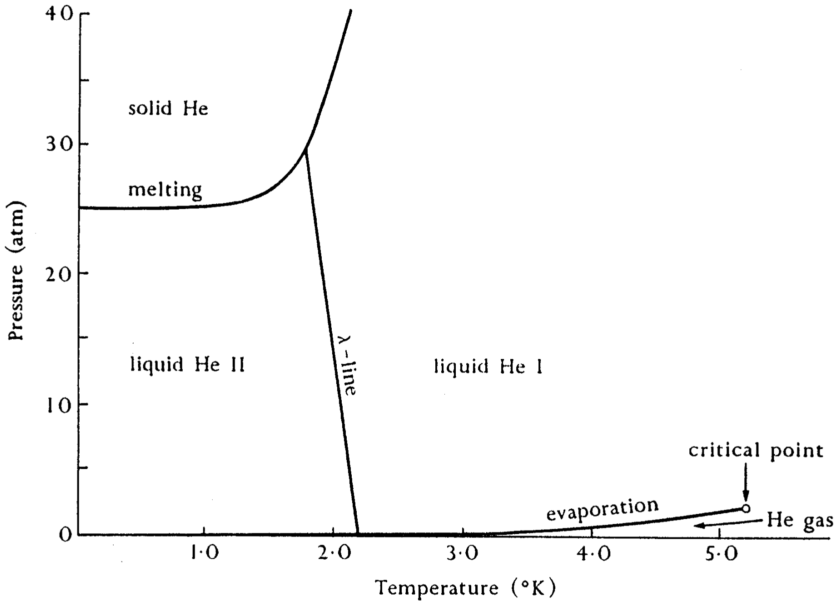

Fig. 1. The phase diagram of ^{4}He at low temperatures, after London[6].

1.1. Superfluidity and dissipation

At its so-called *Lambda transition* at T_λ (= 2.17 K under the saturated vapour pressure), liquid ^{4}He undergoes a phase transformation corresponding[7] to Bose-Einstein condensation. The P–T phase diagram, shown schematically in Fig. 1, illustrates the facts that the liquid persists to the lowest temperatures and that its properties above and below the Lambda transition are utterly different. The He I phase above T_λ is a classical liquid, albeit of unusually small density and viscosity; He II, on the other hand, possesses superfluid properties such as zero viscosity allowing frictionless flow, quantization of circulation (i.e. of rotational motion, and the ability to climb out of open-topped containers by syphonic action via a creeping superfluid film.

In fact, He II behaves as though it were composed of two distinct interpenetrating but non-interacting components: a normal fluid component with classical properties, which carries all the entropy (thermal energy); and a superfluid component with zero entropy and zero viscosity that remains effectively at absolute zero. The densities of these two components, ρ_n and ρ_s respectively, vary with T as shown in Fig. 2. The normal fluid component can be regarded as a gas of excitations[8,9] (phonons and rotons) moving in the "vacuum" provided by the superfluid component. Near T_λ (say $T > 1$ K),

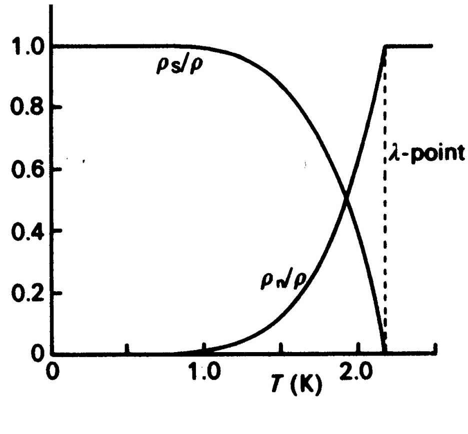

Fig. 2. Temperature dependences of the normal and superfluid densities, ρ_n and ρ_s, divided by the total density ρ of the liquid, as deduced from Andronikashvili's experiment[14].

the mean free path of the excitations is very short, so that they behave like a classical liquid. For the temperatures far below T_λ that will be the regime of our primary interest, the density of the excitations is so low that their mean free path exceeds the dimensions of the container, and they then behave much like a classical Knudsen gas.

In addition to the phonons and rotons, quantized vortex lines[10,11] can exist in the superfluid component. They have narrow cores around which the superfluid component flows at a velocity v_s that decreases inversely with the radial distance from the centre, such that the circulation

$$\oint \boldsymbol{v_s} \cdot \boldsymbol{d\ell} = n\frac{h}{m_4}$$

where m_4 is the ^{4}He atomic mass. In practice, the radius of the core is of vanishingly small (atomic) dimensions and $n = 1$. Vortex lines provide a means by which the superfluid can effectively rotate[12], simulating solid body rotation for large enough angular velocities where a dense array of parallel lines is aligned with the axis of rotation. Vortex lines represent metastable states of the stationary superfluid, but are known to have very long lifetimes when pinned between protuberances: seemingly, any sample of He II however prepared, always contains a low density of such *remanent vortices*[13].

It is now understood that the inviscid properties of the superfluid component persist only while the liquid is being treated fairly gently. If characteristic critical velocities are exceeded, the superfluidity breaks down and there is an onset of dissipation through one of three main mechanisms –

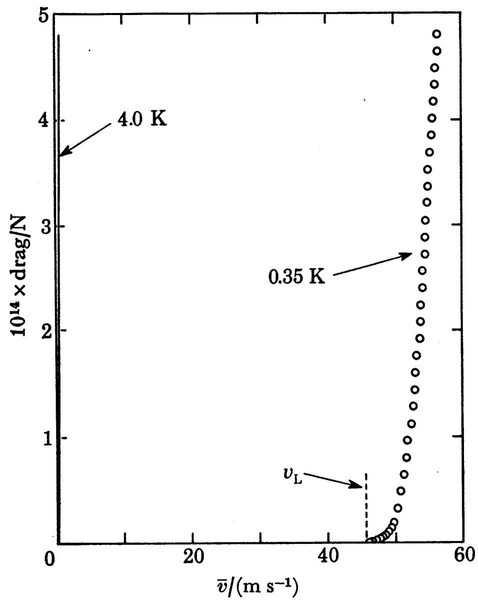

Fig. 3. The measured drag force on a negative ion[15] moving through He II at 0.35 K, as a function of its speed $\bar{v}$. It is found that drag occurs only when $\bar{v}$ is equal to, or exceeds, the Landau critical velocity for roton creation, v_L. The behaviour of a negative ion in He I at 4.0 K is also plotted, in order to emphasize the profound qualitative difference between the two cases.

1. If a moving object (or the flowing liquid) exceeds the Landau critical velocity $v_L \simeq \Delta/p_0$, where Δ and p_0 are the energy and momentum of excitations at the roton minimum in the dispersion curve, then it dissipates kinetic energy through the emission of rotons[15,16] via the mechanism predicted by Landau[8,9]. In experiments, it is found that there is no measurable dissipation for $v < v_L$, and that a sharp increase occurs above v_L as shown in Fig. 3.

2. It is also possible for a moving object to create quantized vortices in the superfluid, a process that requires an energy barrier[17] to be surmounted. Experiments on negative ions[18] have revealed the existence of a vortex nucleation mechanism whose rate increases exponentially with T, attributable to thermal activation over the barrier, as well as a T-independent mechanism representing a form of macroscopic quantum tunnelling through the barrier. Measurements of the thermally activated rate allow the height of the energy barrier to be determined and, as shown in Fig. 4, it is found to be in remarkably good agreement

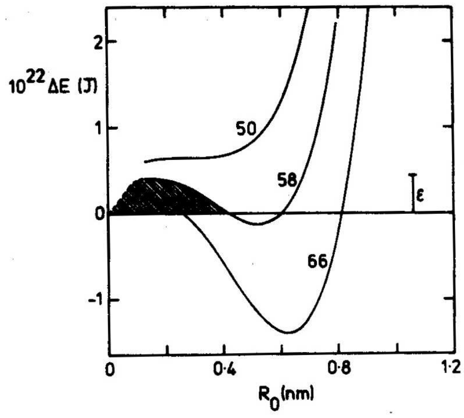

Fig. 4. Comparison of the energy barrier ϵ opposing vortex nucleation by a negative ion in superfluid ^{4}He as calculated[19] by Muirhead and Vinen (shaded) and measured experimentally[18] (the bar on the right). ΔE is the change in energy that is calculated to occur when a vortex loop of radius R_0 is created at constant impulse, and the numbers adjacent to the curves indicate the initial velocity of the ion in m s^{-1}.

with the value estimated[19] by Muirhead, Vinen and Donnelly by calculation of the energy change at constant impulse when a small vortex loop is created *ab initio*.

3. Finally, is possible for remanent vortices[13] (see above) to expand in the flowing liquid, eventually tangling, reconnecting[20,21], and producing a body of unpinned QT moving relative to the flow tube at a velocity determined by a balance between the superfluid and normal fluid velocities v_s and v_n (the vortices lead to a mutual coupling of the fluids because their cores can scatter excitations and thus are part of the normal fluid, whereas their flow fields and kinetic energy exist in the superfluid).

Of these three mechanisms, the third is by far the commonest in practice, at least for flowing superfluid. One reason is that the corresponding critical velocities ($\sim$ cm or mm s^{-1}) are very much smaller than those ($\sim$ 50–60 m s^{-1}) for the first two mechanisms.

1.2. Quantum turbulence

We now introduce the idea of QT and summarise its salient properties; a more detailed introduction to QT will be found in the companion paper[22] by Vinen. Despite the huge differences between a superfluid and a classical fluid (the two-fluid behaviour, inviscid flow, and quantisation of circulation), it has become clear during the past four or five years that QT can often exhibit behaviour that is similar in significant respects to that observed in the appropriate classical analogue[23,24]. Perhaps the simplest case is provided by steady flow through a uniform grid: in the classical case such flow generates a form of fully-developed turbulence that is, at least approximately, homogeneous and isotropic, the detailed study of which has been important in the development of our understanding of classical turbulence. The question arises, therefore, as to how fully-developed grid turbulence is modified if the classical fluid is replaced by superfluid ^{4}He.

Almost all existing experiments relating to this question have been carried out at temperatures above 1K, where there is a significant fraction of normal fluid. Experiments have shown that, on length scales larger than the spacing between the quantized vortex lines, the behaviour of the helium is quasi-classical, in the sense that the two fluids have a common turbulent velocity field similar to the single velocity field observed in the inertial range of length scales in classical turbulence (i.e. the range in which there is negligible viscous damping); the evolution of this velocity field is characterized by a Richardson cascade and a classical Kolmogorov spectrum. It can be argued, however, that this behaviour may be dependent on the presence of the normal fluid. Thus it becomes important to establish whether it persists at the very low temperatures where the fraction of normal fluid has become negligible.

Classical turbulent flow, including that produced by flow through a grid, is ultimately dissipated on a small length scale by viscosity. Such a process cannot operate in the superfluid component. Yet without such dissipation there cannot be a Richardson cascade and its associated Kolmogorov spectrum. There is evidence[25] that at temperatures above 1K, turbulence in the superfluid component is dissipated on small length scales (less than or of order the vortex-line spacing) by mutual friction – the force that is exerted on the core of a quantized vortex line when there is relative motion between the line and the normal fluid. A dissipative process of this kind obviously cannot operate at the lowest temperatures, so another motivation for investigation of QT at these temperatures is to establish the nature of the dissipative processes that are able to operate.

These low-temperature dissipative processes have attracted much theoretical speculation[25], and they are of fundamental importance to our understanding of QT. From computer simulations, it seems likely that there are three significant mechanisms, all arising from the vortex reconnections[21] that can occur when two sections of vortex line come close enough to each other. First, the vortex reconnection process itself probably leads to the emission of phonons and/or rotons; it is therefore inherently dissipative, although the magnitude of the associated dissipation rate is probably relatively small in ^{4}He. Secondly, reconnections lead also to the generation of sharp kinks on the vortex lines, which can be regarded as superpositions of Kelvin waves. These waves can dissipate energy by phonon radiation, but only at a very small rate unless they have very high frequencies, higher than are generated directly by reconnections. However, repeated reconnections lead to the generation of Kelvin waves of high amplitude. Non-linear interactions can then transfer energy, through a "Kelvin-wave cascade", to waves with frequencies high enough for effective phonon radiation[26–28]. Finally, reconnections can lead to the generation of small vortex rings, which can escape from a turbulent region of finite extent. Experimental evidence should enable this picture to be tested because it leads to clear predictions about the rate of decay of the turbulence[25].

1.3. Early experiments pertaining to QT for $T \to 0$

In fact, interest in how QT decays preceded by several years the theory sketched in outline above. Experiments using negative ions to measure the Landau critical velocity [15,29,16] involved periodic injection of ions from field emission tips, and the ions were necessarily subjected to strong electric fields before they entered the drift space. Under these conditions, they are known to have a finite probability of creating charged vortex rings[30,31,18,32] even for pressures above 10 bar using ^{4}He that has been isotopically purified to eliminate complications[33] caused by tiny traces of ^{3}He present in natural helium. The reason is that the distribution function of ionic velocities possesses a high velocity tail[34] that crosses the threshold for vortex creation despite the latter being considerably above v_L. In extremely high electric fields, close to the field emmitter, production of vortex line is continuous[35] without trapping of the ion; in lower fields, metastable charged vortex rings[36] are created and these may be expected to grow, decelerate, and eventually to tangle with each other. Thus there must have been continuous creation of QT in such experiments. If there had *not* been simultaneous QT decay processes, then the density of QT would have increased until the trapped spacecharge prevented continuation of the experiments. That this did not occur implied

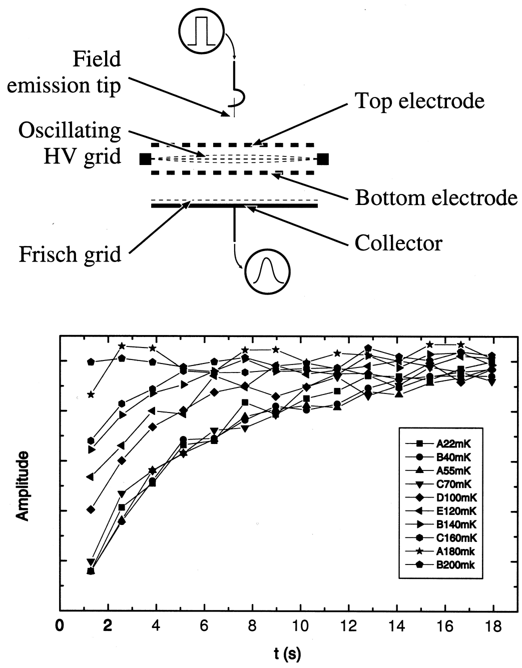

Fig. 5. Top: experimental arrangement (schematic) for measuring QT decay times. Bottom: evolution of the collector signal with time following a burst of grid oscillations.

the existence of QT decay mechanisms, effective even at mK temperatures. It was noted[29], however, that the decay times could apparently be very long.

The first attempt to measure the QT decay time directly[2] used the apparatus sketched in the top section of Fig. 5. The grid is held at a high constant potential. The perforated top and bottom electrodes complete the double capacitor, and oscillations of the grid are excited by application of an alternating potential to the lower electrode. In the absence of any dissipation from the superfluid, the oscillation amplitude increases until a critical velocity for vortex production is reached. A sequence of ion pulses is then generated by the field emission tip. Some ions from each pulse get trapped on vortices, thereby reducing the signal arriving at the collector. The Frisch grid screens the collector from the approaching charge. The bottom section of Fig. 5 plots the evolution of the collector signal with time following a burst of grid oscillations, for different values of T. It is evident that the recovery of the signal amplitude to its vortex-free level takes several seconds, attributable to the time taken for the vorticity to decay. This experiment was illuminating in that it clearly demonstrated the production and decay of QT in the $T \rightarrow 0$ limit, but it suffered from two disadvantages. First, the absolute value of the vortex density was unknown: the vortex/ion trapping cross-section has not yet been measured at these temperatures, although it is evidently small[37]. Secondly, the distribution of QT – whether it diffuses widely within the cell, or remains concentrated near the grid that produces it – is also unknown.

2. THE OSCILLATING GRID

The dynamics of the grid oscillations themselves also turns out to be of considerable interest, given that they take place in a pure superfluid. The experimental situation still includes features that require resolution, although the general picture is clear. We now describe the linear and nonlinear resonances that have been observed, measurements of the hydrodynamic effective mass, and our observation that the threshold velocity for vortex creation is temperature dependent.

2.1. Linear v. nonlinear resonances

For small amplitude oscillations in the $T \rightarrow 0$ limit where $\rho_n \simeq 0$, the liquid exerts essentially no damping. The liquid is expected to behave much like a vacuum, except that backflow will result in the classical hydrodynamic contribution to the effective mass (see Sec. 2.2.). Recent experiments[38] confirm this general picture. Typical resonance curves are plotted for a number of different driving amplitudes in Fig. 6, which shows the amplitude of the oscillating response voltage induced on the upper electrode: note the logarithmic ordinate scale and the highly expanded abscissa. It is found that, for driving voltages of 20 mV or less, the response varies linearly with the drive and the resonances are Lorentzian. For larger drives, the response amplitude on-resonance increases much more slowly, and the curves become flat-topped. These phenomena are attributable to the onset of dissipation via vortex production once the grid exceeds a characteristic critical velocity of ~4.5 cm/s.

The initial experimental investigations[39,40] yielded data that differ in certain respects from the results plotted in Fig. 6. In particular, although the resonance curves for large and small drives are very similar, there was a range of intermediate driving levels and responses where the resonant frequency (i.e. the frequency of maximum response) decreased with rising drive amplitude; and the resonance curves were of a re-entrant shape and displayed hysteresis. Nonetheless, the on-resonance response amplitude remained pro-

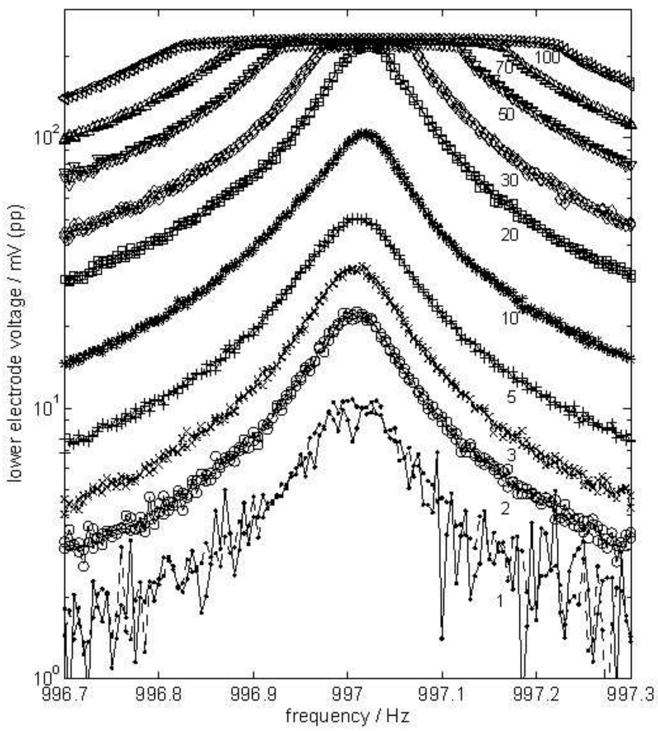

Fig. 6. Resonance curves for the vibrating grid in He II at 20 mK, for the driving amplitudes (mV) indicated by the number adjacent to each curve.

portional to the driving amplitude until the vortex production threshold was reached. It is not known why the more recent data (Fig. 6) do not reproduce these phenomena. The difference may possibly be associated with the surface preparation and quality of the gold plating of the grid which, in the earlier work, was darker and less golden in appearance. Further experimental work is needed to resolve these issues.

2.2. The effective mass

From classical hydrodynamics, the inertia associated with the backflow around a moving object implies an effective mass enhancement of $\Delta M = \beta V \varrho$. Here β is a geometrical constant, V is the volume of the displaced fluid, and ϱ is the fluid density. An important feature of ^{4}He is that adjustment of the pressure p causes significant changes in ϱ (on account of the high zero point energy and the resultant low density: see Sec. 1.). Thus it is possible to adjust the hydrodoynamic effective mass. If we consider the system as linear oscillator driven by a periodic force $F_0 \cos(\omega t)$, we can write

$$M\ddot{z} + D\dot{z} + Kz = F_0 \cos(\omega t) \tag{1}$$

where M is the mass, D specifies the (small) damping, and K is a spring constant. The response is maximum for $f = \omega/2\pi \cong \frac{1}{2\pi}\sqrt{K/M}$. Because $M = V\varrho_{Ni} + \beta V\varrho_{He}$, where ϱ_{Ni} and ϱ_{He} are respectively the densities of the nickel grid material and the liquid ^{4}He,

$$4\pi^2 f_1^2 = \frac{K}{V\varrho_{Ni} + \beta V\varrho_{He}(p)} \tag{2}$$

$$4\pi^2 f_0^2 = \frac{K}{V\varrho_{Ni}} \tag{3}$$

Here f_0 is the frequency of vacuum resonance and f_1 is that of the helium resonance. Dividing the last two equations,

$$\left(\frac{f_0}{f_1}\right)^2 = \frac{\varrho_{Ni} + \beta\varrho_{He}(p)}{\varrho_{Ni}} \tag{4}$$

whence

$$\frac{1}{f_1^2} = \left(\frac{\beta}{f_0^2\varrho_{Ni}}\right)\varrho_{He}(p) + \frac{1}{f_0^2} \tag{5}$$

Hence a plot of $1/f_1^2$ *versus* $\varrho_{He}(p)$ should yield a straight line from which β may be extracted. It was found[40] that a straight line was indeed obtained within experimental error, and that this passed through the vacuum resonance point (for which $\varrho_{He}(p) = 0$). Its gradient gave $\beta = 3.01 \pm 0.05$. This value is to be compared with $\beta = 0.5$ for a sphere, or $\beta = 1$ for an infinite cylinder, or $\beta = 3$ for an elliptical cylinder with major and minor axes equal to the width and thickness of a grid wire.

Subsequent more accurate measurements were found[41] to yield a straight line that did not pass precisely through the vacuum point. The reason is not yet known, but it could possibly be an indication of an additional contribution to the effective mass, a feature that has also been seen in experiments on vibrating wire loops[42].

2.3. Temperature dependences

Measurements at a range of temperatures[38] show a broadening of the resonances above ∼200 mK. The broadening increases up to ∼1 K, where it

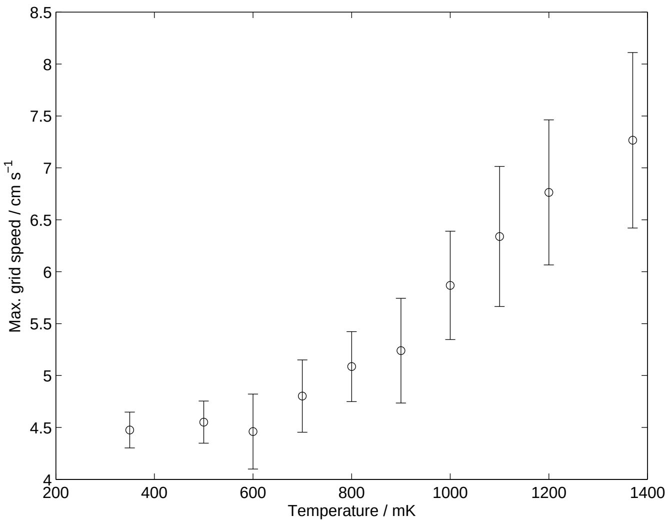

Fig. 7. The resonant response at the onset of dissipation as a function of temperature, at a pressure of 15 bar.

passes through a maximum. This added damping can be attributed to an increasing fraction of thermally excited quasiparticles, i.e. increasing ρ_n as T rises. At temperatures below 1 K the mean free path of the quasiparticles is much larger than the mesh of the grid, and the dissipative drag arises from the ballistic scattering of quasiparticles from the grid. This type of behavior was reported by Jäger *et al.*[43] for the case of an oscillating microsphere, and they showed that it leads to a drag coefficient that increases monotonically with temperature, in agreement with our own results. At temperatures above 1 K the mean free path of the quasiparticles falls below the diameter of the wires in the grid, so that the damping can then be described in terms of a viscous drag. As in the measurements of Jäger *et al.*, this change leads to the maximum in the damping *versus* temperature that we have observed.

The temperature dependence of the critical velocity associated with the onset of vortex production is plotted in Fig. 7. It is interesting to note that the critical velocities associated with the vibration of a single wire[42,44], or with the oscillation of a microsphere[43], are very similar in magnitude and that, at least qualitatively, there are similar temperature dependences. These similarities exist notwithstanding the very different geometries. In the case of wires it seems not to depend much on the surface roughness of the wire, in spite of the fact that this roughness might be expected to have a strong influence on any remanent vortices.

3. DISCUSSION: WHERE NEXT?

The experiments discussed above lead naturally to two lines of future development. First, studies of the grid resonances have revealed many unexpected and poorly understood features that need to be investigated in more detail. Secondly, and as discussed below probably of greater long-term importance, we have helped to open up the possibility of studying QT in pure superfluids in the $T \to 0$ limit. Before discussing possible future experiments, we should take note of the increasingly important role being played by numerical simulation[45,25,46,47], which has evolved into a highly sophisticated technique that is able to provide answers to questions that experiments are not yet able to address.

3.1. Grid resonances

As noted above, the onset of dissipation due to vortex production by the grid seems to be very similar to that in the case of oscillating wires and spheres. Many features of the results at lower grid velocity remain mysterious, however. These include the presence[39,40] or absence[41,38] of re-entrant resonance curves, the observation of bursts of modulation in the response amplitude[39,40], and the occurrence of small resonant shifts in very low amplitude oscillations[38] that occur discontinuously without measurable changes in damping. The possible effect of surface preparation of the grid needs to be investigated. The seemingly anomalous contribution to the effective mass needs to be checked and understood. There is also a parasitic resonance[38] (not mentioned above) whose origin needs to be identified. Thus studies of an oscillating grid in superfluid ^{4}He in the $T \to 0$ limit should still be considered as work in progress with the possibility of some additional surprises in store.

3.2. Quantum turbulence experiments in ^{4}He

The preliminary QT experiments discussed above in Sec. 1.3. need to be developed and improved, to create well-characterised spatially homogeneous QT. Improved QT detection techniques are also needed because ion-trapping is inherently insensitive on account of the small cross-section[37] of the process.

A drawn-grid technique for the production of homogeneous QT at mK temperatures is currently under development in Gainesville[48] and Lancaster[41]. Although conceptually similar to the earlier $T > 1$K QT drawn-grid apparatus in Oregon[24], there are daunting technical problems to be overcome, given the need to avoid extraneous heating of the sample. A superconducting linear motor is being used to move the grid which, together with levitated superconducting magnetic quadrupole bearings will eliminate frictional effects. At the time of writing, the machine has been tested at $T > 1$K, where it performs according to expectation.

Two alternative methods for detecting QT are under active investigation. The first is the possible use[49,50] of small charged vortex rings[36] as probes of QT. They have a much larger interaction cross-section than bare ions, but an obvious concern is that it may prove difficult to create monoenergetic rings of known radius (or energy) in a controlled way. The second QT detection method depends on measurement of the rise in temperature of the helium as the turbulence decays: the idea that turbulent energies can be high enough to produce measurable heating at low temperatures was suggested by Samuels and Barenghi[51] and developed by Vinen[52]. The thermal capacity of the thermometers required must of course be small compared to that of the surrounding helium; their response time must be relatively fast (1 ms); and they must have adequate sensitivity (1 μK at an ambient temperature of 10 mK). Suitable thermometers meeting these stringent criteria are now available[53,54]. The theory of the decay of quantum turbulence at low temperatures, outlined in Sec. 1.2., leads to predicted time-dependent temperature rises, which can in principle be compared with the results of the experiments. Relevant factors include the rate of transfer of energy down the Richardson cascade (known from the theory of classical turbulence), the rate of build-up of the Kelvin-wave cascade and of energy flow within it (known from simulations[25,28], the rate of phonon radiation from the high-frequency Kelvin waves (known from theory[55–57], and the rate of thermal equilibration within the radiated phonons (which may also involve the vessel containing the experimental helium, about which less is known).

Unfortunately, neither ion-trapping nor calorimetry will provide the kind of detailed information about turbulent flow patterns that is available in the case of classical turbulence. Such patterns can be obtained in classical fluids by observing the motion of seed particles in the flow, and a sophisticated development of this type of observation is provided by Particle Image Velocimetry (PIV). Experiments on PIV in thermal counterflow in superfluid ^{4}He above 1K have recently been reported[58], as have preliminary observations of the trapping of micron-sized particles of solid hydrogen by vortex lines[59]; the first steps towards a theory of particle motion in a superfluid have been made[60]. It is becoming clear that a simple interpretation of this observed particle motion is dependent on the right choice of particle size[61]. It has recently been suggested by McKinsey[62] that the neutral triplet-state He_2 excimer molecule, produced along with other excitations by charged particle bombardment of helium, might be useful as a tracer, especially as laser-induced fluorescence might be used for extremely sensitive detection, possibly even at the level of individual molecules. It turns out that these molecules form bubble states that are of the appropriate size for trapping on vortices. Development of the use of these tracer particles might in the long term revolutionize the study of quantum turbulence.

3.3. Quantum turbulence experiments in ^{3}He

Our discussion above has referred to ^{4}He, but it is becoming increasingly apparent[22,63] that QT in ^{3}He-B has much in common with QT in ^{4}He, although there are also some differences. The ^{3}He-B experiments of Bradley et al[3–5] have investigated QT production by looking at the dynamics of vibrating loops and grids, and have detected the QT by means of Andreev reflection. It has been possible to detect the decrease in QT density with distance from the creation region, and to show that a characteristic generation rate must be exceeded for QT to be created rather than individual vortex rings which escape rapidly. These results are highly illuminating, but – as with any oscillating object – they are difficult to compare in detail with theory because the QT is spatially inhomogeneous and not well-characterised in length scale. It is possible in principle that a drawn-grid experiment, similar to that being planned for ^{4}He (see above) might be attempted.

4. CONCLUSION

We conclude that the experimental study of QT in pure superfluids is important, and has made significant progress recently, but that the subject is still in its infancy. Nonetheless, a coherent picture is already starting to emerge. Considerable development may be anticipated over the next few years, both experimentally using both isotopes, numerically, and theoretically.

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