STUDENT PRIMARY TEACHERS' PERCEPTIONS OF MATHEMATICS – A PHENOMENOGRAPHIC STUDY

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Declaration

This thesis results entirely from my own work and has not been offered previously for any other degree or diploma

Signature

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Abstract

This study is situated at a time of political and educational change, whereby a need for improvement in the provision of mathematics education in British primary schools is identified. Undertaken from a phenomenographic perspective, it focuses on mathematical perceptions of student primary teachers (SPTs) as they embark upon Initial Teacher Training (ITT), and considers the potential influence of mathematical perceptions upon their ITT learning and future teaching. Research suggests negative perceptions of mathematics amongst adults, Higher Education students, teachers and student teachers, but the range of variation of mathematical perceptions of SPTs at the outset of ITT has not been previously examined. A phenomenographic study, conducted with thirty-seven SPTs due to begin ITT, led to the development of four qualitatively different ways in which SPTs perceive mathematics. The hierarchical variation is examined in relation to pedagogical associations via a conceptual framework based on a non-dualist perspective of mathematics being constituted of a learner's relational understanding through experience. Potential implications for SPTs' development within ITT are explored and recommendations made regarding how these might be addressed. Whilst ITT provision is an obvious factor in students' development, this research is based on a premise of learners taking responsibility for their own development, especially with regard to intangible and often unconsciously held perceptions. The study offers insight into the range of perceptions SPTs may hold and its association with pedagogy, in order to both raise awareness and to provide a framework for reflection in SPTs' formation of personal philosophy of mathematics upon which to plan learning goals for ITT and associated aspirations for their practice as primary mathematics teachers.

Chapter 1 Introduction And Background

1.1 Context

This research is situated in Initial Teacher Training (ITT) within British Higher Education (HE) and constitutes an exploration of perceptions of mathematics amongst student primary teachers (SPTs). It posits that perceptions are a result of experience, and that they influence the way students learn and teach. It argues the necessity for determination of the range of variation in SPTs' mathematical perceptions at the outset of ITT, to facilitate an exploration of pedagogical associations and the identification of potential implications for their learning in ITT and future practice as teachers of primary mathematics, in order to enable informed reflection upon their preparation for ITT.

1.2 Rationale

As a primary mathematics HE lecturer I am presented with constant concerns from SPTs regarding their understanding, anxiety of and ability to teach mathematics, which ally with theoretical reports of the unsatisfactory nature of provision for mathematics education in primary schools. Small scale doctoral studies (Jackson, 2008; Jackson, 2007) preceding this thesis confirmed the existence of negative perceptions towards mathematics amongst SPTs that could potentially affect their ITT learning and future practice, and the need for more in-depth study, especially since research in this area is sparse. It is also pertinent to consider the preparation of SPTs in their future provision of primary mathematics education at this time of British political change and curriculum alteration.

1.3 Aims

Hence, the aims of this study were to:

- ascertain the range of variation of perceptions of mathematics amongst student primary teachers at the outset of Initial Teacher Training
- consider pedagogical associations relating to that variation
- identify potential implications for SPTs' ITT development
- consider how these potential implications could be addressed

1.4 Research Questions

The research questions set to achieve these aims were therefore:

1. What is the range of variation of perceptions of mathematics amongst student primary teachers at the outset of Initial Teacher Training?

2. How does the range of variation of perceptions of mathematics relate to primary mathematics pedagogy?

1.5 Purpose

In light of my own concerns, those expressed in practice by SPTs, theoretical indications of the need for improvement in primary mathematics education and the anticipated changes in government provision of curriculum, the purpose of this study is to provide a reflective tool in facilitation of SPTs:

- identifying and analysing their own perceptions of mathematics
- raising their awareness of a range of perceptions of mathematics
- comparing and contrasting their personal perceptions of mathematics with the range of variation
- analysing pedagogical associations with their perceptions of mathematics
- identifying potential implications of their mathematical perceptions on ITT learning and future teaching
- creating personal mathematical philosophy on which to base their ITT development and future practice
- identifying and planning necessary change

1.6 Conceptual Framework

The conceptual framework for this study is that mathematics is a non-dualist human conceptualisation dependent on relationality (Marton and Booth, 1997). It is argued that learning is dependent on the individual's relationship between themselves and what is learnt (Marton, 1986).

With regard to SPTs' ITT development, although ITT providers can design courses to attempt to meet the wide needs of SPTs, the fundamental responsibility and power for learning lies in the hands of the learner, and SPTs therefore need to be aware of their learning needs and prepared for what they identify as necessary for their development. Hence, the aims for this study were formulated in the interests of SPTs being in the best position possible at the outset of ITT in terms of their expectations, preparation and aspirations for their ITT development and future practice as teachers of primary mathematics.

1.7 Methodology

Capturing perceptions that are intangible, and potentially unconsciously held, led to a choice of qualitative phenomenographic methodology whereby experience is described and interpreted to determine the range of variation of mathematical perceptions across a typical group of SPTs beginning ITT. Through pooling collective meaning, a hierarchical range of mathematical perceptions is created with the pragmatic use of a reflective tool for SPTs in making conscious their own mathematical perceptions, raising awareness of others' perceptions, comparing and contrasting differentiated perceptions and formulating a personal philosophy of mathematics. The phenomenographical outcome space is examined in terms of differing perspectives of mathematics pedagogy to enable further reflection for SPTs' personal philosophy of mathematics regarding how mathematics is learnt and taught.

Potential implications for ITT development are identified from the pedagogical exploration of the range of variation of SPTs' perceptions of mathematics at the outset of ITT in terms of potential change SPTs may identify to be necessary and recommendations are made for how these might be addressed.

1.8 Organisation Of Thesis

The theoretical context for the study is presented in Chapter 2, outlining the historical nature of difficulties in primary mathematics education and elements underpinning practice. The choice of methodology is discussed in Chapter 3 and the rigorous method adopted is framed. The findings of the phenomenographical study are presented in Chapter 4, where the outcome space pertaining to the range of variation in SPTs' perceptions of mathematics at the outset of ITT is provided. This outcome space is discussed in Chapter 5 in relation to pedagogical associations and potential implications for SPTs' ITT development are identified. Further discussion of these potential implications with recommendations are contained in Chapter 6. The study is concluded in Chapter 7 with research questions revisited as the range of variation in SPTs' perceptions of mathematics at the outset of ITT is considered in relation to pedagogical associations and potential implications for SPTs to address in seeking to provide the best primary mathematics education they can for the children they will teach.

1.9 Originality And Contribution

Although there is wide theoretical recognition of mathematics anxiety amongst adults and concern for the quality of mathematics education, there is little research specific to student primary teachers in Britain with regard to their mathematical perceptions and potential impact upon their ITT development for future primary mathematics teaching.

This study is unique in its determination of the range of variation amongst British SPTs of mathematical perceptions at the outset of ITT and its specific examination of its associated potential pedagogical implications regarding SPTs' ITT development.

1.10 Summary

It is argued that raising awareness of the range of variation of perceptions of mathematics amongst SPTs at the outset of ITT is an important starting point in consideration of the nature of mathematics and its pedagogical associations in SPTs setting goals for their ITT learning and future practice, identifying need for change and planning for change. SPTs, as primary mathematics teachers of the future, have a major role to play in the improvement of provision for children's mathematical learning. To be the best teachers they can be, awareness and preparation are crucial, based on an informed and consciously constructed philosophy of mathematics on which to base their ITT development and future practice.

Chapter 2 Theoretical Context

2.1 Introduction

This study focuses on student primary teachers (SPTs) and their Initial Teacher Training (ITT) development concerning their future practice as teachers of primary mathematics. As will be demonstrated in this chapter, mathematics education has proved difficult for over a century and continues to be a subject for review and development in terms of provision for children's learning needs, this being a particularly crucial time in history as the British government is revising the National Curriculum (NC) (DfEE, 1999a) in an attempt to raise mathematical standards in primary schools. Indications are that further improvement is needed in primary mathematics education and underlying factors warrant further scrutiny, not least teachers' provision of mathematical learning opportunities for children. This is, therefore, an important area of research with regard to SPTs' learning in ITT regarding their future practice as teachers of primary mathematics.

In this chapter it is argued that past experiences of mathematics influence perceptions of mathematics that in turn influence mathematical engagement and mathematical understanding, which have associated potential implications for ITT learners. It is also posited that specific perceptions of the nature of the subject itself can affect the way people learn and the chosen pedagogies of those who teach mathematics. It is concluded that perceptions of mathematics arising from past experience raise potential implications for SPTs' learning in ITT with regard to their subsequent teaching of mathematics and that there is therefore a need to explore SPTs' perceptions of mathematics at the outset of ITT. A methodological approach is established for the exploration of perceptions via scrutiny of existing research and the basis of success in previous phenomenographic study in enabling determination of intangible perceptions. The rationale is to facilitate consideration of mathematics pedagogy associated with SPTs' perceptions and determine the potential implications for their development in ITT.

Research questions concerning the determination of perceptions and associated potential pedagogical implications regarding SPTs' development in ITT are thereby posed, with the objective of providing a framework for reflection by SPTs regarding awareness of others' perceptions and identification of their own perceptions, in order to facilitate challenging these, planning for necessary change and identifying learning needs for ITT. The aim is for SPTs to become the best primary teachers of mathematics they can be, in the ultimate best interests of the children they will teach.

2.2 Research Questions

1. What is the range of variation of perceptions of mathematics amongst student primary teachers at the outset of Initial Teacher Training?

2. How does the range of variation of perceptions of mathematics relate to primary mathematics pedagogy?

2.3 Conceptual Framework

From a dualist perspective, mathematics is viewed as existing in its own right as a body of information imposed on a learner as an "external body of truth" (MacNab and Payne, 2003) and is, therefore, perceived as a phenomenon which exists to be explored separately from human perception since the reality of the world and an individual's understanding of it are separate.

Alternatively, mathematics can be perceived as "man-made as polystyrene" (Owen, 1987, p17), based on human interpretations of phenomena. This non-dualist perception views mathematics as a conceptualisation where "our world is a real world, but it is a described world, a world experienced by humans" (Marton and Booth, 1997, p113), involving the ways in which individuals interact with phenomena.

Epistemologically, the opposing perspectives have potential implications for learning. This study posits that the dualist perception of mathematics teaches by instruction and transmission of facts, explanation and practice of procedural method and leads to recalled and mechanical mathematical knowledge as opposed to relational understanding. In contrast, the non-dualist perception is one whereby mathematical understanding is created through teachers facilitating active engagement with hands-on, practical, contextual problem-solving and posing.

The ontological framework for this research is based on the non-dualist perspective of mathematics being a creation, based on the way individuals relate to phenomena, where mathematics does not exist without the human, since mathematics **is** a human perception created of understanding as phenomena are interpreted. Hence, learning mathematically involves a qualitative experience dependent on the interpretations learners put on their experiences based on the "internal relationship between the experiencer and the experienced" (Marton and Booth, 1997, p113). The creation of mathematics is formulated by humans in their attempts to understand their world, to communicate their understanding and work with what is around them, as well as for intrinsic enjoyment and challenge – and its make up is the social construction of ideas arising from interest, activity and practical need (Thompson, 1992). This man-made perception of mathematics is one where problems are posed and solutions sought (Szydlik, Szydlik and Benson, 2003) and involves an active process (Hersh, 1986) whereby mathematical activity is crucial for learners to engage in problem-solving to reason, think, apply, discover, invent, communicate, test and critically reflect (Cockcroft, 1982) and enjoy the challenge and wonder that mathematical engagement and awareness can bring.

Humans throughout history have sought to understand the world around them, to utilise the resources of their environment and to communicate in a variety of ways through the spoken word and symbols. Humankind hence created the discipline of mathematics to be passed on and learned in order to be used by all in the understanding, utilisation and communication of the world around them. Hence, naturally occurring phenomena were understood through the sharing of a created phenomenon – and therein lies epistemological difficulties. As will be demonstrated in this chapter, learning mathematics is not straightforward in the sense that what began as a creation became a discipline of instruction - the learner's understanding thus being dependent on another's teaching, and the last century documents the inadequacy of mathematical learning via various permutations of this process.

In the interests of primary mathematics education, it is proposed that there is a need for SPTs to be aware of differing perspectives in order to identify their own perceptions and to set goals for their ITT learning and future practice as teachers.

2.4 Difficulties Within Primary Mathematics Education – An Historical Overview

Mathematics teaching has historically proved difficult. The Victorian perception was that mathematics could be learnt by rote via drilling facts into learners (Sharp, Ward and Hankin, 2009). Dissatisfaction with this method led to a wider curriculum in schools and in the 1930s a prescribed mathematics curriculum was mooted (Mathematical Association, 1955). In post-war 1946, perceptions moved to encouragement of mathematical thinking through "constructive play, experiment and discussion" (MA, 1955, pV), whereby awareness of mathematical relationships and structures would be developed, with mathematics viewed as a science and a language to be learnt both for its necessity within society and use in the world, and also for intrinsic pleasure (MA, 1955).

The child-centred perception of learning mathematics continued into the 1960s (CACE, 1967), yet by the 1980s it was reported to be "a difficult subject both to teach and learn" (Cockcroft, 1982, p67), with advice that mathematics should include "exposition by the teacher...discussion between teacher and pupils and between pupils themselves...appropriate practical work...consolidation and practice of fundamental skills and routines...problem solving including the application of mathematics to everyday situations...investigational work" (Cockcroft, 1982, para 243). Subsequently, the 1988 Education Reform Act brought about a statutory

National Curriculum in England and Wales in 1989. The responsibility for mathematical teaching content was removed from teachers and replaced by a prescribed mathematical curriculum as had been suggested, but rejected, over forty years earlier.

However, standards in primary mathematics continued to be criticised (Askew, 1998) and the statutory National Curriculum (NC) was revised in 1999 (DfEE, 1999a), one aim being to encourage primary teachers to engage children further in the use and application of mathematics. The non-statutory National Numeracy Strategy (NNS – DfEE, 1999b) was also introduced into primary schools at this time, extended later into secondary schools and superseded by the non-statutory Primary National Strategy (PNS) (DES, 2003) with encouragement for teachers to consider more creativity within mathematics. Yet, whilst Ofsted claimed these had a positive impact upon teaching and learning (Ofsted, 2005), the PNS (DES, 2003) is now decommissioned and the NC is under further review. The Williams Report (DCSF, 2008a) raised the issue of mathematics teachers needing specialist support in primary schools and the Rose Report (DCSF, 2008b) recommended a new NC to include a stronger focus on mathematical understanding. Amendments to curriculum by the new UK government are due to be announced (QCDA, 2011), but they have stated that the current NC is too prescriptive (DfE, 2010) and intimate "allowing schools to decide how to teach" (DfE, 2010, p10) and that "teachers must be free to use their professionalism and expertise to support all children to progress" (DfE, 2010, p42).

Hence, despite over a century of changing perceptions concerning mathematics education, SPTs enter ITT amidst a societal and political climate of difficulty with regard to quality mathematics provision for primary children. Whilst pedagogy has been identified as an underlying reason for the problem (Ryan and Williams, 2007), addressing this issue is not straightforward. Elements contributing to teachers' practice are many for, as so aptly described by Desforge and Cockburn (1987, p2), "the problem of mathematics education is a many headed monster." To attempt to tackle this decades-old problem with its underlying multi-faceted aspects is, therefore, no mean feat, but necessary when one considers the raw deal that children may be facing in some primary classrooms, since, as Bibby, Moore, Clark and Haddon (2007, p16) purport, "something is going wrong for learners in mathematics classes and…this needs remedying."

The basis of this study was evidence of SPTs expressing negative perceptions of mathematics that potentially linked directly to their ITT learning and future practice as teachers of primary mathematics (Jackson, 2008; Jackson, 2007) and concerns arising from practice were such that a theoretical review to ascertain the nature of such perceptions was warranted. Although this study's context is British, it is apparent that difficulty in teaching mathematics is international (Goulding, Rowland and Barber, 2002), with MacNab and Payne (2003) suggesting that SPTs' insecurities in teaching mathematics are widespread internationally. Hence material is used in this study from across the globe to investigate the topic further. UK and international papers were sourced through education databases using key search words within titles and abstracts of *mathematics, perceptions, conceptions* and *beliefs* which were initially also refined to *student*, *pre-service*, *initial teacher training*, *initial teacher education* and *teacher*, but as little research was evidenced exclusive to SPTs, the search was widened to include adults generally and main citations were subsequently sourced for relevant research.

2.5 Attitudes Towards Mathematics Amongst Adults, Teachers And SPTs

According to government policy for ITT, "teachers can and do make huge differences to children's lives...indirectly through their...attitudes" (DfES, 2002, p2) and it is considered essential that teachers present a positive attitude towards the subject if this is to be encouraged in learners (Akinsola, 2008). A positive attitude towards mathematics is regarded as beneficial to SPTs' effective teaching (Mooney and Fletcher, 2003), with the Advisory Committee On Mathematics Education (ACME, 2006, p4) going so far as suggesting that "excited and enthusiastic" teachers of mathematics are needed if primary mathematics learning and teaching is to be improved.

However, negative attitudes towards mathematics amongst adults are well documented (Bibby, 2002a). On the one hand, mathematics has been described as an unemotional subject (Paechter, 2001), and yet to engage with mathematics can be viewed as "intensely emotional" (Bibby, 2002a, p706), evoking "real emotional turbulence" (Brown, 2005, p21). Emotions associated with mathematics include dislike (Ernest, 2000), tension (Akinsola, 2008), anxiety (Ernest, 2000), dread (Buckley and Ribordy, 1982), anger (Cherkas, 1992), terror (Buxton, 1981) and fear (Akinsola, 2008); with evidence of learners lacking confidence (Pound, 2008) and feeling foolish (Haylock, 2010), bewildered (Buxton, 1981), shamed (Bibby, 2002a), guilty (Cockcroft, 1982) and inadequate (Brown, McNamara, Hanley and Jones, 1999), leading to frustration (Haylock, 2010), distress (Akinsola, 2008) and panic (Buxton, 1981).

Negative feelings about mathematics correlate with social expectations, including the apparent need to do well in mathematics (McLeod, 1992), a fear of looking stupid in front of others (Buxton, 1981) and either high parental expectations (Haylock, 2010) or parents passing on their own mathematical anxiety to their children (Haylock and Thangata, 2007). In contrast, however, there is some parental expectation for children not to succeed mathematically if they didn't do so themselves (Haylock, 2010) and an acceptance and admission of mathematical inadequacy amongst adults seems to be regarded as socially acceptable (Haylock, 2010). There is apparently no clamour for people to describe themselves as mathematicians. Pound and Lee (2011, p1) suggest that "while illiterate adults adopt all manner of strategies to hide their inability, innumerate adults will happily declare that they can't do mathematics to save their lives" and it would seem, as so eloquently purported by Lockhead (1990, p543), that "mathematics has the unique privilege of being the only school subject in which the majority of educated adults proudly claim incompetence."

Perhaps such social acceptance is a result of the apparent assumption that mathematics is reserved for a select group of society. It is sometimes perceived as a male domain (McVarish, 2008), gender having been identified as a possible cause of mathematics anxiety (Cooper and Robinson, 1989) with the notion that males are better at mathematics than females (Furner and Duffy, 2002), the suggestion that girls receive less help and more ridicule when experiencing difficulties (Brady and Bowd, 2005) and Tobias (1978) purporting that girls believed they would invite social unpopularity if they were seen to be good at mathematics. Mathematics is also frequently seen as an intellectual subject only for the gifted (McVarish, 2008) and clever (Sowder, 2001), reliant on having a mathematical mind (Furner and Duffy, 2002) or mathematical brain (Schuck, 2002) and not for people who excel in 'art' subjects (Tobias, 1991).

Whilst there is no assumption made that only negative attitudes exist towards mathematics, in reference to the concerns of this study there is no doubt that there are strong perceptions and pervasive emotions associated with mathematics (Perry, 2004) and these have been recognised amongst teachers (Ernest, 1991), primary teachers (Wilkins, 2008) and SPTs (Haylock, 2010). Indeed Briggs (2009, p100) goes so far as to state that "many people teaching mathematics in the early years and primary age range do not have positive feelings about the subject." Mathematicians seem to be seen to stand apart from the rest of society as elite and teachers themselves have professed uncertainty as to whether they believe themselves to be mathematicians (Battista, 1999). Children in primary schools deserve to learn mathematics in the company of teachers who are not debilitated by negative attitudes and anxiety towards mathematics, but despite the wealth of general evidence regarding adults' negative attitudes towards mathematics, there is limited research exploring this area specifically with SPTs.

2.6 Potential Effects Of Negative Attitudes Upon Mathematical Engagement

The impact of negative feelings towards mathematics is not confined to emotional response, since there are also apparent physical effects on mathematical engagement, ranging from uneasiness at having to partake in mathematical activity (Smith, 1997), to illness and faintness (Smith, 1997), manifested through sweating, nausea and palpitations (Krantz, 1999), churning stomach (Maxwell, 1989) and difficulty breathing (Akinsola, 2008). Learners have described a resulting feeling of helplessness (Akinsola, 2008), crying whilst struggling to learn multiplication tables (Ambrose, 2004), not being able to cope (Akinsola, 2008) and an inability to perform on tests (Smith, 1997). Impact upon mathematical performance includes concentration being difficult (Tobias, 1978) and the ability to remember (Kogelman and Warren, 1978), to the extent that learners can become "paralyzed in their thinking ...and... prevented from learning" (Morris, 1981, p413).

As a result, for some the answer is to evade engagement in mathematics – by avoiding mathematics classes (Smith, 1997), choosing to teach younger children assuming that the mathematics required is easier (Tobias, 1978), avoid it wherever possible (Brady and Bowd, 2005) or to develop coping strategies for everyday life, such as using cheques instead of money (Cockcroft, 1982) or copying (Maxwell, 1989). Some learners become disaffected (NACCCE, 1999), give up (Skemp, 1989) or drop out (Papert, 1981) under their assumption of mathematical inability (Metje, Frank and Croft, 2007).

For SPTs, however, avoidance is not an option, since mathematics is a core primary school subject - despite Haylock and Thangata's (2007, p14) assertion that "many trainees start primary teacher training courses with considerable anxiety about having to teach mathematics." Their work suggests that SPTs' problems originate from past learning experiences - indeed, a range of research points to the educational environment as the major source of infliction of mathematical pain. Teachers have been documented as being unsympathetic (Briggs and Crook, 1991), showing hostile, gender-biased, uncaring, angry and unrealistic behaviour (Jackson and Leffingwell, 1999) and creating a classroom environment of hostility, impatience and insensitivity (Brady and Bowd, 2005). Research carried out with learners of mathematics suggests an expectation to understand after brief explanations of concepts (Brady and Bowd, 2005) and feeling a nuisance in their attempts to understand (Haylock, 2010). In such environments, teachers are seen to be correct and learners accept blame for not understanding (Miller and Mitchell, 1994), being too afraid to ask questions (Haylock, 2010), demonstrating low self-esteem (Akinsola, 2008) and embarrassment (Brady and Bowd, 2005) with the fear of 'being found out' by someone judgemental and in 'authority' (Buxton, 1981).

Experiences of being taught mathematics leading to feelings of inadequacy have been shown to affect attitudes towards the subject (Perry, 2004), with some learners "mentally scarred by past experiences of failure" (Suggate, Davis and Goulding, 2006, p2). Some learners describe feeling "written off by their mathematics teachers" (Haylock 2010, p5) with a tendency to believe teachers who indicate they lack mathematical ability (Miller and Mitchell, 1994). Past mathematical experiences appear to be far-reaching, as perceptions formed at primary school have lasted into adult life (Houssart, 2009) and a single humiliating incident (Ernest, 1991) or a single teacher's judgement (Perry, 2004) can form a lasting conviction of mathematical inability in a learner. Such evidence of adults carrying what Haylock (2010, p5) describes as the "emotional baggage" of feeling a mathematical failure, is a consideration for SPTs in overcoming any potential baggage of their own that may debilitate their learning in ITT.

2.7 Perceptions Of The Nature Of Mathematics And Associated Mathematics Pedagogy

Research clearly suggests the existence of negative attitudes towards mathematics, with potential physical effects on mathematical engagement, but learners' past experiences of mathematics have also been shown to shape perceptions of the subject itself. It is purported that beliefs about the nature of mathematics are linked to attitudes (Swars, Smith, Smith and Hart, 2009) and these in turn affect "the way we learn mathematics, the way we teach it, and will affect the way the children we teach view mathematics" (Ernest, 2000, p4). In consideration of SPTs' mathematical learning in ITT it is therefore crucial to explore various perceptions of the nature of mathematics and associated pedagogical perceptions of teaching and learning mathematics. To do so, a review was made of the range of literature from the library and electronic sources that are recommended to SPTs on ITT courses in a UK university in order to extrapolate the wealth of material that confronts them in attempting to make meaning of this aspect of mathematical theory.

2.7.1 Dualist Perspectives

This study posits that one source of negative perceptions of mathematics is a teaching approach whereby learners take a passive, receptive role as teachers impart what is viewed as correct mathematics – a dualist perception where mathematics is seen as existing as a fixed set of facts to be remembered, rules to be followed and procedures to be undertaken. Ernest (1989) describes teachers within this Instrumentalist view as 'instructors' using a transmission approach frequently followed by practice by the learners (Askew, Brown, Johnson, Rhodes and Wiliam, 1997).

On the surface, transmission is outmoded pedagogical practice – indeed over sixty years ago, Polya (1945, p19) advocated that if a teacher drills "his students in routine operations he kills their interest, hampers their intellectual development and misses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for and some means of independent thinking." Some would hope that the transmission approach is no longer used - Anghileri (1995, p74), for instance, purporting that "the mathematics classroom has changed from the days when the teacher told pupils what to do and how to do it," but there is evidence to suggest otherwise. Ernest (2000, p8) for instance, claims that "too often the teaching and learning of mathematics involves little more than the practice and mastery of a series of facts, skills and concepts through examples and problems" and recent literature evidences expectations of learning rules and procedures by rote without understanding (Haylock, 2010), teacher explanation followed up by learners' practice, with a lack of flexibility in strategies for either teaching or for problem-solving (Schuck, 2002) and Ofsted (2008, p5) reporting that "too often, pupils are expected to remember methods, rules and facts without grasping the underpinning concepts, making connections with earlier learning and other topics, and making sense of the mathematics so that they can use it independently."

Government guidance (DfE, 2003) suggests that remembering information is an important factor in learning mathematics. Memorisation of facts has been shown to extend to memorisation of rules that are presented as 'rigid' (Akinsola, 2008) and rule-based procedures (Boaler, 1997) applied in an equally rigid way, also to be remembered (Boaler and Greeno, 2000), as recalled facts are applied mechanically

(Lampert, 1990), described by Boaler and Greeno (2000) as 'knowing the tricks' whereby use of different approaches is not encouraged.

This rote learning approach has been shown to be a factor of mathematics anxiety (Cornell, 1999). The view of mathematics being a fixed entity to be transferred concentrates on the product or answer being the goal (Cross, 2009) and is reminiscent of Skinner's (1954) behavioural theory as the learners' apparent goals are to achieve required answers and be rewarded with a tick, leading to anxiety about getting answers wrong (Haylock and Thangata, 2007) and believing that correct procedure must be followed – even to the extent that if not, the answer cannot be right (Bibby, 2002b). Learner expectations become transmission of knowledge at a set pace resulting in competition between individuals (Boaler and Greeno, 2000), often carried out at speed alongside a mysterious need for efficiency (Bibby, 2002b) and writing neatly (Boaler and Greeno, 2000), with Ofsted (2005, para 64) noting that "in mathematics, teachers sometimes place too much emphasis on pupils' recording and presentation of their calculations, deflecting their attention from the necessary mathematical reasoning." It is an approach without investigative open-ended mathematical thinking (Oxford and Anderson, 1995), learners motivated by closed questions with set, correct answers (Boaler and Greeno, 2000) that the teacher already knows and since there is thought to be one, and only one, right answer (Lampert, 1990), mathematics is perceived to be logical at the expense of intuitive thinking (Frank, 1990). It has been shown to lead to limited, fragmented understanding (Mji, 2003) as the facts remembered and rules followed are not necessarily understood (Grootenboer, 2008), resulting in "'rule-bound' adults possessing half-remembered rules without having any idea of how and why they work" (Davis, 2001, p137). As the "teachers have children playing a passive, receptive role as learners" (Desforges and Cockburn, 1987, p7), they follow a structured curriculum that is taught in a linear fashion (Oxford, 1990) and in discrete components (Tobias, 1993) where connections are not made, concepts are not understood and learners consequently fall behind (Shodahl and Diers, 1984).

Classrooms have been described as non-participatory environments (Akinsola, 2008) where mathematics is perceived as solitary and performed in isolation of others (Lampert, 1990) with teachers espousing practical mathematics yet not using manipulative apparatus (Foss and Kleinsasser, 2001) and it is suggested that learners expect to be "spoon-fed whatever information the teacher deems appropriate" (Howell, 2002, p116/7) with resulting 'victim mentality' (Hwang, 1995) whereby students blame others for a lack of learning rather than seeing it as a process for which they take responsibility.

Existing literature thus paints a picture of people engaging with mathematics without really understanding why they are doing what they are doing - expected to be compliant and passive, their learning reliant on memorisation of facts (Wong, 2002), using rules without understanding (Nunes and Bryant, 1996) and learning being limited to following procedures (Kyriakides, 2009). Such a teaching approach is likely to result in a surface approach to learning (Cano, 2005) where learners perceive mathematics to be a predetermined set of knowledge to be absorbed without understanding and mathematical activity to be "externally imposed" (Trigwell, Prosser and Ginns, 2005, p351). Research suggests that some teachers' own previous experiences of learning involved individual and rote learning, leading to an assumption that mathematics is reliant on memorisation (Ambrose, 2004) and concerns for SPTs are supported in that their perceptions, based on past experience, need scrutiny to ensure poor mathematics pedagogy is not perpetuated.

Another dualist perspective is that which Ernest (1989b) terms the Platonist view. Although still based on the view of mathematics being a static body of knowledge, this focuses on both content (the body of knowledge) and understanding (by the learner) (Cross, 2009). Rather than instructing, the teacher attempts to give explanations to enable learners to 'discover' the existing body of knowledge, making logical connections to develop meaning and conceptual understanding (Ernest, 1991) through description of mathematical objects and relationships. Learning is, therefore, dependent on receiving knowledge and though there is more active construction than the instrumentalist model, through understanding explanations and the inclusion of problems and activities in textbooks, the focus is not on mathematical process.

Whilst the concept of mathematics being discovered by learners has its merits, not least in the advocation of the use of practical apparatus (Brown, 2000), there have been claims of this more independent approach leading to underachievement (Boaler, 1997) and criticism in the notion of discovery since children are in fact discovering a body of knowledge that has already been discovered by others (Papert, 1981). There also remains the element of received knowledge through teacher explanation and practise of skills and procedures using schemes or textbooks (Boaler, 2002) and what has been termed an 'over-reliance on worksheets' (Ofsted, 2005). It is an approach that has received criticism regarding the limitations of reproducing teachers' demonstrations (Desforges and Cockburn, 1987) that create learner dependency (Burton, 1994) with a lack of communication amongst learners (Anghileri, 1995) and also with a lack of connection to the real world (Romber and Kaput, 1999) and to other mathematics (Hopkins, Gifford and Pepperell, 1999).

Similarly based on an existing body of proved knowledge is the Absolutist view (Ernest, 2000) in which mathematical use and application is promoted. It is widely recognised that there is agreed mathematical knowledge (Koshy, Ernest and Casey, 2000) that includes skills, such as drawing a measured straight line with a ruler; concepts, such as negative numbers; procedures that guide the use of these in solving problems; rules, such as BODMAS; and that these are linked to attitude and understanding. The current English National Curriculum (DfEE, 1999a) sets out the required mathematical content for learning in schools and places strong emphasis on the use and application of mathematics (UAM). The aim of such an approach is for children to be able to "confidently apply their knowledge of mathematics to a range of situations in their subsequent working and domestic lives" (Hughes, Desforges and Mitchell, 2000, p118) and it is suggested that integration of mathematical learning in other curriculum contexts both develops their ability to use and apply mathematics and to relate it to real life (Coles and Copeland, 2002).

However, research suggests that using and applying mathematics in new contexts is problematic (Hughes, Desforges and Mitchell, 2000) with indication that mathematical topics may be taught in isolation and that learners are unable to transfer skills to new situations due to a lack of understanding (Carpenter and Lehrer, 1999). Whilst the government review of curriculum (DfE, 2010) suggests more teacher autonomy, guidance from the past two decades has been prescriptive and objective-led, and regarded as a contradiction to learner-focused pedagogy (Brown, Hanley, Darby and Calder, 2007). This raises the issue of how government policy is interpreted by teachers, since the NNS and PNS were non-statutory and the statutory NC always contained encouragement to follow the more progressive mathematical philosophy of problem-solving. This was included in the 'Using and Applying Mathematics' (UAM) sections of the curriculum, but proved a difficulty in practice, with revision of the NC (DfEE, 1999a) incorporating UAM into programmes of study (PoS) instead of its original separate PoS in an attempt at teacher engagement and recent guidance (DSCF, 2008a) recommending more emphasis on UAM.

Despite these changes, as so clearly represented in the literature, difficulties with mathematics education persist and in terms of the problem-solving aspect of UAM, there is evidence of a reduction of problem solving to calculations wrapped up in word problems (Jones, 2003) and textbook problems being used that are closed tasks with little autonomy for the problem-solvers (Brown and Walter, 2005). Time is needed for a problem-solving approach to allow learners to think, reflect, make connections, recognise relationships, develop ideas and communicate (Carpenter and Lehrer, 1999), but in practice, the problem-solving aspect of UAM is sometimes seen as a 'bolt on' as opposed to an integrated teaching and learning approach (Fairclough, 2002), perhaps due to pressure of getting through the curriculum content and from parents (Orton and Frobisher, 1996). An added difficulty is that its active and practical nature can result in little written evidence which can put some teachers off in terms of accountability (Jones, 2003).

Hence, in addition to past experience, SPTs have to contend with government guidance that during frequent changes over the last two decades has, to date, not resulted in satisfactory outcomes for learners of primary mathematics. Perhaps the inherent difficulty in all of this is the fundamental perception herein of the dualist view – that mathematics is a fixed body of knowledge, prescribed by government through curriculum content and not engaged with in practice in a relativist way.

2.7.2 Non-Dualist Perspectives

Whilst new government ministers grapple with the task of reviewing the school curriculum, teachers await new educational guidance - which on the one hand seems a positive step for mathematics education, but on the other relies on the hope that teachers will feel able to facilitate such an approach: from the perspective of the guidance itself, having previously been criticised on the basis of stilting creativity through structured lesson formats (Mooney, Briggs, Fletcher, Hansen and McCullouch, 2009); from the perspective of autonomy, since research suggests that government initiatives are met with acceptance in schools, with a tendency to conform rather than critique and challenge (Andrews, 2007); and from the perspective of teachers' confidence in embracing such an approach (Haylock, 2010). With espoused improvements to mathematics education in mind, it is, therefore, of value to consider the non-dualist view that encapsulates more creative teaching and learning approaches through viewing mathematics, not as a fixed body of knowledge to be transmitted to the learner, but as a creation in itself from the human mind.

Problem-solving constitutes a perspective of its own, described by Ernest (1989) as a product of creation whereby mathematics is viewed as an active element in society and culture. From the problem-solving perspective, mathematics is dynamic and continually open to expansion, involving a process of enquiry to reach understanding. However, it is not a finished product since the notion of mathematics being a human creation leaves it always open to revision, for as White and Gunstone (1992) suggest, understanding is a continuum. Indeed, the notion, previously evidenced, of mathematics being constituted of right or wrong can be contradicted in infinite ways for children learning mathematics - a simple example being a young child believing that a large integer cannot be subtracted from a small integer, but subsequently learning that this is possible, once the concept of negative numbers is understood. The problem solving perspective is "learner-focused" (Kuhs and Ball, 1986) as it is based on an individual's construction of knowledge. The teacher in this case does not directly transmit knowledge, but is instead a facilitator of knowledge acquisition, with an emphasis on process (Mikusa and Lewellen, 1999). However, as suggested previously, there is a perception presented in practice that "the aim when doing mathematics is to get the right answer and thus please the teachers" (Cockburn, 1999, p108) as opposed to engaging in the process of mathematics. For teachers to focus on the latter, recognition is needed that mathematics is not just a body of knowledge, it is a "disorganised and untidy, creative activity" (Orton in Orton and Wain, 1994, p11).

Whereas transmitted instrumental learning is learnt through habit, the problem-solving approach is one of relational learning (Skemp, 1989). In contradiction of dualist perspectives is the constructivist approach whereby mathematical knowledge is subjectively internalised, constructed and reconstructed by individuals. Schemes and texts may be used, but there is more teacher and school autonomy in the mathematics curriculum, with "provision of meaningful problems designed to encourage and facilitate the constructive process" (Schifter, Twomey and Fosnot, 1993, p9). Constructivist theory concerns itself with construction and modification of knowledge in the light of experience (Bruner, 1966) involving active participation as opposed to transfer of knowledge. Piagetian constructivist theory (Piaget, 1953) purports the development of schema as new experiences are assimilated into existing cognition, with accommodation when modification and reorganisation is necessary (although Piaget's accompanying claims to age-related development is open to criticism (Clemson and Clemson, 1994), despite the plethora of age-related objectives set out in the PNS (DES, 2003)).

With a problem-solving perspective, the construction of knowledge takes place in the learner's mind (Skemp, 1989) through experience and creation, and socially through communication that promotes the active construction of understanding by the individual in a community of talk, interaction and shared meaning (Vygotsky, 1978). A social constructivist perspective of teaching encourages "social discourse involving explanation, negotiation, sharing and evaluation" (Kamii and Lewis, 1990, p35). In a socially constructive learning environment there is an ethos of shared understanding where learning is scaffolded by teachers and learners with support in developing understanding (Yackel and Cobb, 1996) and the classroom is "characterised by a lively mix of discussion, questioning, debate and reasoning that can enhance interaction and as a consequence improve the quality of the children's mathematics understanding" (Bottle, 2005, p77). In contrast to the anxiety described earlier in this chapter, where mathematical performance can be affected by fear of the teacher (Cockburn, 1999), a constructive learning environment is one whereby children can "feel free to try things out and make mistakes without any shame, fear or feeling the need to hide them, so that they can correct them and continue to learn without the interference of any bad feelings" (Ernest, 2000, p16).

Discussion plays a central role in the social construction of mathematical concepts (Askew, 1998), as children explain their mathematical thinking, to themselves and to others (Burton, 1994) both verbally and in written form (Floyd, 1981). This, however, relies on the careful development of mathematical language, which has been shown for some to be "inaccessible" (Wilson, 2009a, p95). Children, therefore, need the opportunity to develop their own means of communicating mathematically (Anghileri, 1995), gradually being introduced to associated formal language (Anghileri, 2000) which, as it is refined, enables them to more accurately explain their mathematical thinking and justifications (Nelson-Herber, 1986) and in turn develop understanding. Also key is collaborative working, whereby children describe their thinking to others as problems are tackled, making sense of both their own and others' reasoning (Anghileri, 1995), trying out ideas "in a non-threatening environment" (Burton, 1994, p112) and, according to Billington, Fowler, MacKernan Smith, Stratton and Watson (1993, p40), "take greater risks in posing questions...develop better strategies...support one another in their learning...are more likely to openly express doubts about their understanding."

Hence the process is one of active learning, meaningful mathematical constructs being created through doing (Atkinson, 1992) and where children are engaged in "playing around with and getting a sense of, noticing and describing, discussion and showing, articulating, asking questions, testing out, convincing, practising and consolidating, developing new situations and contexts" (Delaney, 2010, p77/78) as "active participation in problem solving through practical tasks, pattern seeking and sharing understanding" (Anghileri, 1995, p7) enables children to make sense of relationships that underlie mathematical knowledge. However, it is important to note that, as Kelly and Lesh (2000, p28-29) purport, "mathematical thinking does not reside in problems; it resides in the responses that students generate to problems" and is encouraged through the use of various methods as learners construct meaning and make connections, spot patterns and recognise relationships (Pound, 2008), linking the mathematics they engage with in school with their outside lives (Anghileri, 1995). Rather than the learning of isolated facts through an instrumentalist approach, learners are given the time and space (O'Sullivan, Harris and Sangster, 2005) to make connections between mathematical facts and concepts (Suggate, Davis and Goulding, 2006) as they build up a network of understanding related to their range of experience (Haylock, 2010). One of the benefits of the PNS (DES, 2003) was the encouragement of probing children's mathematical thinking and the promotion of different approaches (Pound and Lee, 2011) whereby children can be enabled to learn new strategies and relate these to their existing understanding. Through working collaboratively, they can be introduced to different ideas from peers as well as the teacher (Burton, 1994), although care needs to be taken that strategies are not taught in isolation and as abstract procedures (Anghileri, 1995) that result in instrumental instruction of mathematics without understanding and meaning.

The problem-solving approach is seen as "one of the core elements in the development of mathematical thinking" (Pound, 1999), viewed by some researchers as fundamental to mathematics since it was created to solve mysteries, utilise that which is around us and communicate understanding (Ollerton, 2010). For the learner, it provides a "purpose and reason to mathematics and allows children themselves to see why and how mathematics is relevant to their lives" (Jones, 2003, p88) and encourages them to become problem finders (Pound, 2008, p59), especially since tackling problems leads to "some reformulation of the original problem that is essentially a problem generating activity" (Brown and Walter, 2005, p126). It is also an opportunity for learners not to merely perceive mathematics as difficult, as previously described, but to accept that seeking solutions to problems and understanding our environment is challenging and that "children need to come to terms with the frustrations and disappointments as well as the pleasures and satisfactions when they explore new territory" (Orton and Frobisher, 1996, p32). One of its criticisms, however, is teachers' belief that "time set aside for problem solving will eat into the time they have available for teaching facts and skills" (Jones, 2003, p90) – an indication that the full nature of this integrated approach to teaching is not fully comprehended. Indeed, some of the advantages of planning mathematics learning through this medium are facilitation of an holistic approach of integration of mathematics into cross-curricular areas (Sakshang, Ollson and Olson 2002) and differentiation for varied learning needs through providing different levels of challenge, maintaining learners' interest and making connections to various aspects of mathematics and other curriculum areas. It has been suggested, however, that teachers can feel out of control if mathematics is presented as open-ended and learners use different methods, find different solutions or do not reach a solution at all (Jones, 2003, p89). Perhaps one of the difficulties in teachers taking this perspective on board, especially if their own learning experiences involved mechanistic methods, is that it may mean "we are challenged to think differently" (Sakshang, Ollson and Olson, 2002, pvi) – indeed, there is evidence of teachers who have "had to make a shift in their own thinking and mathematical practice" (Fairclough, 2002, p85).

The perspective of recognising the importance of mathematical process as opposed to focussing on a product that may be correct but constitute little understanding, is also reflected in the Purist view which Ernest (1991) describes as non-threatening and supportive, since it views all mathematical learners as equal in achievement of personal potential. Based on the purity of creativity, it encapsulates the non-dualist notion of the formation of mathematics where "inventors doodled, made mistakes galore, agonised over problems for hours, days, weeks, even years, disposed of hoards of paper and chalk, and made haste slowly" (Dawson and Trivett, 1981, p125) and contradicts the instrumentalist views of formal mathematical procedures, rule following and right/wrong answers. However, whilst there is support for more progressive approaches to mathematics education where "pupils need to see mathematics as a process that they can be actively and creatively involved in rather than a body of knowledge that 'belongs' to someone else" (Anghileri, 1995, p9), there is indication that "creativity and mathematics or creative mathematics appears for many to be a contradiction in terms" (Briggs, 2009, p94) and, according to Briggs and Davis (2008), teachers who regard mathematics as 'right or wrong' are unlikely to recognise the creativity inherent in mathematics.

The Purist perspective focuses on the learner's development, based on construction of understanding. However, although teachers need to be aware of learners' existing understanding (O'Sullivan Harris and Sangster, 2005) there is a danger of deciding what they should learn next, for mathematics is not necessarily the linear progression of content that guidance such as the PNS (DES, 2003) prescribed, nor should it be limited to Piagetian theory that suggests age-related learning. Development is based on the learners' construction and reconstruction as their understanding facilitates, and whilst ceilings should not be set on children's learning, teachers should also recognise their innate ability, for as Desforges and Cockburn (1987, p4) suggest, "before children come to school they are inventive mathematical thinkers" yet according to Skemp (2002, p75), "children come to school having already acquired, without formal teaching, more mathematical knowledge than they are usually given credit for."

Other non-dualist perspectives where mathematics is not externally imposed, but is socially constructed include what Ernest (1991) terms the Public Educator view which focuses on society. From this perspective, ability is not fixed, children's learning being affected by their environment and culture for, as a human creation, mathematics is part and parcel of the culture in which it is produced (Nunes and Bryant, 1996). Rather than being passive recipients of knowledge, children take an active part in their learning through making their own decisions, the teacher's role being "to provide opportunities in classrooms and throughout the day that require observation, wonder and time for children to make decisions on their own" (McVarish, 2008, p8). This approach involves children being encouraged to question, for as Pound and Lee (2011, p25) advocate, "mathematics is actually about raising questions as much as it is about solving them. The ability to shape (or ask) and to solve mathematical problems is the essence of constructing mathematical reasoning." Potential implications arising for practice here are the encouragement of mathematical enquiry amongst learners and the associated need for a classroom ethos where children are confident in asking questions of the teacher (O'Sullivan, Harris and Sangster, 2005) and teachers who are sure of their own mathematical competence since "it takes confidence to deal with questions from children to which you do not have a ready answer" (Boaler, 2009, p52).

Non-dualist perspectives are not new - Dewey's Pragmatist theory of the early twentieth century, based on the usefulness of mathematics and a focus on the practical and everyday life (Hickman and Alexander, 1998), being an example. Rather than transmission of a body of knowledge, the pragmatic approach focuses on "creating worthwhile learning experiences" (Mason, 2000, p346) and values the relationship between the learner and the mathematics that leads to mathematical understanding. The non-dualist perception of mathematics is a view of the relational aspect between the learner and the object, since the creation that is mathematics could not otherwise come into being, being formed of that relational understanding. Despite mathematics therefore being all-encompassing, it has been described by some learners as pointless, a perception identified as contributing to the difficulties experienced in learning mathematics (Pound, 2008). It is argued that it is important for learners to connect mathematics with the world around them (Nunes and Bryant, 1996), yet learners have been shown to find difficulty recognising mathematics that is relevant in their everyday lives (Bottle, 2005) and using school mathematics outside the classroom (Boaler, 1997). It is purported that school mathematics should be set in "meaningful situations" (Atkinson, 1992, p169) yet researchers warn of the dangers of imposing adult contexts that are outside the realms of children's interest (Ollerton, 2010), forcing them to "suspend reality and accept the ridiculous" (Boaler, 2009, p45).

It is suggested that focusing on the practical aspect of mathematics can help learners to "rationalise their experience" (Edwards, 1998, p8), through the "accessible, real and tangible" (Lee, 2006, p15), providing objects to touch and move to help them describe what happens (Anghileri, 2003, p90) and images and contexts to reach abstract mathematical concepts (Askew and Wiliam, 1995). Whilst it is important to realise that "there is no mathematics actually in a resource" (Delaney, 2001, p124), it is regarded that kinaesthetic experience can aid mathematical engagement although, as Ball (1992, p47) warns, "understanding does not travel through the fingertips and up the arm" and as such learning will not automatically happen through the manipulation of resources. Resources are used as a focus for discussion, for modelling, explanation and demonstration (Bottle, 2005) although, rather than being limited to presentation by the teacher, they need also to be accessible for children to make their own choices about what might prove useful (Burton, 1994). Practical apparatus can be used by learners to create visual and mental images that can help reach understanding of abstract concepts (Moyer, 2001). However, there are indications that children do not necessarily make the link between the materials they use in the classroom to the outside world (Aubrey, 1997) and also that their understanding remains in the concrete (Andrews, 2007). As Askew (1998, p15) suggests, "practical work is not at all useful if the children fail to abstract the mental mathematics from the experience" and it is therefore important to be aware that "concrete embodiments do not convey mathematical concepts" (Gravemeijer, 1997, p316) and to ensure that teaching encourages children to use objects in a way that creates an understanding of representation (Harries and Spooner, 2000) through associated mental reflection and the development of mathematical thinking.

One of the major reasons for mathematics generally being perceived as difficult (Pound, 2008) is its abstract nature (Orton and Frobisher, 1996) and learners' associated difficulties in imagining (Pound, 1999) and communicating its concepts (Skemp, 1989). As such, there is strong argument for the non-dualist perspective that supports a social construction of understanding via the relationship between mathematics and learner through opportunities to question, pose problems, look for patterns (Pound, 2008), to learn to use abstract symbols, mathematical language and develop generalisations as they work with physical objects and practical situations (Bottle, 2005). Hence, although children may come to learn the mathematics of "great abstractedness and generality, achieved by successive generations of particularly intelligent individuals each of whom has been abstracting from, or generalising, concepts of earlier generations" (Skemp, 1981, p83), they do so not by having a recognised body of mathematical knowledge presented to them for absorption, but by forming their own relationship with ideas about their world by reaching a mathematical understanding that is unique to their experience and relation with the subject as an individual learner. Rather than passive receipt, such learning derives from collaborative work and active construction (Von Glaserfeld, 1990, p22), and is dependent on learner autonomy developed through exploration, interest and engagement in mathematical activity as learners are encouraged to explain, reason and use a variety of methods to form relational understanding (Skemp, 1981), making connections between mathematical knowledge and methods to build on previous understanding (Nathan and Koedinger, 2000) and develop a "strong conceptual knowledge base" (Garofalo and Leicester, 1985, p88) where learners "engage with what is being learnt in a way that leads to a personal and meaningful understanding" (Trigwell, Prosser and Ginns, 2005, p351).

Whilst the perspectives presented here are not intended to be an exhaustive list of potential views of the nature of mathematics, they encompass a range and one whereby mathematical purpose includes its use in everyday life and the wider society, an attempt to understand that which is around us, a means of communicating that understanding and an essential element of culture. With regard to the early observation in this chapter of the ideal for 'enthusiastic' teachers of mathematics, an added perception of mathematics is that of its intrinsic value. It is evident from literature that mathematics is perceived by some to be pointless and difficult, with evidence of disaffected learners. The notion described above of problem-solving being included in the school curriculum as a 'bolt-on' is supported by Briggs and Davis' (2008, p16) observation that "part of the problem with mathematics is that it can seem like you only get to the interesting parts of the subject after you have completed all the dull stuff" and it is purported that, "if children …feel…that mathematics is boring, limited and about sums and that is all, it is small wonder that they begin to see mathematics as something not very pleasant or meaningful" (Clemson and Clemson, 1994, p10). There is a wealth of evidence contained herein that points to mathematics not being considered in the least bit fun. As Owen (1987, p17) purports, "throughout history there has been conflict between mathematics seen as a subject growing out of economic and social necessity and the view that mathematics has a purity which transcends mere practicality" for mathematics can, for some, be a source of pleasure (Andrews and Hatch, 1999), wonder (Haylock and Thangata, 2007), power and enjoyment (Skemp, 1989) and "intrinsic interest" (Pound, 2008, p8). To learn mathematics without experiencing this aspect is, according to Koshy, Ernest and Casey (2000, p8) "superficial, mechanical and utilitarian" and is summed up proficiently by Ernest (2000, p8) in that "to neglect the outer appreciation of mathematics is to offer the student an impoverished learning experience...when an outer appreciation is neglected, not only does school mathematics become less interesting and the learner culturally impoverished, it also means that mathematics becomes less useful, as learners fail to see the full range of its connections with daily and working life, and cannot make the unexpected links that imaginative problem solving requires."

The issue here for SPTs is a crucial one. Faced with a plethora of theory on their ITT course concerning mathematics pedagogy relating to different philosophies of mathematics and connected to different psychologies of learning, it is little wonder that they present with anxiety (Haylock, 2010). SPTs currently enter ITT at a time of political and educational change, in the aftermath of teachers following what Hughes (1999, p4) describes as "undoubtedly the most prescriptive approach to primary mathematics ever developed in this country" with an indication that teachers have recently "been positioned more as technically competent curriculum deliverers, rather than artistically engaged, research-informed curriculum developers" (Pound and Lee, 2011, p ix) as they have followed the government-set curriculum and, according to

Brown (2010) followed a procedure-based pedagogy as they teach with national testing in mind. There is hence a strong basis to argue that SPTs need to be in a position to clarify underpinning principles for learning and teaching primary school mathematics and ascertain their own perceptions and values (Lang, 1995) in order to be the best teachers they can be and deal with whatever curriculum changes may be brought about in the near future.

This study therefore posits that effective teachers of mathematics need "to be able to view it and appreciate it, from a range of perspectives" (Cockburn, 1999, p43). However, to be in such a position SPTs would first need to explore both their past experiences of mathematics and associated perceptions, and also reflect on their pedagogical aspirations based on their personal philosophy of mathematics. The range of theory available to SPTs to engage with within ITT is challenging in itself, particularly in light of the changing face of ITT where time spent on learning primary mathematics is reduced (Brown, 2010). Their perceptions are likely to have gradually developed over time and may not have been explicitly articulated, and, as Orton and Frobisher (1996, p34) argue, "they cannot be changed overnight, but they can be challenged." It is argued, therefore, that there is a need for SPTs to reflect upon their personal perceptions and perspectives of mathematics at the outset of ITT to form their own mathematical philosophy that links to the kind of primary mathematics teacher they aspire to be.

2.8 Links Between Experienced Mathematics And Mathematical Understanding

A crucial factor in establishing mathematical philosophy is an individual's mathematical understanding and one which is closely linked to the effectiveness of mathematics teaching. Mathematics subject knowledge is identified in the UK as "one of the main differences between the most and least effective...mathematics lessons" (Ofsted, 2005, p14) with "weaknesses in teachers' subject knowledge...[continuing] to detract from the quality of teaching" (Ofsted, 2005, p14). Evidence is provided by ACME (2006) of teachers, including headteachers, lacking confidence in mathematics subject knowledge. This observation is not confined to the UK, as international studies also raise concern about the quality of teachers' mathematical knowledge affecting the proficiency of teaching (Chapman, 2007). It is surely non-contestable that "teachers require a sound understanding of the mathematical concepts which they teach and an appreciation of how children think and learn" (Cockburn, 1999, p3), and that SPTs need "to be fully competent and confident about your [sic] own mathematics subject knowledge, skills and understanding" (Mooney, Briggs, Fletcher, Hansen and McCullouch, 2009, p69) - but achievement of this is no simple matter.

British SPTs currently need a Grade C GCSE or equivalent in mathematics to be accepted into ITT and have a test to complete before completion in numeracy skills (although these criteria are currently under governmental review) and to achieve Qualified Teacher Status, are required to "have a secure knowledge and understanding of their subjects/curriculum areas and related pedagogy to enable them to teach effectively" (TDA, 2007, p9). Whatever tests are passed, however, assumption cannot be made that a level of mathematical knowledge is sufficient to be able to teach (Golding, Rowland and Barber, 2002) since learned knowledge for the purpose of passing examinations does not equate to confident and secure mathematical knowledge needed to be able to teach the subject (McNamara, 1994). Whilst it is recognised that improving mathematical knowledge is a valued component of ITT (Goulding, Rowland and Barber, 2002), it is crucial to not limit the nature of this knowledge to a superficial level, as teachers need a deep understanding of mathematics to include "how it interconnects within the subject and how it relates to applications outside it" (ACME, 2006, p6). According to Fennema and Franke (1992, p151), ''when a teacher has a conceptual understanding of mathematics, it influences classroom instruction in a positive way,'' yet research suggests that conceptual understanding needed for effective teaching can be lacking (Mewborn, 2001).

There is therefore a clear need for SPTs to "confront the nature of their own mathematical understanding" (MacNab and Payne, 2003 p67) but there are indications that little research has been carried out in the field of Higher Education generally with regard to subject matter and its relation to students' learning (Prosser, Martin, Trigwell, Ramsden and Lueckenhausen, 2005, p139) and, according to Beswick (2007), recent research has been limited in its scope for use in improving mathematics education. Exceptions include Bibby (2002b), who looked at teacher identity in relation to mathematical beliefs, Mji's (2003) South African study linking mathematical conceptions to learning and phenomenographic research by Gullberga, Kellnera, Attorpsa, Thorena and Tarnebergb (2008) investigating prospective teachers' conceptions about pupils' understanding of science and mathematics. However, no existing research is apparent that focuses specifically on determining the range of variation of SPTs' perceptions of mathematics as they begin their ITT courses and the potential links to their ITT learning in relation to their future practice as teachers of mathematics in primary schools.

This review highlights that SPTs' perceptions of mathematics, arising from their past experiences, could affect their learning within ITT since negative attitudes towards mathematics have been shown to affect students' learning (Townsend and Wilton, 2003) and beliefs about a subject can affect understanding and can impede learning (Hofer and Pintrich, 2002). It is purported that if "they lack confidence and dislike the subject they may find it difficult to work up the enthusiasm to teach mathematics in an effective manner" (Cockburn, 1999, p15). In the seventies, approaches to learning were studied and differentiation made between surface and deep approaches, the former constituting a quantitative conception about learning that is superficial and the latter involving students who "have a deep idea, or qualitative conception, about learning" (Cano, 2005, p206). Experiences outlined above of teaching by transmission where students have learnt mathematical facts and rules without conceptual understanding and the ability to make connections to apply to problem-solving situations, is an example of a "teacher-focused" (Trigwell, Prosser and Ginns, 2005, p352) and 'surface' approach (Prosser et al, 2005). In contrast, a 'deep' approach would endeavour to develop conceptual change and be "student focused" (Trigwell, Prosser and Ginns, 2005, p352) where students "engage with what is being learnt in a way that leads to a personal and meaningful understanding" (Trigwell et al, 2005, p351) and have "awareness of and control over their own learning processes" (Biggs, 1987, p5).

This study, therefore, considers that past experience can lead to students adopting a learned response to mathematical engagement dependent on their familiarity with mathematics as an accumulation of facts passively received, or with mathematics as an active construction of conceptual understanding since their perceptions will determine how meaning is made (Hofer and Pintrich, 2002). Thus there are potential implications for SPTs' ITT learning related to their prior experiences of being taught mathematics in terms of their perceptions of mathematics pedagogy, since HE students' perceptions of learning have been shown to vary, related to prior experiences (Prosser and Trigwell, 1999, p58). Applying this concept to the focus for this study, a link is suggested between students' perceptions of mathematics and their approaches to learning, suggesting that 'fragmented' conceptions (mathematics as numbers, rules and formulae, applied to solve problems) lead to a surface approach to learning (reproducing parts), and that 'cohesive' conceptions (mathematics as a complex logical system and way of thinking used to solve complex problems and providing insight for understanding the world) lead to a deep approach to learning (understanding wholes) (Prosser et al, 1998). For mathematics to be taught effectively, awareness is needed of how mathematics is learnt (Speer, 2005) and it is suggested that teachers may tend to teach in the way that they themselves were taught (Wilkins, 2008).

2.9 Potential Implications Arising For SPTs' Learning In ITT

Rather than an irrational phobia (Hodges, 1983), mathematics has been shown here to cause rational anxiety and negative perceptions amongst some learners that in turn can affect their ability to engage in mathematics. Since there is apparently "an urgent need to teach mathematics differently" (Hogden and Askew, 2007, p470) and it has been established that teachers themselves have an enormous influence on what is taught and on learning (Cross, 2009), to consider the role of primary teachers of the future is a valuable enterprise as they embark upon ITT with a range of perceptions concerning mathematics (Ambrose, 2004) that are likely to influence their practice (Nespor, 1987). The premise of this study is to facilitate improvement in mathematics education through potential development of positive perceptions (Noddings, 1992) since these have shown to have a strong influence on understanding how to teach effectively (Hofer and Pintrich, 2002).

It is recognised that "students' prior conceptions of the nature of the subject matter they are studying needs to be taken into account in the design and teaching of courses in higher education" (Prosser et al, 1998, p94) and establishing the range of variation of mathematical perceptions amongst SPTs could, therefore, be useful in raising awareness amongst ITT providers. However, whilst there is a need for ITT providers to be aware of these perceptions in order to consider what to include in their courses (Swars, Smith, Smith and Hart, 2009), teachers have different experiences, attitudes, knowledge and pedagogical understanding of mathematics and flexible opportunities for development are needed to meet individual needs (Smith, 2004). Perceptions are a personal and intrinsic entity and it is considered here that direct involvement on behalf of the learner is needed in terms of taking responsibility for learning (Tolhurst, 2007). As such, it is argued that it is essential for SPTs to examine their own perceptions of mathematics at the outset of ITT and to take control of a subject that may have caused them anxiety in the past by providing them with the means to reflect on their own mathematical experience and practice (Cooney and Krainer, 1996). Gattegno's (1971) notion that 'only awareness is educable' is valid here, in that the starting point for addressing and potentially changing perceptions as may be warranted, needs to start with the SPT, for perceptions cannot be taught (Ernest, 2000, p7) and as such lie in the hands of the students themselves.

With regard to improving mathematics teaching in primary schools, it is suggested that negative perceptions can be challenged (Uusimaki and Nason, 2004), that "adults can get over a negative disposition towards maths" (Pound and Lee, 2011, p16), that mathematics anxiety is learned and as such can be unlearned, (Ashcraft and Kirk, 2001) and that SPTs' concerns regarding mathematics can be improved during ITT (Hopkins, Pope and Pepperell, 2004). However, change is always difficult, not least changing people's ingrained beliefs, but it is argued here that being aware of others' perceptions is an effective starting point for considering one's own and heightening awareness in such a way is supported through existing research (Houssart, 2009). Mathematical perceptions can be challenged (Edwards, 1996) but first need to be identified which is a process entirely personal and unique to an individual who is the one "with the capability to influence their environment and determine their own actions" (Christou, Phillipou and Menon, 2001, p44). Although students' perceptions of mathematics are a result of a lifetime of experience and may be difficult to change (Liljedahl, 2005) it is a worthwhile process to engage with in order for the individual student to determine their personal philosophy of mathematics to be the best teacher they can be, for as Pound and Lee (2011, p16) suggest, "we owe it to them (children) to do all that we can…to develop our own enthusiasm and, in the process, our expertise."

However, that process of development is far from straightforward since mathematical perceptions are "the indirect outcome of a student's experience of learning mathematics over a number of years" (Ernest, 2000, p7), developed over considerable time, "implicitly and unintentionally" (Bishop, 1991, p195) with individuals perhaps unaware of them since they may never have given them any conscious consideration (Ambrose, 2004). It is argued here that identification of variation in the range of

SPTs' mathematical perceptions can enable reflection and facilitate the opportunity for SPTs to "examine these beliefs and consider their implications" (Schuck, 2002, p335). Such a reflective process may confirm their perceptions of mathematics as being valid and worthwhile, expand awareness and gain understanding (Valderrama, 2008) and provide an opportunity for critical reflection leading to change. Whilst it is recognised that there can be "great psychological difficulty for teachers of accommodating (restructuring) their existing and long standing schemas" (Skemp, 1978, p13), according to Ernest (1989, p249), "teaching reforms cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change."

Hence, it is acknowledged that challenging and potentially changing established perceptions is problematic for "beliefs tend to be highly resistant to change" (Cross, 2009, p327), being considered particularly painful with regard to the teaching of mathematics (Clarke, 1994). There appears to have been little engagement with philosophising about primary school mathematics in terms of teachers questioning what their views are of mathematics and what they are teaching (Bibby, 2002b) and their associated perceptions of mathematics (De Corte, Op 't Eynde and Verschaffel, 2002). Perhaps the lack of research into eliciting mathematical perceptions is due to the difficulty that lies in capturing something so elusive (Hofer and Pintrich, 2002). Wilkins (2008) suggests that large scale studies are needed to advance the theoretical perspective, as, according to Adler et al (2005) most recent research in this area has been qualitative in nature. However, since perceptions rely on interpretation of mathematical experiences (McLeod, 1992), including affective considerations such as attitudes (Liljedahl, 2005), a qualitative approach was considered most apt for this study.

Although their focus was on HE statistics, Petocz and Reid's (2005) phenomenographic work exploring conceptions of the subject and associated learning is of interest here. Their findings suggest that students were unaware of variations in thinking about the nature of the subject, learning the subject and using it professionally and that they tended "to assume that their fellow students think in the same way that they do" (Petocz and Reid, 2005, p798). They recommended that students' awareness of the range of variation could be a useful first step in assisting development of a broader view. Hence, for the purpose of this study, ascertaining a phenomenographic outcome space of SPTs' perceptions of mathematics with regard to their perceptions of mathematics based on past experiences was considered a prospective tool to facilitate SPT reflection on a range of variation of mathematical perceptions in order to evaluate their own perspectives, identify their personal aspirations for teaching primary mathematics and establish their associated learning needs for ITT.

The phenomenographic approach is a methodology that embraces the perspective of there being a relation between a phenomenon being experienced (in this case mathematics) and the individual experiencer (the mathematician), resulting in meaning of the phenomenon. Based on a non-dualist conceptual framework that mathematics is a construct whereby humans attempt to understand and describe their world through using previous and current experience of phenomena to make mathematical sense or meaning, a phenomenographic approach lends itself to exploration of the perceptions SPTs bring to an ITT course from their previous experience of learning mathematics. The result of phenomenographic method in this case is an outcome space of categories of description of the range of variation of mathematical perceptions amongst SPTs. By providing a hierarchical framework of mathematical perceptions in this way, SPTs can be enabled to form their own mathematics philosophy by engaging with the range of variation arising from a group of peers, make explicit their own mathematical perceptions, make comparisons, identify aspirations and plan what is needed from their ITT learning to reach their goals for teaching primary mathematics. There is certainly support for further research into this domain (Grootenboer, 2008) if students' reflection is facilitated and potential change is enabled in relation to the impact perceptions have on teaching (Cross, 2009), as "research on teacher beliefs, although fraught with pitfalls to avoid and difficulties to surmount, has great potential to inform educational research and practice and is therefore worth the effort" (Leatham, 2006, p91).

The premise of this study is that, in order to teach mathematics effectively and be in a position to stimulate learners of mathematics, mathematical and related pedagogical understanding is dependent on SPTs' own perceptions of mathematics. This study, therefore, argues that SPTs at the outset of ITT need to establish a mathematical philosophy whereby they ascertain what mathematics means to them, determine their understanding of mathematics, acknowledge their attitudes towards mathematics and consider their intentions regarding the way it should be taught, so that their perceptions are identified in a way that can be acted upon during and beyond their ITT in terms of developing effective practice.

2.10 Summary

In summary, this review raises concerns regarding the quality of primary mathematics education. SPTs embark upon ITT amidst a sea of political change, altered educational guidance and a wealth of conflicting theoretical perspectives regarding the learning and teaching of mathematics in the primary school.

This study is based on a non-dualist relativist conceptual framework whereby understanding is believed to be constructed by an individual's relationship with mathematics, as opposed to mathematics and learner being seen as separate entities. From this conception of mathematics being a human construction, created through active, creative learning as opposed to transmission of a body of facts, rules and procedures, it is argued that perceptions of mathematics are formed through learners' past experiences of mathematical engagement, although they may not be consciously held.

It is posited that these perceptions affect attitudes towards, understanding of and engagement with mathematics and result in potential implications for SPTs' learning in ITT which in turn has an impact on their future practice within primary mathematics education. It is argued that reflection is needed in order for SPTs to make their mathematical perceptions explicit, identify their aspirations for their future practice as teachers of primary mathematics, ascertain their personal philosophy for mathematics and determine their associated learning needs within their ITT course.

Phenomenography, as a non-dualist methodology – outlined in more detail in the next chapter - is chosen to explore these issues further, by ascertaining SPTs' mathematical perceptions at the outset of ITT and enabling exploration of related potential implications for possible change in the interests of becoming the best primary mathematics education practitioners they can be. Hence, this study, in accordance with the research questions above, seeks to establish the range of variation of perceptions of mathematics amongst student primary teachers at the outset of Initial Teacher Training, determine how that range relates to primary mathematics pedagogy, ascertain the potential implications for student primary teachers' development within ITT and consider how potential implications might be addressed.

Chapter 3 Methodological Approach

3.1 Introduction

This study is set in the context of primary mathematics education in Britain and student primary teachers' (SPTs) learning in Initial Teacher Training (ITT). It seeks to determine the range of variation of perceptions of mathematics amongst SPTs at the outset of ITT, consider how this range relates to primary mathematics pedagogy, explore potential implications for student primary teachers' development within ITT and ascertain how potential implications might be addressed. A phenomenographic approach was chosen for this study due to the qualitative nature of exploring SPTs' mathematical experiences and perceptions, the methodology's non-dualist stance in focusing on the relational aspect of constructing mathematical understanding through experience and its potential for provision of a hierarchical outcome space formed of categories of description of mathematical perceptions. In this chapter, a detailed rationale for this choice of methodology is provided.

3.2 Relationality

In order to ascertain SPTs' mathematical perceptions arising from their past experience, a phenomenographic approach is taken. As established in Chapter 2, SPTs' mathematical perceptions encompass the nature of mathematics, attitudes, understanding, intentions with regard to teaching mathematics and aspirations for development that is needed through ITT. Exploration of these interpretative aspects of mathematics warrants a qualitative approach and one whereby examination of a group (Dunkin, 2000) is facilitated. Phenomenography provides this, alongside the means of determining different understandings of the phenomena (Marton, 1986) of mathematics as experienced by SPTs, forming a continual relationality, as illustrated in Figure 3.1:

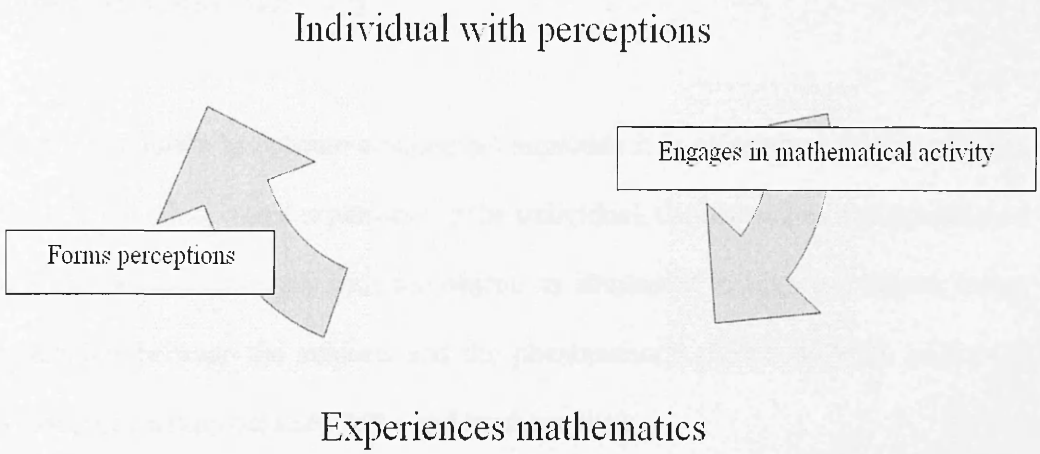

Figure 3.1 Relationality – Perceptions And Experience

Since the study is concerned with SPTs' perceptions, a phenomenographic approach was chosen as one which can be used "to uncover the individuals' own views of an aspect of the world or how they function within that world" (Dall'Alba, 2000, p95) and "describe an aspect of the world as it appears to the individual" (Marton, 1986, p33). The non-dualist methodology of phenomenography was necessary for the focus of this study whereby the relationship between the object [mathematics], and the subject [the person engaging in mathematical activity] are not considered separate (Marton, 2000), since the focus is the relational aspect between mathematics and student through a "non-dualist view of human cognition that depicts experience as the internal relationship between human and the world" (Pang, 2003, p147). The intention is not to provide reasons for mathematical perceptions but to concentrate on the relation between the experiencer and the phenomenon (Marton and Booth, 1997). Thus, rather than taking a first order perspective of describing the world of mathematics (Marton, 1981), this study takes a phenomenographical second order approach via exploration of experienced mathematics "based on a relational view of the world" (Bowden, 2005, p11).

Within this non-dualist phenomenographic approach it is recognised that in addition to the internal relation that is personal to the individual, the researcher's perspective as interpreter is also involved, with the object, as illustrated in Figure 2 below, being "the relation between the subjects and the phenomenon" (Bowden, 2005, p12) – in this case, the relation between SPTs and mathematics:

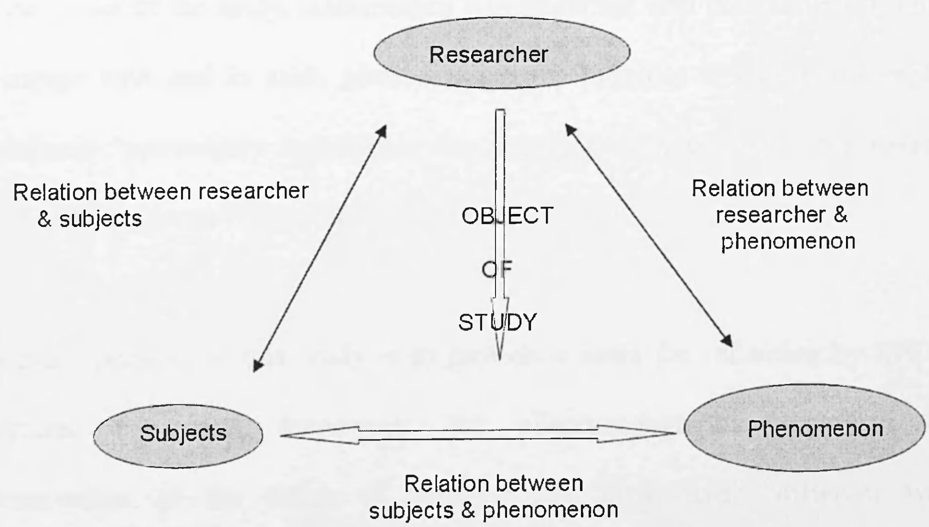

Figure 3.2: Phenomenographic Relationality (Bowden, 2005, p13)

Seeking to ascertain mathematical perceptions arising from past experience is not straightforward, not least because they are not necessarily part of conscious thought (Cross, 2009), neither can they be observed directly (Rokeach, 1968). Hence, the phenomenographical approach taken enables SPTs' descriptions to be elicited in terms of "the relation between an individual's prior experience and their perceptions of the situation" (Trigwell and Prosser, 2004, p410). These perceptions differ (Åkerlind, 2005a) and can be difficult to articulate (MacNab and Payne, 2003), and through the researcher's involvement are reliant on inference (Leder and Forgasz, 2006), but using phenomenography "a perspective in which the person perceiving and his/her conceptions of the world are integrated" (Säljö, 1997, p174) is facilitated.

3.3 Exploration Of Experience

At the outset of the study, mathematics was identified as a potentially difficult arena to engage with and as such, phenomenography provides a vehicle for exploration considered "particularly appropriate for engaging with complex, controversial or deeply held issues or viewpoints" (Cherry, 2005, p62).

Since the purpose of this study is to provide a basis for reflection by SPTs via a structured theoretical framework, the phenomenographic approach enables determination of the range of variation of "qualitatively different ways of experiencing" (Linder and Marshall, 2003, p272), providing the means to "move up conceptually" (Green, 2005, p35) through analysis beyond individual experience and contexts (Green, 2005) to form a structured and hierarchical outcome space to form a reflective tool for SPTs embarking on ITT to ascertain their personal mathematical philosophies and identify their learning needs to develop as necessary through ITT and beyond.

Existing research into British SPTs' perceptions of mathematics at the outset of ITT linked to their personal mathematical philosophy is not apparent. However, Petocz and Reid's (2005) phenomenographic study of HE statistics provided insight into determining variation from fragmentary to holistic understanding (Petocz and Reid, 2005, p798), their phenomenographic outcome space providing a range of conceptions which they deemed useful in enabling students to develop a wider outlook – considered worthwhile for this study's determination of SPTs' mathematical perceptions via categorisation of perceptions to provide a "useful tool" (Speer, 2005, p224) for reflection.

3.4 Pragmatic Choices

The intention here was to explore perceptions of mathematics in a typical group of SPTs embarking on ITT to ascertain the range of variation in perceptions in line with what Marton (1986) termed "pure" phenomenography whereby "the qualitatively different ways of understanding a phenomenon or aspect of the world are seen as a main outcome of the research" (Dall'Alba, 2000, p98). However, the study also extends to provide a structure for reflection intended to be an "educational tool to improve teaching and learning" (Åkerlind, 2002) whereby students may clarify their personal mathematical philosophy and identify their associated ITT learning needs and hence potentially "facilitate the transition from one way of thinking to a qualitatively 'better' perception of reality" (Marton, 1986, p33). Hence, whilst phenomenography does not claim to provide generalization (Bowden, 2005), this study does constitute 'developmental' phenomenography (Bowden, 2000a) with a "pragmatic perspective" (Bowden, 2000a, p16) and the intention to provide a practical outcome (Green, 2005).

It is worth noting other considerations leading to the choice of phenomenographic methodology for this study. Since outcomes of reflection were not part of the research, action research and practitioner research methodologies were ruled out. Grounded theory was mooted, but its emergent focus (Glaser and Strauss, 1967) did not suit the intended approach to specifically determine SPTs' perceptions. Phenomenology was also considered, an approach which also examines experience, but which tends towards the individual perspective, as opposed to the intention herein to reach a collective meaning (Barnard, McCosker, and Gerber, 1999). Whilst phenomenological and ethnographic approaches might fit the relational aspect of this context, phenomenography allowed "mapping qualitatively different conceptions" (Dall'Alba, 2000, p97) to form a hierarchical structure to use as a reflective tool.

Phenomenography was therefore a considered choice, including acknowledgement of the argument that analysis should be carried out as a team (Bowden and Green, 2005; Walsh, 2000). Phenomenographic analysis involves the formulation of categories of description arising from transcripts of interviews, and to be of worth, Marton (1986, p35) argues that "it must be possible to reach a high degree of intersubjective agreement concerning their presence or absence if other researchers are able to use them." To work within a team was neither logistical nor welcomed as this research was an individual project for a doctoral thesis, but the individual approach taken is supported by Green (2005) and Åkerlind (2005a, p70) who confirm that "an individual researcher can....make a substantial contribution to our understanding of a phenomenon."

Notwithstanding this confirmation, it was recognised that independent phenomenographic study warranted a critical and rigorous approach (Åkerlind, Bowden and Green, 2005), especially since there has been criticism of phenomenographic method (Ashworth and Lucas, 2000) largely due to a lack of recording of the actual process. Also, phenomenography, based on SPTs' descriptions of their experiences and interpretation of their perceptions, may be open to critics who argue that qualitative research may be unreliable and invalid (Kvale, 1996). Whilst it is suggested that "there are different ways of going about the process" (Cherry, 2005, p60), to ensure a rigorous approach, decisions and careful plans were made from the outset (Green, 2005, p45) to uphold the study's validity and reliability, and avoid any potential criticism for a lack of theoretical background (Marton and Tsui, 2004).

3.5 Pilot Study

Interviews are the most common method of obtaining phenomenographic data (Walsh, 2000) and deemed in this study to give the richest means by which students' perceptions could be explored via their accounts of experience. A pilot study was carried out with a group of final year undergraduate students since they were accessible and "similar to the intended interview sample" (Bowden, 2005, p19) in that they were ITT students, though these were at the end of, rather than the beginning of their course. Since it was the questions being piloted, rather than the responses, this proved a useful sample to ascertain that the questions were sufficiently open-ended for the purpose. Small scale analysis was used to ascertain any modification of questions, which was minimal, and the pilot interviews were then "discarded and not used as part of the research study" as advocated by Bowden (2005, p19).

3.6 Participants

SPTs new to ITT were invited to participate in the actual study since it was perceptions of mathematics prior to beginning ITT that were to be examined. Thirty-seven post-graduate SPTs subsequently took part as follows:

Reason for inclusion in study	Number of SPTs
Post-graduate SPTs whose ITT places were confirmed in July were approached, affording time for interviews to take place prior to ITT courses beginning in September	200
SPTs who subsequently expressed an interest in participation	50
SPTs subsequently available for interview between July and September	40
SPTs who subsequently gave written consent for their involvement	38
Interviews that were subsequently untainted by researcher discussion that may have affected SPT's responses	37

Table 3.1 Research Participants

The sample of thirty-seven SPTs formed a cross-section of ages, gender, cultures, degree specialisms, previous occupations and ITT institutions, and hence maximised the likelihood of variation in perceptions being determined. Bowden (2005, p17) advises that enough interviews need to be undertaken "to ensure sufficient variation in ways of seeing but not so many that make it difficult to manage the data", with Trigwell (2000) suggesting that between ten and twenty interviews are sufficient for analysis. Despite being aware of the wealth of data from interview analysis (Åkerlind, 2005a) and that "the limiting factor at the upper end is the volume of data produced" (Trigwell, 2000, p66), the number of participants was deemed manageable and all thirty-seven were included in the study.

3.7 Interviews

Interviews consisted of semi-structured questions that were open-ended so that responses were more likely to refer to the participant's perceptions relevant to their experience, and "designed to be diagnostic, to reveal the different ways of understanding the phenomenon" (Bowden, 2000a, p8). These were used to invite SPTs' responses regarding the nature of mathematics, mathematical understanding, attitudes towards mathematics, intentions regarding the way it should be taught, and their expectations of their forthcoming ITT. This method was chosen to allow scope for interviewees to provide data "through their own discourse" (Tan and Prosser, 2004, p269) and "uncover their lived world" (Kvale, 1996, p1). SPTs were asked to describe their experiences of mathematics in previous educational forums and in everyday life and were encouraged to share their thoughts about mathematics and hence "express their qualitative understanding of the phenomenon under investigation" (Bowden, 2000a, p10).

In order to explore SPTs' experiences, initial questions were used as triggers (Trigwell, 2006) and subsequent prompts such as *Why?* and *How?* were used to encourage elaboration on some aspects, but additional questions were not used in order to avoid potentially leading responses (Green, 2005) and for the SPTs' perspective to remain the central focus (Dall'Alba, 2000). To maintain the flow of the individual discourse, the semi-structured questions were not necessarily asked in the same order each time, but were used for each SPT to ensure consistency in that "the interviewees are all talking about the same phenomenon" (Åkerlind, 2005b, p113). Interviews continued until the position was reached where the experience and perceptions had been described (Trigwell, 2006).

3.8 Phenomenographic 'Bracketing'

A second order perspective was maintained throughout this study, an essential element of phenomenography, from the setting of the research focus, through the whole process to the reaching of conclusions, in that the emphasis was on trying to see mathematics through the SPTs' eyes. As described by Marton and Booth (1997, p121), "at every stage of the phenomenographic project the researcher has to step back consciously from her own experience of the phenomenon and use it only to illustrate the ways in which others are talking of it, handling it, experiencing it, and understanding it." This is not straightforward (Prosser, 2000), and is an aspect open to criticism by those who claim it to be impossible to set aside one's own assumptions and preconceptions in order to remain open and unbiased to others' descriptions and reach an understanding of what they say (Ashworth and Lucas, 2000).

The requirement of phenomenography is to 'bracket' via the setting aside of presupposed notions, but "the question is whether this is possible" (Dunkin, 2000, p147), because the researcher is "acquainted with the subject matter in question" (Säljö, 1988, p41) and there is a danger of making immediate analysis of interviewees' responses (Ashworth and Lucas, 1998). It is inevitable, and fully recognised, that one has personal views and thoughts on the phenomenon, as well as some relationship to the SPTs, and so a conscious decision was made to focus on their descriptions and not impose researcher views within the interviews. A brief outline of interest in SPTs' perceptions of mathematics at the outset of ITT was necessary when introducing the research to potential participants, but beyond that, personal perspectives were not shared once they had indicated an interest in taking part in interviews. Instead, a strong focus was maintained on "trying to understand the meaning of the phenomenon for the interviewee" (Åkerlind, 2005b, p108) and focusing on the individual experience.

3.9 Ethical Considerations

The ethical guidelines of both the university where I work and my doctoral university were implemented throughout the research. It was made clear at the outset that no SPTs were under any obligation to take part in the research and were free not to take up the invitation. I ensured that these were not SPTs I would be teaching so that they did not feel under any pressure to appease their tutor (Richardson, 1999). Their anonymity was assured (Cohen, Manion and Morrison, 2000, p279), the nature and purpose of the research was shared with all (McNiff, Lomax and Whitehead, 1996), and the use of the data was explained, as advocated by Bell (1999), but this was kept brief in order to avoid any potential influence.

Signed consent for participation was received and permission given for interviews to be recorded and, although it was recognised that this might cause constraint upon the interviewee (Cohen et al, 2000, p281), as an alternative to note taking it helped to maintain the flow of dialogue and clarity of responses, as well as enabling time to be kept to a minimum. Interview transcripts were provided for those SPTs who requested these, together with a second consent form for all SPTs having been given further time to consider their data being used, as it was deemed important "to ensure that the interviewees feel comfortable and that their willingness to co-operate is never abused" (Bowden, 2005, p31).

The interviews were arranged at a date, time and place convenient to participants (Green, 2005, p39) in "a comfortable and relaxed atmosphere that would encourage frank discussion" (Åkerlind, 2005b, p106) and they were reminded of the confidential nature of the process. Potential "eagerness of the respondent to please the interviewer" (Borg, 1981, p87) was considered with the interviewer taking responsibility for overcoming "the problems of the likely asymmetries of power in the interview" (Cohen et al, 2000, p279) and ensuring no imposition on my part (Ashworth and Lucas, 1998).

Time was given and silences accepted in order to allow thinking time (McNiff et al, 1996) and SPTs were assured that they could decline to answer any of the questions, since I was mindful that mathematics can be an emotive subject for some and it was important that there was trust between the interviewee and interviewer, especially since, according to Åkerlind (2005b, p115) "phenomenographic interviews are potentially uncomfortable for interviewees, in that they invite them to reflect deeply and attempt to integrate issues that they have often not reflected on or attempted to integrate before."

3.10 First Stage Of Analysis

The interviews were analysed using a phenomenographic approach. SPTs were asked in interviews to describe what mathematics means to them, their experiences of learning mathematics, how they felt about mathematics, how they think it should be taught and what their expectations were of their forthcoming ITT course, providing a source of data to analyse into phenomenographical categories of description that, according to Marton (1986, p33) was *content oriented* (mathematics), *relational* (between mathematics and the students), *experiential* (based on students' past mathematical experiences) and *qualitative* (based on the students' descriptions of their experiences and perceptions).

Each was transcribed verbatim (Trigwell, 2006), the laborious and time-consuming nature (Marton, 1986) of this being avoided via use of a transcriber, with consent given by interviewees, based on assurance of confidentiality. Tapes were coded so that only the researcher knew the SPTs' identity, none of the SPTs were known to the transcriber, tapes were deleted once interviews were transcribed, and transcriptions were deleted from the transcriber's computer once passed to the researcher. Transcriptions were completed immediately after the interviews so that they could be quickly checked against the recordings whilst fresh in the researcher's mind, errors amended, "tainted data" (Green, 2005, p40) omitted and transcripts then provided for interviewees to check as required.

Transcripts were read and re-read to gain a sense, within context, of an overview of what SPTs were describing (Dunkin, 2000). Phenomenographic researchers use different analytical processes (Walsh, 2000) including the use of whole transcripts and large excerpts, claiming that use of smaller excerpts run the danger of using quantitative frequency of responses being assigned to categories or what Prosser (2000, p45) terms a "shopping basket" of conceptions. Care was taken in this study to use a qualitative mapping process, with exact wording used so that "the concepts and terminologies of the interviewees speak for themselves" (Barnacle, 2005, p49). Excerpts were identified that represented particular meanings, as suggested by Marton (1986), and coded for reference, with whole transcripts continually revisited to check context and meaning, as the excerpts began to be categorised in terms of qualitative similarities and differences. At this stage, as Cherry (2005, p61) explains "I take the people out of the equation, and think of the categories as constructs in their own right." Thus, individual responses from transcripts formed categories that described meaning from the whole set, a process supported by Marton and Booth (1997).

3.11 Second Stage Of Analysis

The formation of relational links between categories involved judgements based both on the empirical evidence of the data, and, inevitably, on logical decisions made by the researcher, for it is recognised that "the categories don't exist independently of the person who's doing the analysis" (Walsh, 2000, p22) since analysis is dependent on the researcher's background, knowledge and ideas. However, central to phenomenographic rigour, the data were <u>not</u> used to fit pre-existing themes (Barnacle, 2005). To do so, it was necessary to be aware of one's own ideas in order to "actively challenge" (Patrick, 2000, p133) any expectations one might have, with a conscious effort made to remain as objective as possible and true to the data, without pre-determining categories in advance of the analysis (Marton, 2000).

Although Walsh (2000, p21) indicates that "some see it as valid to omit some categories that emerge from the data and to discount others" and that "if the data is found not to conform to predetermined logical relationship, then it is discarded" (Walsh, 2000, p26), every piece of data was included in the analysis and contained within a category of description, even if only one individual had provided that data. Hence, as advocated by Åkerlind (2005b), all data were used in formation of the categories – the categories being formed from the data as opposed to data being 'fitted in' to categories. However, limited use of data excerpts was included in the presentation of findings (see Chapter 4) due to the volume of data and word limitation of this thesis.

Categories were revised and changed as the transcripts were re-read and contextual meaning confirmed through a long period of iteration (Dall'Alba, 2000). As Bowden (2005, p26) observes, analysis began with an "overwhelming number of variations" but through a process involving sorting and defining (Marton, 1986), ways of experiencing mathematics that qualitatively differed from others (Åkerlind, Bowden and Green, 2005, p82) was eventually established.

3.12 Formation Of Phenomenographic Outcome Space

Neither the SPTs themselves, nor the actual experience, were analysed, perceptions being interpreted from the interview data to form the phenomenographic 'categories of description.' Transcripts thus provided "pools of meaning across individuals" (Green, 2005, p39) and were analysed to determine the "variation in the range of experience across the whole set" (Bradbeer, Healey and Kneale, 2004, p19).

Analysis resulted in four categories of description. Whilst there are suggestions of the number of categories usual in phenomenographical research (Marton and Booth, 1997) an open mind was kept throughout the analytical process with the number of categories being wholly dependent on the data.

To form a framework for SPT reflection on clarifying a personal mathematical philosophy, consideration was given to structuring the categories of description into a hierarchy, an aspect of phenomenography that enables learning development (Marton and Säljö, 1976). Whilst a hierarchically structured outcome space is not a phenomenographic essential (Green, 2005), it is a recognised part of phenomenographic method (Marton and Booth, 1997), the rationale here being to show structure in the variation, including an overview, key aspects and relationships between them (Prosser, Martin, Trigwell, Ramsden and Lueckenhausen, 2005). Hence, whilst categories were refined to constitute qualitatively different meanings, a logical structure between the categories was also identified (Åkerlind, 2005b). Analysis in this case involved both the empirical evidence from the SPTs' accounts of mathematical experiences, and logical support in constituting relationships between the categories, as supported by Åkerlind (2005b). As with formation of categories not being pre-determined, neither were the structural relationships between them (Åkerlind, Bowden and Green, 2005), with the focus remaining throughout the analytical process on the "relation between the subject and the phenomenon" (Bowden, 2005, p16) with avoidance of imposing my own ideas (Ashworth and Lucas, 2000) and a desire to "stay as faithful as possible to the individual's conceptions but without being claimed to be equivalent to them" (Bowden, 2000a, p16) in forming draft categories of description first before considering links between them (Bowden, 2005).

3.13 Reliability

Reliability in a positivist sense, based as it is on accuracy of data collection and concept measurement (Henn, Weinstein and Foard, 2009), is neither possible nor logical for a qualitative study such as this. Phenomenographic method is not based on the premise that categories are out there awaiting discovery (Bernard and Ryan, 2010) as the process is far more interpretative, with a social construction of reality based on the relational aspect between the phenomenon, the experiencer and the researcher whose role "constitutes an integral part of the findings themselves" (Wilson, 2009b, p116). Reliability for this study hence lies in the rigour of the process undertaken that is transparently accounted for in this chapter and the way in which the questions posed are subsequently answered (Denscombe, 2002). The method was prevented from influencing the results by trustworthy adherence to the interviewees' responses as recorded and quoted verbatim and kept in the context of the meaning of whole transcripts, with every attempt made to avoid researcher subjectivity by avoidance of leading questions or two-way discussion during the interviews, which in turn followed the same semi-structured questions that were justified through theoretical review (2009). There is no claim for reliable repetition, but this is defended in such a qualitative study (Richards, 2009) since it would be impossible to replicate the same interviews with the same interviewees since their perceptions will necessarily have changed with interim experience.

3.14 Validity

Similarly, validity is upheld both via the reliability of the data, the transparent process of examination of the data and the conclusions drawn that match the intentions of the study (Henn, Weinstein and Foard, 2009) which is soundly constructed of logical progression (Richards, 2009) and documented here. The researcher's preconceptions were limited to the choice of questions asked, enabling interviewees to speak for themselves in describing their experience and expressing their perceptions freely. The subsequent analytical process is considered valid in interpretative research whereby "the object of the research is...understood through the consciousness of the researcher" (Oliver, 2010, p73) as the findings do result in achieving the aims set. Positivist external validity is not appropriate for a context-bound qualitative study such as this (Wilson, 2009b) which does not set out to generalise, but its internal validity is trustworthy in that the findings are credible and true to the process undertaken and outlined.

3.15 Summary

The aim of this study was to determine the range of variation in SPTs' perceptions of mathematics at the outset of their ITT courses, and in so doing, phenomenography was used to represent the range of meanings of students from the range of a sample (Åkerlind, 2005b). The phenomenographic analysis used all the transcriptional data to produce categories of description and an outcome space that did not reflect individual perceptions, but used all data as a pool of meaning to ascertain the range of variation for the group. For the group of SPTs involved, the categories do not claim to be equivalent to any individual SPT (Bowden, 2000a) and there is no expectation that any one category will link specifically to a transcript, and vice versa, since phenomenography seeks to "capture the range of views present within a group, collectively, not the range of views of individuals within a group" (Åkerlind, 2005b, p118). No individual SPT would expect their perceptions to match a single category

(Barnacle, 2005) as the categories of description are "compositions, formed out of an aggregate of similar perceptions" (Barnacle, 2005, p50) incorporating "key elements from the statements of a number of people" (Cherry, 2005, p57). The findings were therefore not checked by participants as these move beyond individual meaning (Green, 2005) and to do so would affect reliability, since returning to interviewees would change the data source by introducing new ideas.

The purpose of this study was to form a theoretical framework that could facilitate SPT reflection in forming personal mathematical philosophy. A phenomenographic approach, according to Åkerlind (2005a) can be useful in providing practical application in such a way. Whilst it is recognised that "any outcome space is inevitably partial, with respect to the hypothetically complete range of ways of experiencing a phenomenon" (Åkerlind, 2005a, p70), this research presents categories of description which represent the qualitatively different perceptions amongst SPTs at the outset of ITT, and therefore information for similar SPTs to apply to their personal situations. Different perceptions of mathematics will exist for different SPTs under different circumstances, but this study's outcome space provides information for engagement by all SPTs since it provides an holistic perspective on collective experience, and "the presentation of categories constructed through the phenomenographic process could act as a powerful trigger for such meta-reflection" (Cherry, 2005, p59) by facilitating SPTs to engage in thinking about their mathematical philosophy.

It is not suggested that SPTs move through hierarchical categories of description, since "the categories are constituted from self reports of a group of people, a bit like a snapshot of that group at a particular time" (Trigwell, 2000, p80). However, the information is presented in order that SPTs can reflect on their own circumstances and where they aspire to be, for, as Åkerlind (2005a, p72) suggests, "the aim is to describe variation in experience in a way that is useful and meaningful, providing insight into what would be required for individuals to move from less powerful to more powerful ways of understanding a phenomenon."

To identify the reasons for SPTs' perceptions, or ways of changing them, was not the purpose for this research and is not a phenomenographical aim (Trigwell and Prosser, 1997), and hence there was no speculation "about the motives, intentions, mind-maps – and 'journeys' – that might have produced the data" (Cherry, 2005, p60). However, phenomenography can be "a mechanism to facilitate conceptual change" (Trigwell, 2000, p80) and so the outcome space is discussed to explore potential implications for SPTs' engagement in relation to their learning for ITT, for such a "focus on critical aspects of and structural relationship between different ways of understanding a phenomenon is seen as having powerful heuristic value in aiding insights into teaching and learning" (Åkerlind, 2004, p365).

A detailed account of the categories of description, with evidence from transcriptional data is provided in Chapter 4, together with the relationships among the categories presented by a matrix showing the categories and their relation to each other, followed by detailed discussion in Chapters 5 and 6 that links the findings with mathematical pedagogy associated with differing perceptions of mathematics, SPTs' development within ITT and how potential implications that arise might be addressed.

Chapter 4 Findings

4.1 Introduction

This chapter addresses the first of the research questions for this study:

What is the range of variation of perceptions of mathematics amongst student primary teachers at the outset of Initial Teacher Training?

In accordance with the methodology outlined in Chapter 3, student primary teachers' accounts of their experiences of mathematics are analysed phenomenographically and the resulting outcome space is presented in this chapter. This is structured developmentally so that, in conjunction with the theoretical aspects considered in the next two chapters, SPTs are afforded the opportunity to reflect on these findings in relation to their own learning for ITT.

From an ontological perspective of the way SPTs relate to mathematics, their unique and individual data is used to demonstrate the range of variation in the ways in which mathematics is perceived across the group. It must be stressed that the categories within the hierarchical framework described do not depict any individual student, as the pooled data set was used to determine the whole range. In particular, it must be recognised that all categories of the hierarchy arise from SPTs' past mathematical experiences and, except where expressly stated, are not assumed to equate to their aspirations for their own practice. Despite confidentiality being upheld so that readers are unable to identify individuals, the exception is the participant him/herself, who may be able to identify his/her responses and associated coding. It is, therefore, crucial to emphasise that the inclusion of a response in lower categories is no indication of criticism of the individual. Far from any unintended disrespect, considering the extent of negative emotion associated with mathematics that was shared, individual SPTs are to be admired in their determination to overcome difficulties with mathematics to achieve their goal of becoming primary teachers.

Whilst individual SPTs are not contained in any one category, the range of variation is intended to provide a basis for reflection by SPTs in order to ascertain their own perceptions of mathematics from the potential perceptions of others and, whilst this chapter presents the findings in relation to the first research question for this study, it also enables discussion of pedagogical links (see Chapter 5) and associated potential implications for SPTs' development within ITT (see Chapter 6).

The phenomenographic analysis of SPTs' responses illustrates qualitative differences in structure between mathematical knowledge (for example, of recalled mathematical facts, rules and procedures) and mathematical understanding (for example, of how to work out unknown facts and justify known facts; and of origins of rules and why procedures work). These two structural elements of the outcome space refer to qualitatively different relationships with mathematics that are described, ranging from external relationship with mathematics as learners, internal relationship with mathematics as learners, to teaching mathematics from the perspective of learners' internal relationship with mathematics.

Four qualitatively different ways of describing SPTs' perceptions of mathematics at the outset of Initial Teacher Education are presented in this chapter via an outcome space matrix showing the referential and structural aspects of the categories of ways of describing the perceptions, a brief summary of the hierarchy of categories, excerpts from the group data set and detailed outlines of each category of description, and a summary of the qualitative differences between the categories.

4.2 Coding References

References to excerpts include a coded interviewee transcript number followed by coded transcript excerpt number as follows:

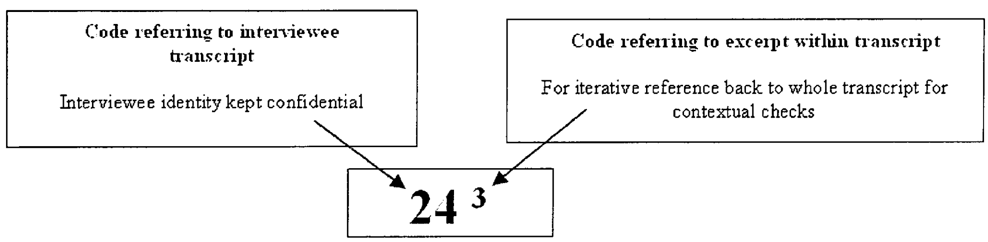

Figure 4.1 Coding References For Transcript Excerpts

4.3 Outcome Space Showing Range Of Variation In Student Primary Teachers' Perceptions Of Mathematics At The Outset Of Initial Teacher Training

Four qualitatively different ways of describing SPTs' perceptions of mathematics at the outset of Initial Teacher Training are presented hierachically as follows:

Student Primary Teachers' Perceptions of Mathematics at the Outset of Initial Teacher Training		REFERENTIAL		
		External relationship with learning mathematics	**Internal relationship with learning mathematics**	**Internal relationship with teaching mathematics**
STRUCTURAL	Knowledge	1	2	
	Understanding		3	4

Figure 4.2 Phenomenographic Outcome Space – Range Of Variation Of SPTs' Perceptions Of Mathematics At The Outset Of ITT

4.3.1 Category Of Description 1:

Mathematics - Knowledge Learned From External Relationship

SPTs' descriptions of their mathematical experiences within this category are consistent with mathematics being externally imposed, by transference to passive learners. The perception is that learners are taught with little evidence of gaining mathematical knowledge beyond recall of memorised numeric facts and that mathematics is an entity to be feared and avoided wherever possible.

4.3.2 Category Of Description 2:

Mathematics - Knowledge Learned From Internal Relationship

Within this category of description, as in the previous, SPTs' descriptions are of gaining mathematical knowledge, with a qualitative difference of teacher-given methods and attempts at forming internal relationship with mathematics through individual practice and working through schemes. As such, mathematical knowledge is demonstrated sufficient to know how to follow a given method to reach required answers, alongside learners' awareness of the limitations of their learning which lacks depth of understanding. As in the previous category, mathematics is perceived as an entity separate to the learner, qualitatively differentiated by the inclusion of given methods and rules as well as facts to be memorised, alongside some individual and internal relational learning. Rather than giving up in the face of mathematical adversity, frustration is described of the apparent inability to understand mathematics, although it is not avoided and there is the desire to achieve.

4.3.3 Category Of Description 3:

Mathematics - Understanding Learned From Internal Relationship

This category describes a focus on learners' internal relationship with mathematics, but is qualitatively different from Category 2 in that SPTs' descriptions in this category focus on and evidence experiences of development of mathematical understanding, including the use and application of mathematics in life, the importance of mathematical process, and notions of mathematics being elusive due to its perceived structured, scientific nature. Varying degrees of confidence and a desire for improvement through ITT learning are described, with a qualitatively differentiated view of mathematics constituting a mixture of a separate scientific and structured entity constituted in given curriculum content to be learnt, and an internal relative understanding constructed through social, active engagement with phenomena.

4.3.4 Category Of Description 4:

Mathematics - Understanding Taught Through Perspective Of Internal Relationship

This category also constitutes SPTs' descriptions of understanding mathematics through internal relationship, but is qualitatively different in its focus on aspirational intentions for future teaching. Development of mathematical understanding is described with interest and excitement in terms of an internal relative experience facilitated via an active learning approach through creative means based on mathematical process. Although there is awareness of curriculum requirements, the approach to learning is qualitatively different in that mathematics is perceived as a way of thinking that is essential for both understanding and shaping the world and which is intrinsically a source of stimulation and sense of mystery.

4.4 Categories Of Description – Findings and Summaries

4.4.1 Category Of Description 1:

Mathematics - Knowledge Learned From External Relationship

Findings

Within this category, mathematics is described in terms of number (35[11]; 9[4]; 19[5]; 26[7]). Methods used to learn about number are described in terms of transmission via teacher demonstration of procedure (2[14]; 36[7]; 30[100]) and recounted as teacher-led (49[10]). The learning environment is described as individual with pupils seated separately (36[7]) and with talking discouraged (43[8]; 19[15]; 40[15]; 40[16]; 30[17]). In this category, mathematics is recalled as being taught by rote, involving recitation and repetition (6[1]; 9[3]; 47[1]; 43[400]; 43[3]; 30[13]; 30[14]; 22[3]; 43[9]).

There are recollections of this mode of teaching being enjoyable (15[3]) and considered an appropriate learning method:

> *it seems to me that the only way to learn times tables is just to practise them and say them and I do think saying them aloud helps…you need that repetition to help it to sink in* (18[9]).

The teaching approach provided motivation (19[12]), forming a useful aid to memory (4[4]), enabling fluency (19[11]) and creating mental imagery:

> *I needed that repetition - you needed to keep going over it, definitely and it was something I liked. Even now I've got pictures in my head of the times tables, so you know like the 9s that they always add up to make 9* (15[8]).

Accounts describe memorisation of given mathematical knowledge which is not evidenced as extending beyond recall of number nor to a wider mathematical vocabulary, described within this category as a language that is beyond comprehension (29[9]; 13[2]) and *a secret code that I don't understand* (12[3]).

Hence, this category includes the attempted transfer of mathematical knowledge consisting of number, using a transmission mode of teaching requiring memorisation and a vocabulary that is not comprehended. Although there is indication that this method of learning worked, accounts within this category describe a perceived lack of purpose (3[19]; 43[14]; 9[7]; 25[14]; 19[18]; 1[17]; 43[15]; 28[12]; 8[12]; 1[16]; 11[7]; 49[23]; 44[7]; 9[8]; 2[30]) with

queri purpose (3[19]; 43[14]; 9[7]; 25[14]; 19[18]; 1[17]; 43[15]; 28[12]; 8[12]; 1[16]; 11[7]; 49[23]; 44[7]; 9[8]; 2[30]) with

what it is, its value, its relation to the world, maths, I know it has some kind of value to it but what it is? I can't remember (1[5]).

Its purpose is unclear other than as needed for becoming a primary teacher (18[1]) with reasons for learning mathematics in school being questioned (9[1]; 49[68]; 22[2]; 3[4]; 1[6]; 24[4]; 8[7]; 46[2]; 49[7]; 8[2]; 24[3]; 15[2]; 26[4]; 1[3]; 24[18]) and perceived irrelevance linked to difficulties in learning (19[2]; 24[5]; 9[2]):

things that I felt that at the time didn't apply to anything, I struggled with (4[1]).

In accordance with the perception of mathematics being a teacher-led doctrine whereby learners are limited to number work involving recitation of facts and repetition of algorithms, lacking understanding of mathematical vocabulary used and questioning the point of mathematics, included in this category are perceptions of mathematics being difficult (45[17]; 11[1]; 12[4]) and something imposed upon learners that could not be escaped (9[22]; 18[4]; 28[3]; 18[7]; 1[10]; 11[4]; 1[4]; 49[15]; 12[9]). It is regarded as the exclusive domain of people who are different in some way (43[1]), described as *swotty* and *clever* (18[2]), *weird* (49[11]) and *geeky* (6[4]).

These emotions include negative perceptions evoking distress in learners:

I never liked any of my maths lessons. I used to get upset and I used to cry...I just did not like maths at all (34[9]).

Feelings of dislike are recalled (36[12]) which transcend into adulthood (41[4]; 5[5]; 30[9]). Learning mathematics is described as *divisive and daunting* (5[3]) and *really detested*

(3[4]). In this category are descriptions of feeling fear (29[7]; 3[3]; 40[1]; 23[3]; 49[100]; 24[100]) *of not being able to do it, and looking stupid* (5[1]; 5[14]) since not being able to do mathematics invited ridicule (2[22]) and feelings of oppression are described:

> *it's almost shaming that I'm not as good at it as I'd like to be…it's like a big black cloud* (49[4]).

This negative view of mathematics is reinforced within this category by descriptions of pressure placed externally – from parents (43[11]; 1[2]; 1[1]) and peers (46[3]; 12[18]; 2[3]; 46[6]). Additional pressure is indicated by the apparent need to be able to do mathematics neatly (46[16]; 28[13]; 28[14]; 26[17]) and at speed (35[19]; 18[6]; 46[5]; 25[2]; 24[10]; 24[9]). The effect on learners was that *it was all moving around too quickly for me* (2[17]), a need to catch up to others:

> *teacher would put you in front of the class, take it in turns and ask you individually and I'd be still trying to get my head round it…feeling put on the spot in front of everybody and not being able to keep up quite as quickly* (8[4]).

Descriptions in this category include external pressures perceived from teachers with *horrible memories of school* (45[1]) recalled, where experiences of being taught mathematics were *awful* (30[1]). Memories are recounted of a *horrible maths teacher* (29[2]) and being *taught in a horrible way from a blackboard* (45[2]) with learning seen as *very off putting* (13[1]) to the extent that some learners were afraid of their mathematics teacher (2[1]; 2[4]; 3[1]; 23[2]; 15[1]). Memories are recalled of being *devastated* (2[7]), of being taught mathematics as *quite soul destroying* (2[8]), and with pressure applied by teachers leading learners to feel stupid and self conscious (41[200]; 29[3]; 35[1]).

Accounts in this category describe mathematics as dependent on finding answers (8[17]; 22[6]; 21[7]) with a particular emphasis apparent on getting the 'right answer' (8[5]; 15[17]; 15[18]; 49[27]). The notion of *there's a right and wrong answer* (8[29]) is described in this category as adding an element of uncertainty regarding the purpose of mathematics:

> *when I was taught maths many years ago, it was all about getting the right answer, a series of numbers that didn't always seem to make sense and I wasn't sure why I was studying it* (49[500]).

Feelings of embarrassment, humiliation, stupidity and fear are described when the right answer was not found (49[5]; 2[2]; 2[11]; 2[39]), which contributed to learners being put off mathematics (41[8]):

> *what switches me off...you have nowhere to hide with maths..you can either do it or you can't... that's the big scary thing with maths - you either have to get it right or everybody's looking at you. That's for me why maths always has been scary for me, there's no room for error* (24[24]).

Accounts in this category suggest that external pressure affected motivation in mathematical engagement in school (2[5]; 2[6]; 22[1]) and included in this category are the apparent longer-lasting effects on confidence within and beyond school years (23[1]; 35[12]; 21[3]; 5[6]; 8[6]; 1[8]; 36[11]; 23[11]; 2[21]; 5[4]; 34[8]; 49[13]) with the desire expressed to *dream about not ever having to worry about it again* (19[9]). Accounts indicate that external expectations and pressure continue into adulthood (19[1]; 49[6]; 25[4]; 36[3]) with a continued lack of self-esteem with regard to mathematical ability (34[23]; 23[6]; 2[12]; 35[4]; 34[2]; 41[2]; 5[2]; 36[2]; 41[3]):

I had all my confidence knocked out of me, because the teacher strongly indicated that I was behind...it's had a strong effect on me now because I really lack confidence (41[1]).

Within this category are descriptions of physical effects manifested through mathematical engagement including tension (40[12]); bewilderment (24[1]), confusion and an inability to think clearly (23[5]; 23[4]; 40[11]). Descriptions in this category are of learners feeling panic when faced with mathematics (49[9]; 8[3]; 40[7]; 49[8]; 40[6]) and mathematical activity being hindered in that the mind ceases to work (29[8]) with descriptions of the mind going blank (23[9]), the brain switching off (35[8]; 1[7]; 36[6]; 24[7]) and feeling lost (25[1]; 30[3]; 30[5]; 30[4]).

Accounts include feelings of mathematical inability (44[2]; 34[4]; 35[9]; 30[2]; 45[4]; 24[8]) with the perception of being unable to improve since learners have been left behind (30[19]; 24[6]) or they cannot be taught mathematics:

if I come across something I can't do, I panic straight away because I think nobody's going to be able to teach it to me and they're going to get angry at me and I'm going to get upset (36[5]).

There is indication that learning would not be encouraged of some learners since it was considered by teachers to be reserved for those who were *good at maths* (49[7]) and an acceptance of the status quo that they would not therefore be challenged is indicated (19[4]; 1[9]; 12[1]), the approach for some in such circumstances being to give up (40[8]; 30[6]; 19[3]; 40[4]).

In this category are descriptions of approaches taken when faced with mathematics to include procrastination (21[1]), relying on mathematical aids such as calculators (35[20]; 40[13]), getting someone else to do it instead (35[5]; 36[4]; 26[6]; 31[9]; 26[5]; 12[8]), and attempting to disguise a lack of mathematical understanding (26[2]; 34[1]; 6[2]; 35[2]; 12[7]):

> *I think I cheated my way through most things…thinking oh my god I've no idea what's going on here* (34[11]).

Within this category, in addition to attempting to get someone else to do any mathematics that presents itself in life, avoidance is described (40[5]; 40[3]) with indications that avoidance tactics, rather than being confined to schooldays, remain in adult lives (35[6]; 23[8]; 34[3]; 35[7]; 21[4]; 34[12]; 40[19]):

> *it's like freezing in the headlights, so I'll just avoid it and not tell anyone I'm not very* good (23[7]).

Accounts consistent with this category describe the implication of fears (34[5]) towards mathematics being carried forward to forthcoming ITT learning (36[8]):

> *that actually really frightens me, is going into college in September and not being able to answer a question in maths or feeling like everybody is looking at me if I got asked a question or I couldn't answer it* (23[10]).

4.4.2 Category Of Description 1:

Mathematics - Knowledge Learned From External Relationship

Summary Of Findings

SPTs' descriptions of their mathematical experiences within this category are consistent with mathematics perceived as a body of knowledge that is taught by transmission. Recounted experiences describe mathematics constituted of number, with passive learning being teacher-led via rote methods. There is some comfort indicated of a level of mathematical knowledge consisting of memorisation of given number facts, with learning of these described as enjoyable, appropriate and motivating, generating recall, fluency and mental imagery.

However, mathematics beyond the recall of facts is regarded as pointless and difficult, understood only by clever people who are seen as weird and associated mathematical language is described as incomprehensible. Mathematics is described as something learners are made to do, with fearful associations including learning experiences and expectations of finding the 'right' answer, with pressure being imposed from parents, peers and teachers. A lack of mathematical confidence is described as affecting self-esteem and mathematical ability through physical and mental feelings of tension, bewilderment, confusion and prevention of clear thought leading learners to switch off, get lost and seem unable to engage mathematically.

Hence, learning beyond recall of numeric facts is limited and SPTs describe the perception that mathematics is not something that can be grasped, indicating acceptance that results in a tendency to give up, procrastinate, rely on others, disguise apparent lack of ability and avoid mathematical situations where possible.

Educational experiences transcend into adult life with confidence detrimentally affected and forthcoming mathematical involvement in ITT met with fear.

Hence, mathematics is perceived as a separate entity external to the learner: composed of facts to be memorised and recalled to present requisite correct answers; an irrelevant and incomprehensible body of knowledge that others understand and which can be avoided; and which is feared, that fear being transferred to forthcoming learning in ITT.

4.4.3 Category Of Description 2:

Mathematics - Knowledge Learned From Internal Relationship

Findings

Accounts of mathematics consistent with this category are that, as in Category 1, it mainly concerns number, a qualitative difference in Category 2 being that a basic knowledge of how number has some relevance is described (3[8]), particularly in real life (31[3]; 5[7]), such as for achievement in assessments (13[7]; 11[6]; 1[14]). However, accounts describe mathematical knowledge for this purpose being limited to knowing enough to pass examinations without further understanding (25[11]; 43[13]), alongside a perception that once it has fulfilled this purpose it can be forgotten (1[15]; 26[16]; 23[17]):

> *You used to learn it and do it in tests and think, well I'll never need that again, so I'd just let it fall out of my head* (35[28]).

Unlike the previous category, in this category connections are made between number and other aspects of mathematics (20[2]), although the extent of such aspects and their relevance to life are not clear (25[5]; 8[8]; 28[2]; 24[12]; 24[11]; 25[3]; 3[10]). In this category, recommendations are made for improvement, including that mathematical learning should be associated with real life (13[20]; 8[35]) and that making connections would have been useful (30[21]; 28[11]; 49[22]; 1[12]; 8[11]; 19[44]; 12[32]; 1[13]) with a variety of methods to use (43[29]; 2[48]; 43[30]; 19[41]; 44[17]; 43[28]; 30[28]; 40[30]; 11[16]; 13[16]; 47[12]; 47[11]) and more autonomy in their use:

> *you should be able to find your own way of working something out, rather than saying that's the way you do it, don't do it like that* (6[16]).

In this category, teaching by rote as identified in Category 1 is defined as unsatisfactory (49[16]) and is qualitatively different in Category 2 since experiences are recalled of teacher explanation of how to do mathematics, usually from a classroom board, followed by learner-focused working using the teacher's method (18[11]; 43[10]; 36[16]), with teacher explanation followed by learners working alone following the example provided (35[25]; 18[13]), and reluctance from the teacher to follow up with further explanations (35[27]; 6[7]; 8[10]; 40[17]; 45[6]; 30[18]; 24[17]; 23[15]; 12[16]; 5[12]; 25[9]; 18[12]; 28[9]; 46[14]; 25[8]; 35[24]; 20[4]; 5[8]; 9[6]; 15[10]; 18[18]; 11[3]; 34[13]; 3[15]; 3[13]; 43[2]):

> *we'd have the books and she would show us how to do it on the board and then it was do it yourself, so it was literally I'll show you once, I'll show you twice if I've got to* (29[12]).

Within this category, recollections of being taught focus on the teacher presenting a single way to be used uniformly by the class (2[16]; 12[6]; 36[10]; 44[3]; 21[2]; 40[9]; 13[4]) and descriptions are of explanation of method considered to be the unique way to do mathematics:

you have to do it their way or it's wrong, you know (6[5]).

Accounts of experience recall reliance on use of schemes (23[16]; 46[13]; 2[27]; 23[14]; 20[3]), and descriptions of a teaching approach which was not proactive (5[9]; 28[10]; 34[17]):

we weren't taught anything, we just had to get on with it ourselves, just a series of worksheets, very, very boring (31[6]).

Through this method of teaching and learning, there are recollections of working individually (29[11]; 46[12]; 28[7]; 28[8]; 43[7]; 2[26]; 19[14]; 46[9]; 12[15]; 31[11]; 9[9]; 23[19]; 13[5]; 25[7]; 18[17]; 46[10]; 8[9]; 34[16]), with the emphasis described as completion of task as opposed to working with understanding (34[19]; 6[8]; 12[19]):

we could ask questions at the end of the lesson and get help then, but it was more about getting the task accomplished, rather than understanding how the maths worked. It was just, have you got it all written down in your exercise book and ticked, rather than knowing how it worked (49[20]).

Similar to the previous category, where the need to find the 'right' answer was described, in this category, learners feel comfortable knowing how to get that 'right' answer (26[23]; 20[8]; 19[43]; 6[12]), although there is uncertainty of why the answer is not enough:

> *that's a weird thing being told to write anything, if you don't understand you will still get a point and you think but maths is supposed to be about right and wrong* (24[25]).

Whereas in Category 1 rote learning of facts was recounted as enjoyable, appropriate and motivating, promoting recall, fluency and mental imagery, in this category there are recollections of an inadequate method of teaching, described as *a limited approach, teaching in a very narrow way* (26[8]), with awareness of teachers' lack of diagnosis for how mathematics is learnt (49[1]; 26[1]), or provision for different learners' needs (19[16]; 49[1]), either for those apparently 'able' to do mathematics (49[3]), or for those who openly struggled:

> *there was no special educational needs…it was just a case of sink or swim, well I sank* (29[5]).

Accounts also suggest that mathematics should be taught more inclusively (49[600]; 12[29]) with differentiation for learning (41[10]) and understanding (26[31]):

> *different levels and different abilities for every child… if a child is struggling with something, take him back a level… If they're not understanding it, don't move on to the next level* (29[21]).

The method of teaching is perceived as dull (49[19]), with recollections of no enjoyment in mathematics (12[10]; 3[6]), with little interaction (3[17]; 2[28]), or use of practical resources (5[10]; 26[15]; 18[10]; 31[4]) or engagement:

shown how it's done on the board and then work through the books and it's not like, it's not particularly like, it's not particularly engaging, it's basically watching somebody do it and not doing it yourself (5[11]).

Whilst in Category 1 there seemed to be a resignation to the mathematics experienced, accounts in Category 2 differ in that the focus is on the inadequacies in learning through the methods used and alternatives are presented, such as regular use (24[15]; 35[29]), with mathematics involving more engagement than repetitive examples (11[2]; 3[14]), and that it could be more interesting (13[3]), fun (34[33]) and exciting (12[30]). Learning opportunities are described that might have helped mathematical learning, with some awareness of how understanding may have been aided, such as the use of collaboration (15[9]; 8[38]; 43[25]; 47[8]; 2[46]; 2[47]; 3[32])...

group work would have helped because you could have talked about the problem and then understand...rather than just thinking it's a dry subject that you've got to do (13[18])

... practical learning activities (4[14]; 4[20]; 34[34]; 15[23]; 31[22]; 45[14]; 22[14])...

there wasn't a lot of actually doing practical ways of working with maths...I think maybe that's why some things didn't click possibly - if it had been more practical then it might have helped me get it, but at the time it was the teacher at the front at the blackboard, this is the way you do it, kind of thing (4[7])

...visual aids (2[51]; 8[34]; 34[35]; 22[13]; 28[17]; 25[1]; 2[4]; 11[21])...

> *it helps you see things - you can actually see things, how they work so like with the divisions you can use the cubes and things* (2[50])

…kinaesthetic working…

> *I can't visualise in my head what I'm trying to do. I think if you can touch things, you know, like I'm always counting on my fingers because it's just easier for me to do that* (23[25])

…and a focus on process as opposed to answers:

> *at school we weren't allowed to question why...in maths you feel like you've always got to get an answer and the answers always got to be right to be good at it. How you got there was irrelevant in the school then, you just had to get the right answer* (1[22]).

In Category 1, mathematics appeared to be presented with little evidence of gaining mathematical knowledge beyond basic numeracy, whereas in Category 2, accounts of educational experiences indicate acquisition, though largely reliant on memory, of some knowledge, of vocabulary (4[9]) …

> *I'm always jotting words down, thinking I haven't got a clue what that is, I'll look it up… you do forget…words keep cropping up and you think, what the heck is that?* (3[24])

…of facts (25[12])…

I knew it was 180° as soon as I was told it again, but facts have slipped my memory (19[20])

...of rules (44[15]: 26[22]: 24[28])...

I've forgotten all those things that I'm going to need to know again...There's lots of things...that I know I've forgotten the rules of and I need to relearn them because you don't use them (18[22])

...and of methods (46[17])...

if I was asked to work out even like long division or multiplication, I might have to look up how to do it (8[16]).

Hence, as opposed to the focus on teacher input of Category 1, in this category, the focus is on learners and learning which extends to a wider mathematical knowledge than basic numeracy. Whereas, in Category 1 there was acceptance of mathematical inadequacies, in this category, both teacher and learner are critiqued (19[10]; 3[2]; 40[22])...

There was always that question, was it me or was it them? What went wrong? (24[19])

...and the link made specifically to perceptions of mathematics being a factor contributing to difficulties encountered...

I don't think it's the maths, just the perceptions of it really (40[23])

…and a specific awareness apparent in this category of mathematical knowledge being insufficient (36[20]; 2[9])…

I'm quite confident in some areas, but I think, you know, I can confidently like do calculations but I never understood it really... I can do it quite quick, but I don't understand why (2[29])

…with awareness in specific areas of shortfalls in mathematical understanding (29[9]; 3[12])…

I knew my tables quite well because I just recited them parrot fashion, not understood why but knew them still (2[15]).

Descriptions in this category are of knowing how to get answers and complete mathematical tasks, alongside awareness that this does not equate to mathematical understanding (12[11]; 4[100]; 2[19]), with memorisation methods described in Category 1 being critiqued in this category through recognition of a lack of understanding (31[1]; 4[3]; 18[16]; 47[3]) that extends to an indication of the need to understand mathematical process (49[24]):

I used to forget the way I was supposed to work things out. Sometimes I found that I could do certain sums, but I didn't necessarily understand why I do things that way (2[32]).

Rather than acceptance of not being able to do mathematics as in Category 1, in this category there is a desire to try to learn. The difficulty with mathematical language of the former category is taken a stage further in this category whereby the vocabulary itself is not the issue, but rather attempting to understand the formulation of mathematical problems (34[14]) and more autonomy is indicated in this category by asking questions of the teacher (44[1]). However, despite the desire to learn, accounts in this category include seeking understanding through querying the teacher that was not successful due to the teacher-led explanation described above being limited to one method (15[4]; 34[6]; 40[10])...

> *there's the reluctance to ask the same question twice. If they've explained it to you once and you don't quite get it then it's I can't ask again because it's already been explained* (12[17])

...and teachers' attitudes were unhelpful to endeavours to learn (36[9]) with descriptions of embarrassment at a lack of understanding (46[1]; 36[1]):

> *bit of embarrassment not understanding...there's only so many times you want to ask someone* (35[3]).

Whereas in Category 1, mathematics was perceived to be imposed upon learners as something they *just had to do* or *get on with*, in this category there is the sense that they would have liked to have learned had teachers been more helpful. Although not quite as '*horrible*' and '*off-putting*' as those described in Category 1, this category outlines perceptions of unapproachable, impatient and disinterested mathematics teachers (15[5]; 35[13]; 18[5]; 35[14]; 18[14]):

you're always the person that's asking the questions so in the end I just didn't ask any. That's the attitude she gave, I had real trouble with it, because she'd literally actually walk away with impatience (44[5]).

With regard to external pressures, as in the previous category, there are recollections of being put off by teacher attitude, but in this category such experiences are recounted in relation to an awareness of a lack of understanding (2[10])...

I couldn't say, I don't understand it, because it was public humiliation...don't say you don't understand it because it makes you look thick (29[6])

...and, as in Category 1, in this category there are accounts of pressures affecting perceptions of mathematics, but rather than these being externally imposed, in Category 2 the pressures are personal (36[14])...

If you're put on the spot to do it, I think, as an adult, I don't know, I expect that I feel like I should be able to do it (35[18])

...making comparison with peers (30[8]; 30[11]) ...

I think everyone else was a little bit better than me, maybe some people were struggling with it, but it never felt like that, it always felt like everyone else was much more confident than I was (35[16])

...and competition with peers (8[1]; 19[8]; 34[7]) ...

you wanted to be ahead, you wanted to be with all the other people because it was so competitive you used to think but they're on Book 5 why aren't I there? There was even a Book 0. I always thought that was wrong. It was terrible and it was so competitive because you'd be thinking I'm not anywhere near there but at least I'm better than so and so, and this person. It was quite nasty really (2[20])

...and a desire to fit into society:

I think in some cases you're made to feel that way yourself which is probably a societal thing. It's really important, like you have to be literate...and be numerate...but if you can't do one of them it's like having three wheels on the wagon (49[14]).

Personal perceptions in this category include the notion that mathematical ability is genetic (35[22]; 18[8]) and that perceptions of not being good at mathematics may be transmitted to children within a family (26[10]; 35[34]; 26[11]) with accounts describing low expectations stemming from a tendency to believe mathematical aptitude runs in families (24[14]; 35[21]).

Whereas in the previous category, mathematics was deemed to be the domain of 'clever' people, in this category there is the qualitative difference that understanding mathematics is perceived to be dependent on having a 'mathematical brain' (6[20]; 46[4]; 46[8]; 6[6]; 46[15]; 36[18]; 46[7]) and the need to be logical (39[2]; 4[23]; 19[47]; 8[39]; 22[19]; 2[54]; 4[25]; 44[20]; 4[24]) with mathematics suggested to remain out of reach to those who do not perceive themselves to be logical (22[16]; 2[53]; 22[17]; 2[52]).

Rather than the fear described in Category 1 when faced with mathematics, this category includes indications that mathematics is difficult and a struggle (3[11]; 2[23]; 12[12]), with a recognition that this does not bode well for ITT learning (30[12]), evidencing that difficulties with mathematics have lasted beyond school education (24[13]), but accounts consistent with this category suggest a desire to make changes and overcome difficulties...

> *I'd like to enjoy it. I'd like to be able to not feel anxious about it* (23[24])

...with indications of how this might be accomplished (49[18]; 23[23]; 29[18]; 19[42]; 44[13]; 29[10]; 36[15]; 43[6]; 49[17]; 43[5]; 23[13]; 30[16]; 40[21]; 23[18]; 40[20]). In recognising that mathematical knowledge does not extend to understanding, frustration is described (30[10]; 35[17]; 3[7])...

> *Terrible about it. Absolutely terrible. I don't think I've ever had to do anything with maths that hasn't resulted in tears, because I find it so frustrating ...sometimes it gets on top of me and I feel like there's always going to be something in everything, that I'm never going to get* (36[13])

...and anxieties about forthcoming ITT include depth of mathematical knowledge (28[4]; 28[6]; 4[6]) including the perception of the need for right answers (9[5]; 4[5]), the need to be able to provide explanations (2[25]; 39[1]; 25[6]) and for the need for understanding a range of mathematical strategies...

> *I think the one thing that worries the most, especially in terms of me teaching maths, is that if I understand how to do something, I will show a child how to*

do it, but then if they don't understand, then I'm not sure I can think of another way of saying it, another way of getting the same point across (11[5])

…to be able to meet a range of children's needs (5[15]; 30[15]; 26[14]; 28[5]; 1[11]; 26[12]) and provide accountability for parents (34[15]):

the thing that worried me about that was nothing's recorded, the children could go and very little would be recorded, so nothing to take home, nothing to look back on and nowhere would they have written down (19[13]).

This category includes descriptions of being *nervous* (12[14]) about learning mathematics in ITT but indications of desire to do something about it (35[23]):

I feel actually really quite worried about it…I don't want to be frightened of it (23[12]).

4.4.4 Category Of Description 2:
Mathematics - Knowledge Learned From Internal Relationship
Summary of Findings

SPTs' recalled experiences consistent with this category, as in Category 1, are that mathematics concerns number, but there is the qualitative difference in this category of some connection being made to the relevance to real life, although this is not transparent.

Accounts describe mathematics learning as teacher-led as in the previous category, but in this category focus on a level of internal relationship with mathematics since learning constitutes a more personal and independent approach since, rather than rote learning through recitation, this category contains description of teacher explanation and repetition through learners' individual worked examples and schemes following given methods. The notion of 'right' answers, described as a source of anxiety in Category 1, is welcomed and mathematics is described in terms of finding answers, although there is a lack of awareness described of mathematical process, as mathematical knowledge is dependent on following teachers' methods and explanations.

Dissatisfaction is expressed with learning opportunities and awareness is described of a lack of understanding and shortfalls in experienced teaching methods are identified, alongside recommendations for improvement to the teaching approaches experienced.

Rather than the previous category's indications of mathematical inability, of giving up and avoiding mathematical situations, where it was deemed for clever people only, in this category the desire to learn is evidenced, although there is a differentiated perception of mathematicians having particular brains and needing to be logical. In this category, there resides the question of whether the reason for a lack of understanding lies with the teaching or learners themselves and student-focused pressures are described of comparison and competition with others as opposed to the external imposition of parents, peers and teachers in the previous category, with feelings towards mathematics causing frustration at the difficulties experienced and lack of understanding rather than the fear of the previous category.

The lack of confidence in Category 1 left SPTs fearful of ITT, whereas in Category 2 SPTs' accounts, albeit suggesting mathematics to be a struggle with anxiety expressed regarding ITT, demonstrate the desire to overcome difficulties, with awareness that an understanding of mathematical process is needed.

Hence, in this category, as in the previous, mathematics is limited to knowledge as opposed to understanding but there is qualitative differentiation with regard to the referential aspect of relational experience since, unlike Category 1, in this category accounts are consistent with some internal and relational engagement with learning mathematics both through individual working through schemes and via awareness of the limitations of mathematical knowledge without understanding.

4.4.5 Category Of Description 3:

Mathematics - Understanding Learned From Internal Relationship

Findings

In the previous category, recollections of school mathematics were generally not enjoyable nor deemed adequate for learning, whereas learning experiences described in this category are qualitatively different as positive memories are recalled (22[7]) where learners could seek understanding through asking questions (12[23]; 18[23]) without fear of ridicule (2[35])...

> *She didn't explain it to you like you were stupid or like you had to do it at the front of the class. There were some people who were fantastic at maths and some who struggled and I saw how she helped them, she never once belittled them and I thought that was fantastic* (9[12]).

Teachers were on hand to help (9[24]; 18[28]; 8[21]; 19[23]), difficulties were diagnosed (26[26]) and a variety of approaches were used to aid understanding (2[40]; 21[14]; 9[13]; 44[10]; 1[20]; 25[13]; 12[27]; 15[20]; 1[19];45[8]; 6[10]), focusing on mathematical process as opposed to the 'right' answer...

> *he used to show us all different ways and then say whichever one was the best and in fact I think he's one of the reasons that I do love maths now...he always used to say look I don't mind how you get the answer as long I can see how* (6[14]).

Similar to Category 2, descriptions of mathematical learning in this category include critique, the qualitative difference here being the focus on mathematical understanding such as *why numbers do what they do* (35[32]), the need to practically engage with mathematics (21[10]), the application of mathematics (49[26]) and the importance of process (18[21]) with a recognition that the requirement of working neatly, described in Category 1, had a reason:

> *helped to see the setting out and helped to follow, the flow of the problem, or the process you were using and I suppose then it was helpful to the teacher to then follow what you've done, so they could then in theory help you where you'd gone wrong* (26[18]).

Whereas in the previous category, shortfalls in teaching methods were described with suggested improvements that could have been made, in this category, accounts describe those being put into practice with some awareness of their benefits for mathematical learning demonstrated, with differentiation for varying needs (12[24]; 29[22]; 19[45]; 46[30]), use of vocabulary (13[9]; 21[6]; 26[19]; 12[21]), visual apparatus (36[19])...

> *bring things much more to life for me to focus on and to visualise and to work with and he started to make me understand stuff* (3[26])

...collaboration (15[12]; 6[9]; 2[36]; 3[33])...

> *if the teacher didn't explain it again you could always turn round and ask the people behind you and sometimes they'd explain it as well* (12[22])

...games...

> *he'd have things like dice that he would get out, and you'd have to roll a number to make the sums and little games* (11[8])

...practical (49[31])...

> *the most successful lessons I've witnessed in maths are the ones where the children have had to weigh things themselves and work it out and count things, so the learning is more concrete* (21[15])

...cross-curricular...

> *incorporating all the communication skills and speaking and listening skills and it works cross-curricular really doesn't it really, yeah, but they're*

incorporating so many different aspects of learning into it that it just makes it more alive (3[37])

...use of logic (19[22]) ...

more practical in that you were given scenarios using your logic and reasoning that tends to be logical puzzles and I quite enjoyed them. That sunk in (4[11])

...the encouragement of mathematical thinking (8[22]; 8[20]; 9[26])...

it's more showing how you can break things down, pull it apart and then put them all together and you've got an answer kind of thing (44[11])...

...and the use of different methods (6[15]; 21[11]; 29[19]):

realising that there are different ways of doing things and that there's nothing wrong with being completely different to how someone else would do it, that's been the most useful technique that I've found, tool that I've come across (36[22]).

Accounts of understanding mathematics in Category 3 goes beyond the use of number outlined in previous categories, mathematics being described as (40[24]; 31[13]):

everything really. It's obviously figures but not just figures, it's kind of things to do about everyday life, working things out - even things like dimensions and shapes or when you go shopping like money (2[37]).

Whereas in the previous category, awareness was demonstrated of the relevance of mathematics to everyday life, in this category, specific reference is made to an understanding of how mathematics is applied to everyday life (22[10]; 13[11]; 41[6]) including home, family, work and social lives (24[23]; 13[12]; 21[9]; 8[28]; 11[11]; 15[15]; 19[30]; 46[21]). Accounts include an understanding of how <u>different</u> aspects of mathematics have relevance in life involving a <u>range</u> of uses of number (19[31]; 34[25]; 4[13]; 31[12]), estimation (35[31]; 1[21]), money (24[21]), measurement (43[17]; 29[15]), time (2[38]; 23[21]), length (18[20]), distance (29[14]), mass (19[28]), capacity (15[16]), conversion (43[16]; 39[3]; 46[23]), shape (26[20]; 29[13]; 5[18]; 34[600]; 24[22]; 20[7]; 15[14]; 31[14]; 34[24]; 19[27]; 40[25]; 46[22]; 18[19]; 9[10]; 11[10]), percentages (19[25]; 30[24]; 19[29]; 21[8]), comparison (19[26]), angle (43[18]), logic (11[12]), trigonometry (13[13]), investigation (4[12]) and making connections between mathematics and other areas of school curriculum (19[32]) with description of why mathematics matters (3[27]; 5[17]; 43[19]; 3[28]; 4[22]; 44[12]):

it's really important....I think well the earth is spinning at a certain speed for gravity to work, if I need to get from A to B, how far is it, days of the week, months of the year (49[33]).

In the previous category, there is indication of learners putting pressure on themselves to do better in mathematics, whereas accounts consistent with this category describe an ambivalent attitude (49[21]; 25[10]; 18[15]; 13[6]; 43[12]; 21[5]; 41[5]; 20[5]) with varying degrees of confidence (34[20]; 36[17]; 22[4]; 11[9]; 13[10]; 39[400]; 46[20]; 8[27]; 24[20]; 19[24]; 22[9]; 6[11]; 46[19]; 8[26];

30[23]; 49[25]; 15[13]; 4[8]). Whilst anxiety about mathematics was manifested in the previous category, in this, there is description of a societal acceptance of not being able to do mathematics (49[300]; 35[10]; 49[12]) with mathematical enjoyment described in disparaging terms:

> *I do like sitting down working out problems, quite sad really* (6[3]).

Unlike the previous category, in this category mathematical understanding is not described as being out of reach, although there are indications of the need to work harder at it than others (44[9]; 46[18]). Rather than mathematical ability being due to having the 'mathematical brain' described in Category 2, in this category a different perception is placed upon mathematics being structured and mechanical (12[25]; 13[8]; 22[18]; 8[14]; 44[8]; 8[13]; 8[15]; 19[19])...

> *there are some people who are, kind of, a lot more creative brains and struggle to understand the processes, the mechanical processes behind maths... maths tends to be very structured and very kind of stage orientated* (12[20])

...and described as *a means to an end rather than something that's fun and creative*[3.20]. Although seen as a separate discipline (11[18]) it is also perceived as a science rather than a creative discipline (3[21]; 43[16]; 2[31]; 44[16]; 2[43]; 2[42]) with the suggestion that rules are necessary to reach mathematical answers...

> *there are definite answers, and a definite way to get to that answer, you've got to learn rules to get through it whereas with English or with history it's more*

dynamic isn't it, I mean your opinion is your opinion and it's neither right or wrong as long as you've got evidence to back it up, whereas with maths, it's very rigid...If you can't follow those rules or you don't understand those rules, the whole subject is blanked off then (11[14])

...and as such it is necessary for the teacher to ensure the rules are known:

I never understood this idea that oh you do your workings out but we don't care if you don't get the answer right and you think, hang on, maths is a science - it either works or it doesn't...in real life if you make a mistake you making it with money and it costs you so there's no room for error... it's only recently where I've suddenly understood that what the maths teachers were trying to was to do was to see if you know the rules, you're going to make a mistake – you might put a 1 instead of a 2 – here it doesn't matter, obviously in commerce it does (24[26]).

Following on from the evident desire to improve mathematically in the previous category, accounts in this category indicate a need to improve for future teaching with self-expectation to change (25[16]; 30[32]). Whilst indicating the hope to receive support from ITT in various aspects of mathematics for primary teaching (35[31]; 2[33]), specific areas of mathematics are described that there are intentions to work on including subject knowledge (19[21]; 3[25]; 19[35]; 35[35]; 43[24]; 9[11]; 8[18]; 34[27]), mental mathematics (34[28]; 34[22]), use of different approaches (30[22]; 30[26]; 31[15]; 15[21]; 13[14]; 15[11]; 11[15]; 20[9]; 2[34]), practical apparatus (5[16]; 25[17]), inclusion (9[29]; 1[26]; 6[13]), diagnosing children's understanding (26[30]; 19[37]) and learning how to teach mathematics (35[30]; 8[19]; 1[18]; 46[27]; 15[19]; 25[15]). In this category, changes which are planned are evidenced (12[26]; 25[18];

34[26]; 47[6]; 43[20]; 43[21]; 30[25]) and the need for understanding mathematical process demonstrated (20[6]):

> *I'm going to have to really practise my maths... I think I learn better doing a problem and like, you know, trying to work it out rather than just writing it out and memorising* (47[7]).

4.4.6 Category Of Description 3:

Mathematics - Understanding Learned From Internal Relationship

Summary of Findings

In the previous category, mathematical knowledge of how to reach answers was deemed insufficient due to awareness of a lack of understanding, whereas Category 3 is qualitatively different in that SPTs' accounts describe an active quest for mathematical understanding through an internal relationship involving questioning, engagement and application alongside interaction with helpful teachers who use a variety of methods to meet learners' needs, putting into practice the kinds of suggestions presented in Category 2 regarding differentiation, the use of visual apparatus, collaboration, games, logic, vocabulary and encouragement of mathematical thinking.

Whereas Category 2 described learners being aware of mathematical purpose in terms of number in everyday life, this category indicates an understanding of the importance of a range of aspects of mathematics applied to specific aspects of life as accounts describe an internal relationship with the purpose of mathematics and understanding of its relevance.

Instead of the perception of the previous category that mathematicians have a particular brain, this category differs qualitatively by suggested reasons why mathematics may elude some learners due to its perceived nature of being structured and scientific with rules to be followed to get answers, with the notion that it is acceptable to not be able to do mathematics very well. This category does not present the negative attitudes of the previous category, instead describing an ambivalent attitude towards mathematics and varying degrees of confidence but, in relation to the desire to understand in the previous category, Category 3 indicates that changes have begun based on awareness of mathematical process having importance in reaching mathematical understanding and accounts describe specific actions planned as part of ITT learning to make improvements in identified areas of mathematics.

Accounts consistent with the previous category described learners knowing how to reach mathematical answers through following teachers' methods and/or working through published schemes which were mainly considered in negative terms and judged as inadequate in terms of associated mathematical understanding. In Category 2 a desire was described to improve mathematically with frustration at not achieving 'right' answers. Category 3 is similar to the previous category in that the focus is on the SPTs' internal relationship with mathematics, but is qualitatively different in that descriptions in this category focus on and evidence mathematical understanding.

4.4.7 Category Of Description 4:

Mathematics - Understanding Taught Through Perspective Of Internal Relationship

Findings

Accounts in Category 3 described an ambivalent attitude towards mathematics whereas in Category 4 an enjoyment of, interest in and liking for mathematics is described (20[10]; 45[5]; 15[22]; 21[13]; 24[27]; 4[15]; 20[11]; 9[18]; 6[17]; 31[18]; 8[31]; 9[19]; 43[26]; 9[15]; 31[16]) with an indication that enjoyment corresponds to confidence in mathematical ability (4[16]; 29[16]; 40[29]; 31[19]; 35[33]). Within this category are descriptions of excitement in mathematical engagement on a personal level (12[31]) ...

> *maths is exciting, you can engage with it, you can take it to whatever level you want to take it to* (3[31])

...and when working with children:

> *seeing the kids getting it I was actually, I was thinking, I remember that and yeah, and I actually did feel myself getting excited. I've observed a few lessons, maths lessons specifically, that I have felt oh excellent, cool you know that's really great, that's grabbing me that* (3[29]).

In the previous category, a quest for mathematical understanding was described with areas identified for development in ITT based on previous mathematical learning. In this category, accounts describe the difficulty inherent in deconstructing and reconstructing understanding...

once you've learnt something it's very hard to relearn it a different way (21[12])

...but suggest a move from the prescribed mathematics experienced to changing mathematical perceptions...

if you don't sort of shut it off and think it is prescribed and that there is just one end result...it's getting yourself into that mindset if you've not been used to that with your background and how you've learnt and how you see maths to be (3[30])

...such as seeing mathematics as logical, as in Category 2, and a science in Category 3, but teaching it creatively with more freedom in the choice of methods...

to do with logic and theories...you can teach maths in a creative way, but I think its still quite a science, it's still quite logical. So it's kind of like you've got your right answer but there can be different ways of working it out (2[55])

...with description of change having been made in perceptions (26[21]; 40[28]; 26[29]; 13[15]; 47[5]; 40[26]; 22[11]; 44[14]):

I wanted to change my mindset and I believe I have done that now and I feel a lot more confident...treat it as a friend rather than an enemy...I'm not fazed over it anymore (40[27]).

Whilst aware of changes and improvements in personal learning that may be needed, as identified in Category 3, in this category accounts centre on aspirations for teaching

(8[40]): to be approachable (41[9]), to encourage children to enjoy mathematics (29[23]), to promote interest, fun and excitement in mathematics (22[15]; 3[16]; 22[12]; 49[28]; 9[23]), to aid confidence (34[700]), encourage children to do their best (18[26]) and to inspire them (46[31]).

In Category 2, accounts described factors considered helpful to aiding mathematical understanding, with descriptions in Category 3 of these put into practice for personal learning. In this category, accounts describe putting these into practice for children's learning, suggesting a creative way of teaching…

> *it should be taught creatively, using loads of different resources, group work, individual work, games, visual aids, and like, using as many different resources as you can* (43[311])

…as opposed to the structured way of learning experienced that was not considered conducive to understanding (12[33]) with a focus in this category of concerns for children specifically:

> *it concerns me that there are those children who are never really going to get to a point to manage outside of school. That's worrying really* (26[13]).

Aims for practice are described in diagnosing children's understanding (45[10]), encouraging independent thinking (40[31]) and providing active learning opportunities for children using different approaches:

you've got to have things that engage them and get them to think about it in the different ways...I would love to develop a way in which it was constantly active and moving and just different approaches (11[19]).

Desired practice is described where children can question (29[20]), collaboration is encouraged (31[21]), where children *are not afraid to get things wrong* (45[11]) and where they are able to work at their own level of achievement:

children understand before I do move on...don't make them aware of that's the lower ability group...if everyone knows which one's the best group, you're Group 4, you feel devastated...not putting children on the spot as well to answer questions...not like picking on people and making them feel really ridiculous (2[44]).

Under this category, there are aspirations to meet children's learning needs and aid mathematical understanding through flexibility (45[12]), provide attention for groups (31[22]), using visual, interactive and practical learning opportunities (15[24]; 49[30]; 4[21]; 4[19]; 9[27]; 3[34]), link with other areas of the school curriculum (34[31]) and plan differentiated tasks to work on:

have the same basic problem to solve, irrespective of the levels and find aspects of the same question that different children can work on (49[32]).

These planned learning opportunities are based on real life relevance (18[25]; 13[19]; 35[36]; 49[29]; 20[12]; 26[28])...

I really hope that I will be able to show children that they don't need to be scared when it comes to maths and that maths is really something that everybody uses everyday and is actually very useful (46[28])

…and include the use of different methods (43[23]; 2[41]; 19[36]; 31[10]; 11[20]; 3[35]; 31[23]; 30[33]; 30[29]; 5[19]; 8[33]; 9[200]; 5[21]; 6[18]; 44[18]; 30[31]; 18[27]; 34[30]; 30[30]; 31[20]):

encourage them to work it out for themselves, and find the best way for them, so if a child isn't taking to a particular method, then maybe they can have a look at a different way of doing it, and so they are never going to be bad at maths, it's going to be a case of one method not working for them, and they can find the right method that helps for them so it just means that everyone's got a chance of being good at maths (43[32]).

Rather than the search for a 'right' answer described in previous categories, in Category 4 mathematics is described as both problem solving (23[22]; 18[24]; 24[29]; 29[100]; 1[25]; 30[27]; 34[21]) and problem posing (20[16]). Accounts describe *the actual doing* (19[40]) of mathematics and facilitation for children to explore and discover their own mathematical meanings…

I think they should be encouraged to discover things for themselves rather than just teaching them this is how you do it…getting them to explore something for themselves…not sitting them down with a book and getting them to learn a set way (43[31])

…concentrating on mathematical processes that children utilise…

> *making sure that children understand how they get to the answer, and the approach might be different for every child so they can come to an understanding of the answer* (47[10])

...and, based on a philosophy that: *it's always better for children to be encouraged to try and work out how to do it themselves first rather than being spoon fed* (8[36]), engaging in mathematical process through active thinking (19[38]) by *encouraging an active mind, proactive thinking, rather than just sitting back and being told how to do things* (8[37]).

Within this category, a critical perspective is taken in realising that putting ideologies into practice is not without difficulty, with recognition that planning for differentiation is not easy (41[11]) and that teachers face constraints in the form of *pressure on your time, fitting everything into the curriculum* (44[19]) and government pressure...

> *pressure coming from above from statistics and government documentation, very difficult not to get into that mould, to forget why you're there, you're there to teach and encourage children* (45[15])

...including testing:

> *testing in particular and the teachers feeling responsible themselves for results at the end of the year... when you've got a teacher who is too hung up on league tables, it's very difficult* (45[16]).

Whilst the previous category included a range of aspects of mathematics applied to specific aspects of life, in Category 4 mathematics is described as essential to life (11[17]; 47[9]; 1[23]; 36[23]; 8[32]; 26[25]; 46[29]; 2[45])...

> *if you didn't have maths everything would collapse, everything is based on maths and people just don't realise even like in the library all the codes, it's all mathematical isn't it?* (6[19])

...and regarded as an aid...

> *You can make more informed decisions. You know, can you afford it? Is it right thing to do? It's just so limited if you close your mind to that subject, it affects so many other things* (26[24])

...with reference made to its significance for children...

> *it's a way of helping children get by in the world, I can see that it's absolutely essential they're good at it* (19[39])

...and the suggestion that a holistic approach should be taken in school to mirror the way in which real life is not separated into subjects:

> *it needs to be like the early years for everything...all over the place...go with the flow – that's the way the world is* (34[32]).

In addition to the notion of mathematics being essential for children to function in the world, in line with the previous category's description of mathematics as a science, it is described here as a science applied to the world in order to model what is understood about the world...

> *it's the applied science of maths – it's understanding statistics, it's being able to measure things, and understanding angles, being able to come up with equations, being able to model the world with equations* (13^{21})

...with science being defined as tested and proven:

> *science for me is something that is backed by a hypothesis that's been tested and there's evidence and stuff like that. My knowledge of science doesn't extend to whether you actually do that in pure maths, I'm assuming when you get to higher levels you probably do. Your hypothesis might be that 4 add 4 equals 8 and so in that respect I suppose it is a science* (22^{5}).

From the scientific perspective, mathematics is therefore described as having agreed facts, methods and rules that are tested and agreed by a mathematical community, but in terms of the described ideology above of learning mathematics creatively there is the notion here of philosophy whereby...

> *it's a way of thinking, it's related to things like science* (19^{46}).

Hence, whilst mathematics is seen as logical, as in Category 2, with structure as in Category 3, in this category it is seen as a tool to be used in thinking and reasoning in order to understand the world and function within it, with mathematics described as:

> *all about logical thinking and reasoning...a way of making sense of the world and operating* (4[67]).

Rather than being relevant to and applied to everyday life, as described in Category 3, accounts consistent with this category describe mathematics as being a man-made framework created for the purpose of understanding and making use of our world...

> *a framework of logical aspects of physical science...the world around us is governed by maths and maths as a subject is trying to understand that, trying to harness why things work* (49[67])

...and a means of communicating that understanding, described as an *all encompassing communication device* (46[26]) whereby the world can be understood (46[25]; 46[24]; 36[21]; 13[17]; 41[7]; 45[9]):

> *mathematics is how numbers fit together to explain things in the world at large...it's like building blocks to patterns and creating... Our developed society, everything is built on maths, everything has to be worked out and measured and that's what maths is, little building blocks...actually physically build something, or create something... make a car or to mix a recipe...you're working through a process of things to get to the other side so I see maths like a little web I suppose - like a network to another level* (39[3]).

In this category the notion that proven scientific principles make mathematics a science, as described in Category 3, is viewed differently since the thinking utilised by the mathematicians who discovered and created the shared framework that is mathematics are themselves seen as creative thinkers, descriptions in this category relating that *it's weird isn't it that most of our great philosophers were mathematicians as well?* (24[31]).

Unlike the previous notion that mathematics provides a 'right' answer, perceptions consistent with this category suggest that mathematics is *like a politician, they can go all round the houses and not give you the right answer* (29[17]) and, differing from the previous perception of there being a set way to doing mathematics is the creative approach described above in descriptions of ideologies for teaching whereby:

> *it's about kind of making it more accessible to people and less scary, so for me personally it was, it was something that has a definite structure and this is what you need to do and you cannot stray outside that and as soon as you don't understand the structure, you feel like you've got it wrong, and I think if it's taught in that more creative way of, you know, when you're writing a story and there's never necessarily a right or wrong way to do it and it needs to have that element of creativity within in as well as being able to kind of rigidly teach those basics* (12[28]).

Hence in this category mathematical perceptions differ from the facts, methods and rules which learners found difficult to understand through transference teaching and mechanical learning, to the perception that although that framework of shared facts, methods and rules exists, it is necessary for children to learn through opportunities to think for themselves, discover and create through investigation (26[27]; 1[27]; 4[18]), an example being whereby through working practically, someone had discovered the value of pi for herself whilst dress making (20[15]).

Mathematics is described as difficult in Category 1, a struggle in Category 2, with the notion posited in Category 3 that creative people have to work harder at mathematics to succeed. In this category, the perception is that mathematical thinking is a creative process and accounts suggest that it may not be easy but the challenge can be enjoyable (9[14]; 9[16]; 45[13]; 4[17]; 9[17]) with a sense of satisfaction for achievement (5[20]; 9[20]; 9[21]; 43[27]; 1[24]) and accounts describe the desire for children to also enjoy this challenge (18[29]; 26[32]):

> *I want the children to find it exciting and a challenge, because if you understand the challenge is there to learn from, it can be a joy and that applies to all different levels, at all different subjects* (45[200]).

In this category, the difficulty of mathematics is not contested, but is seen as a challenge and the description of mathematics having *that sense of mystery to me – that I don't really understand it all* (20[17]) is acknowledged in that mathematics is indeed a mystery, just as the phenomena around us are a mystery but, rather than being the source of fear and anxiety in previous categories where mathematics is described as unattainable and beyond full understanding, its mysterious nature is embraced in this category as *stimulating* (45[12]) and *fascinating how it all works out, like in nature and things like that* (20[13]) with *a sense of wonder about maths* (20[14]) described and its mindblowing nature being a source of interest as opposed to being something to cause anxiety…

it nearly made my brain pop...and what did spur me on was how good the Greeks were – if you take Pythagoras and think what he came up with – you know, a real intellectual and Euclid inventing metric...I found that really interesting (24[30])

...and hence, learning in ITT is described with anticipation...

so excited I'm counting down the days, literally. I can't wait to learn everything we need to learn (9[25]).

4.4.8 Category Of Description 4:

Mathematics - Understanding Taught Through Perspective Of Internal Relationship

Summary of Findings

As opposed to the ambivalent attitude towards mathematics described in the previous category, Category 4 describes SPTs' attitudes of enjoyment and interest in mathematics, excitement in engaging in mathematics and the desire to be inspirational teachers. In so doing, accounts describe the difficulties in changing perceptions of mathematics, based on past experiences and the ways in which it is considered that mathematics should be learnt.

Accounts in Category 3 of mathematics teaching and learning experienced that were considered useful for mathematical understanding are identified in this category as approaches to take in practice teaching children, with aspirations to be creative, approachable, encouraging, promoting interest, fun and excitement and diagnosing children's understanding. The provision of learning environments that promote an internal relationship with mathematical development amongst learners is described. where children can question, collaborate, feel comfortable and work at their own pace. with intentions to meet children's needs through flexibility, group attention, learning tasks, visual, interactive and practical activities, use of different methods, relevance and cross-curricular links. The awareness of mathematical process being important to reaching mathematical understanding described in the previous category is manifested in practice in this category in the descriptions of SPTs' intentions for teaching.

The scientific nature of mathematics, described in the previous category as being a cause of mathematical difficulty, is described through SPTs' accounts in this category as being facts, methods and rules that have been created by mathematicians and which can be explored by children through a creative approach of active learning that encourages development of thinking and use of different ways of solving problems, in addition to posing problems, based on the notion that mathematics is itself a way of thinking. Rather than being relevant to everyday life, as purported in Category 3. mathematics is perceived in this category as essential to functioning in the world, but also viewed as a way of thinking that has been used by humans to make sense of the world and communicate understanding, to be used by children through investigation and an holistic approach.

Accounts describe difficulties inherent in teaching through the barriers that may be faced. Difficulties in learning mathematics are described - not in the negative terms of previous categories, but as a challenge - to be embraced with a sense of fascination for a philosophy of mathematics that represents a framework for trying to describe.

understand and shape the world and that its very mystery, rather than being perceived with anxiety, is a source of stimulation and wonder.

4.5 The Range Of Variation Of Perceptions Of Mathematics Amongst Student Primary Teachers At The Outset Of Initial Teacher Training

In answer to the first of the research questions posed for this study, the range of variation of perceptions of mathematics amongst SPTs at the outset of ITT is determined. Based on a pooled set of meanings from a sample of SPTs at this stage in their teaching career, an hierarchical outcome space is presented that provides an opportunity for SPTs to consider their own mathematical philosophy in accordance with perceived:

1. knowledge learned as a result of an external relationship with mathematics
2. knowledge learned from an internal relationship with mathematics
3. understanding learned from an internal relationship with mathematics
4. understanding to be taught through learners' internal relationship with mathematics

In the first category of description, mathematics is perceived as an external entity that is known by the teacher and taught by transmission via rote methods. Learners have little opportunity to internalise anything other than the absorption of given facts and hence wider aspects of mathematics, including its unique vocabulary, are lost to them. In the second category, whilst there is an element of teacher-led demonstration and explanation that limits mathematical to knowledge as opposed to understanding, there is a qualitative difference of learners engaging at an internal level with mathematics through individual working through examples and tasks. In the third category, learners' internal relationship with mathematics is evident, a qualitative difference being teachers providing opportunities that promote learners' mathematical thinking and understanding. These learning opportunities are also evident in the fourth category, the qualitative difference being that these are viewed from the perspective of use in practice to facilitate children's internal relationship with mathematics.

Linked to the external, imposed notion of mathematics of the first category is the limited view of mathematics being constituted of number, and regarded as both difficult and pointless, whereas the second category illustrates the qualitative difference of some internal connection, although not entirely clear to learners, in relation to real life. Both of these perspectives suggest mathematics limited to knowledge as opposed to the understanding illustrated in Category 3 where the importance of a range of mathematical application to various elements of real life is described. Whilst this relevance is also exemplified in Category 4, there is the qualitative difference of that relevance being considered essential as opposed to useful.

The level of mathematical knowledge constituting memorisation of externally given facts in Category 1 differs qualitatively from an awareness of the importance of mathematical process in Category 2 as learners attempt to internalise mathematical rules and procedures in their attempts to understand, this differing in turn from Category 3 where that understanding is evidenced as a result of learning opportunities that focus on mathematical process and have afforded the chance to internalise. Once more, those learning opportunities, designed to facilitate learners' internal relationship with mathematics via a process of active questioning, collaboration, practical activity, use of different strategies and development of mathematical thinking, is exemplified in Category 4 as aspirations for teaching.

Category 1's perspective of mathematics as facts to be remembered leads learners to perceive the pressure to provide 'right' answers that is exacerbated by external pressures felt from teachers, peers and parents. The externally imposed body of mathematics to be learnt in this way links to a perception of fixed mathematical ability and leads learners to physical and mental debilitation as they panic, cannot think clearly, become confused and switch off from what they see as the external entity that is mathematics. The notion of 'right' answers is a perception shared in Category 2, the qualitative difference being that there is some pleasure to be found in 'knowing' that there is an answer to be found – despite this being dependent on following teacher-given rules and procedures and hence unaccompanied by mathematical understanding. However, there are internal pressures associated with finding the answers as learners compare themselves with their peers and compete to complete tasks, despite an awareness of not understanding why the answers are 'right'. Category 3 illustrates the active quest for mathematical solutions but differs in that it is not via the means of rule- and procedure-following since it is based on an internal relationship with mathematics that seeks understanding through questioning, engagement and application. The qualitative difference between the quest for a 'right' answer of Categories 1 and 2 and the possible solutions of Category 3 is the notion in Category 4 of mathematics being a way of thinking about, understanding and communicating understanding of the world and therefore is an internalised problem-solving view of mathematics in which the asking, not just the answering, of questions is crucial.

The perceived imposition of mathematics as an external entity and associated lack of understanding links to the notion in Category 1 that it is the domain of others who are clever and weird, perceived somewhat differently in Category 2 where learners do not disassociate from mathematics but do regard its need for a mathematical and logical brain. This is described in a qualitatively different way in Category 3, mathematics being seen as scientific and hence perceived in terms of understanding, using and applying associated facts, rules and procedures and as such regarded as separate to the 'arts' and hence not for the creatively minded. Also, since there are apparently good reasons for not being very good at mathematics, it is seen as socially acceptable to admit to that. However, in Category 4 there is the qualitative difference of acknowledgement of and challenging previous perceptions of mathematics that may have arisen from past experiences in order to potentially make changes in light of the needs of ITT and future practice.

From the perspective of Category 1 where mathematics is viewed as requiring little more than the recall of facts, the experience can be described as enjoyable, but the external embodiment of mathematics imposed upon learners acutely aware of their limited knowledge is frightening. Category 2 constitutes learners aware of limited mathematical knowledge, but attempts at an internal relationship with mathematics to elicit understanding leads to frustration at the difficulties experienced and the learning opportunities presented. In the third category, as understanding is evidenced from an internal relationship with mathematics, learners demonstrate ambivalence and varying degrees of confidence with mathematics, whereas the fourth category comprises learners who enjoy and find interest in mathematical engagement to the point of excitement at forthcoming teaching experience, viewing difficulties faced with mathematics as a challenge and a source of fascination, stimulation and wonder in humans' attempts at understanding the world's mysteries.

The debilitation experienced in the first category leads to an external body of mathematics being perceived as lying out of learners' reach and hence a resignation to avoiding it wherever possible. Whilst learners constituted in the second category are aware of a lack of mathematical understanding, they are not tempted to give up during their school experience and instead evidence a desire to learn, qualitatively differentiated from the third category where change has begun with regard to expanding awareness and focusing on mathematical process as a means to strengthening an internal relationship with mathematics. This focus on mathematical process is further exemplified in Category 4 through the internal relationship intended to be fostered in learners by ensuring that mathematics is not seen as an external entity, but approached in a creative way – emphasising that scientifically proved facts, rules and procedures of the known body of mathematical knowledge could not have been discovered and justified without humans creatively engaging with and thinking about phenomena and hence reaching personal internal understanding.

The fear of Category 1 stemming from experience is demonstrated in adult life through lack of confidence and self-esteem regarding mathematics, leaving SPTs fearful of ITT involvement. Nervousness as opposed to fear constitutes the second category, differing in terms of espousal to overcome difficulties, whereas Category 3 comprises actions planned for improvements identified as necessary during ITT. Category 4 depicts mindfulness of what learning is needed, but differs in the excitement towards ITT and future practice in school.

4.6 Conclusion

As anticipated, some SPT experiences appeared to have not been previously consciously considered (Cross, 2009; MacNab and Payne, 2003) and the method used was successful in its enablement of SPTs' described experience to be articulated in light of the impossibility of perceptions being directly observed (Rokeach, 1968) and in consideration of viewpoints being at times complex and emotive (Cherry, 2005), its qualitative nature was particularly beneficial in this area of affective considerations (Liljedahl, 2005; McLeod, 1992).

As is the nature of phenomenographical methodology, the categories of description do not depict any one individual SPT, but accounts of experience of the whole sample were interpreted to provide the potential range of perceptions described. The method was successful in the facilitation of interpretation through analysis of relationality (Leder and Forgasz, 2006; Säljö, 1997) to move beyond the individual description to the context of variation across the group (Green, 2005; Bradbeer et al, 2004) to achieve a better understanding of the range of mathematical perceptions (Dall'Alba, 2000; Marton, 1986).

With regard to phenomenographical 'bracketing' (Ashworth and Lucas, 2000; Dunkin, 2000; Säljö, 1988), whilst researcher knowledge of mathematics and the conceptual framework for the study were used in analysis of the outcome space, the interview process and data analysis were conducted without preconceptions of what SPTs had to say (Patrick, 2000; Ashworth and Lucas, 1998), focus maintained on what they said (Åkerlind, 2005b), not appropriated into pre-existing themes or categories (Barnacle, 2005; Marton, 2000), and all data were used in forming the categories of description (Walsh, 2000; Åkerlind, 2005b).

These findings include an overview of the range of variation, key elements of differentiation and the qualitative relationship between the categories, as befits phenomenographical research (Prosser et al, 2005; Åkerlind, 2005b), the logical structure between categories also not having been pre-determined (Åkerlind et al, 2005). The four categories of the phenomenographic outcome space presented differ qualitatively in ways that are complex, but which enable consideration of perceptions arising from this group of SPTs to enable reflection upon how others may perceive mathematics and to identify and make explicit personal perceptions.

Through non-dualist phenomenographic method, the relational aspect between the experiencer (SPT) and the experienced (mathematics) (Bowden, 2005; Pang, 2003) was examined, the premise of this study being that SPTs' relation with mathematics is the key to their own mathematical understanding, the way in which they will learn in ITT, and the shape their own practice as future teachers of primary mathematics will take. As such, using this outcome space to reflect upon in order to establish personal mathematical philosophy, based on previous experience and aspirations for the future, can facilitate SPTs ascertaining their personal learning goals and needs for ITT and determining the potential changes that are needed in order to fulfil their mathematical philosophy in practice.

The phenomenographical methodology has included SPTs' accounts of their previous learning of mathematics and aspirations for their future practice as teachers, both of which suggest a particular need for SPTs to consider the varied pedagogical approaches associated with the findings presented here. This variation ranges from mathematics viewed as an external entity to be learnt via imposed methods with learners being limited to mathematical knowledge that lacks associated understanding, to mathematics viewed as a creative product of learners' active engagement with and thinking about phenomena that leads them to ask questions and where a focus on mathematical process based on mathematics as a way of thinking is key to asking and seeking solutions to problems. In consideration of this, in Chapter 5 the findings of this chapter are discussed in relation to a range of associated pedagogical approaches and these in turn are discussed in Chapter 6 with regard to potential implications for SPTs' development in ITT.

Chapter 5 Discussion Of Findings – Pedagogical Associations

5.1 Introduction

The range of variation of perceptions of mathematics amongst student primary teachers at the outset of Initial Teacher Training is established in Chapter 4. In this chapter that variation is considered in relation to pedagogy in consideration of the second research question for this study:

How does the range of variation of perceptions of mathematics relate to primary mathematics pedagogy?

The underlying concern that led to this research is the difficulty within primary mathematics education in providing effective mathematical learning opportunities for children. Central to this study are student primary teachers who will play a vital role through their future practice as teachers of primary mathematics in that provision. It is argued here that their perceptions of mathematics will affect the way they learn – and hence have potential implications for their forthcoming learning in ITT – and the way they will teach – and therefore also have potential implications for their development in ITT. Thus, there is discussion in this chapter of the study's outcome space and associated pedagogy, in order to identify potential implications for development in ITT, which are then examined in more detail in Chapter 6.

There is limited research in this field specific to SPTs in Britain. As such, much of the discussion of this study's findings is confined to use of theory associated with primary mathematics education generally and literature pertinent to adults' attitudes towards mathematics. This research is unique in its determination of the range of variation of mathematical perceptions specifically amongst SPTs at the outset of ITT and its associated identification of potential implications for SPTs' development in ITT.

5.2 Conceptual Framework

This study, aligned with a non-dualist conceptual framework, examines the variation in the ways that SPTs perceive mathematics, ranging from gaining mathematical knowledge through an external mathematical relationship; gaining mathematical knowledge via an internal mathematical relationship; developing mathematical understanding through an internal mathematical relationship; to intentions to facilitate children's construction of an internal mathematical relationship that promotes mathematical understanding.

The first three categories of description of the variation refer to pedagogical approaches experienced as learners of mathematics whilst the highest category refers to SPTs' aspirations for their own teaching and associated pedagogy. The range of variation constitutes perceptions of:

- experienced and intended pedagogical approaches
- the relevance of mathematics
- mathematical understanding
- mathematical engagement

- feelings about mathematics
- beliefs about the nature of mathematics
- beliefs about the nature of mathematicians
- expectations for learning about primary mathematics education in ITT

These elements of differentiation are examined in this chapter in terms of how they relate to pedagogy, with existing theory used in analysis, and is the first to examine the range of variation of mathematical perceptions of SPTs at the outset of British ITT in such a way.

5.3 Experienced And Intended Pedagogical Approaches

The outcome space presented in Chapter 4 indicates a range of experienced and intended pedagogical approaches amongst SPTs. At the lowest end of this range of variation is experienced dualist pedagogical practice whereby a transmission approach is used, learners being apparent passive receivers (Howell, 2002; Desforges and Cockburn, 1987) of an external body of facts with little opportunity to construct meaning. Mainly confined to number facts, learnt through recitation and repetition, experience is similar to the Victorian methods outlined by Sharp et al (2009) and leaves learners disassociated from a wider mathematics vocabulary.

The second category, qualitatively different from externally imposed factual knowledge, constitutes experienced pedagogy dependent on a body of mathematical knowledge known to and explained by the teacher through whole class demonstration of set rules and procedures, as seen in primary schools by Askew, Brown, Johnson, Rhodes and Wiliam (1997), that are then used by learners, through limited internal relationship with mathematics, to reach a set of 'correct' answers, through reproduction (Desforges and Cockburn, 1987), to closed questions (Brown and Walter, 2005).

Further variation is aligned in Category 3 with mathematics experienced via a socially constructed learning environment with scaffolded support based on learners' identified needs from approachable teachers (Yackel and Cobb, 1996) available to provide help and of whom learners feel confident to ask questions as advocated by O'Sullivan, Harris and Sangster (2005). An internal relationship with learning mathematics is enabled, whereby teachers facilitate opportunities for mathematical thinking, doing and process.

This non-dualist pedagogy comprises the highest category as espoused practice for an active learning environment conducive to children internally relating to mathematical understanding through engagement. Included is concern for children's mathematical learning and a desire for inspirational pedagogical practice to effect approachable teachers who encourage enjoyment of learning mathematics and promote interest, fun and excitement amongst learners, encouraging children's confidence to do their best, through the kind of environment espoused by Billington et al (1993) where children feel free to take risks with questions and strategies, support each other, query their understanding, work at their own pace and are not 'afraid to get things wrong' as advocated by Ernest (2000). However, detail of the way in which such an approach promotes relational mathematical understanding is limited amongst SPTs' accounts constituting this category.

Whilst existing theory relating to primary mathematics aids understanding of the variation in SPTs' experienced and aspired pedagogical mathematical perceptions, this study is unique in its determination both of the range of variation and exploration of potential implications for development. The findings indicate that experience within the lower categories constitutes dualist practice that has limited formation of an internal relationship with mathematics and equates to limited mathematical knowledge. Such a dualist pedagogical approach to mathematics has been considered in theory to be confined to history (Anghileri, 1995), yet in the more recent past it has been suggested otherwise (Ofsted, 2008; Ernest, 2000) and this study confirms that it constitutes the experience of current SPTs.

If theory is correct in the assertion that perceptions can influence practice (Nespor, 1987), then there may be implications concerning the potential reproduction of such pedagogy in SPTs' future practice, since dualist experience potentially limits the ways that mathematics learning and teaching is perceived, which in turn potentially has implications for learning in ITT.

In addition, although the range of variation in described pedagogy includes experience and aspirations for learners developing internal, relational mathematical understanding, specific description of how this is brought about is not transparent and hence requires further examination.

5.4 The Relevance Of Mathematics

The lower category of variation describing an external relationship with mathematics constitutes mathematical knowledge limited to basic numeracy. A lack of connection is made with other aspects of mathematics (Hopkins, Gifford and Pepperell, 1999) and real life (Bottle, 2005; Romberg and Kaput, 1999), other than the necessity for learning to become a teacher. The need for learning aspects of mathematics is questioned, with indication that a perceived lack of relevance contributes to difficulties in learning mathematics.

There is qualitative difference in the variation whereby, in the second category, pedagogy constituting set rules, procedures, content, tasks and mathematical ability leads to some internal connections made between mathematics and real life, an element of mathematical learning identified as important in theory (Boaler, 1997; Nunes and Bryant, 1996; Atkinson, 1992), but these connections are not entirely transparent.

Further variation is demonstrated in Category 3 whereby a view of mathematics is presented as extending to something more than number, with different aspects of mathematics acknowledged. The usefulness of mathematics (Mason, 2000; Hickman et al, 1998) is recognised, its relevance described and links made between mathematics and other school curricular areas.

Perceived usefulness of mathematics is differentiated in the final category, in accordance with Ernest's (1991) Public Educator view, of mathematics being regarded as essential, with aspirations for mathematics to have relevance for children, for cross-curricular connections to be made, and school-based mathematical activity to be linked to children's real life as advocated by Anghileri (1995). In conjunction with Nunes and Bryant's (1996) assertion that mathematics is an integral part of the culture from which it originates, this category constitutes the difference of mathematics viewed not as a separate subject, but from a holistic viewpoint which promotes a non-dualist pedagogical approach that supports children's innate thinking aligned with the way they view their world – an approach advocated by Sakshang et al (2002) whereby mathematics is integrated within the school curriculum.

Hence, variation in SPTs' perceptions reflects the varied perceptions of mathematical relevance that have been recognised within existing theory amongst primary mathematics pedagogical practice, although this study is unique in its concentration on those perceptions as held by SPTs at the outset of ITT. These findings raise potential implications for SPTs' development with regard to perceptions of the relevance of mathematics for learning in ITT, for the purpose of teaching mathematics, and for consideration of the way in which mathematics can be taught holistically as opposed to a separate subject in the primary school in order for children to engage with its holistic relevance.

5.5 Mathematical Understanding

The lowest category of variation depicts a dualist pedagogical approach constituting memorisation of externally given facts, limiting mathematical knowledge to recall, which links to perceptions of mathematics being a difficult subject, due to learners' lack of understanding, as supported in theory amongst learners by Pound (2008). However, this experience has associated recollections of being enjoyable and useful, analysis of which suggests that description of repetition enabling the facts to 'sink in' indicates an absorption of facts, as also observed in the work of Ambrose (2004) and Wong (2002). Although recent government guidance (DfE, 2003) supports memorisation of mathematical facts, SPTs' described experiences raise concern regarding the difference between recall of externally presented facts and recall of mathematical knowledge with which the learner has previously internally engaged to reach understanding.

Perceptions of mathematical understanding vary in Category 2 amongst learners given more autonomy in internalising mathematics as they attempt to make sense of given rules and procedures via individual mathematical work. Similar to aspects of the Platonist perspective (Ernest, 1989), there is some opportunity for learners to 'discover' the mathematics provided by teachers via working through given examples. Whilst there is some active learning and teacher interest in learners' mathematical thinking, an emphasis on process is not transparent and is not actively encouraged by the open-ended tasks advocated by Oxford and Anderson (1995). Procedures are taught in isolation (Anghileri, 1995) and, whilst some mathematical knowledge including vocabulary, facts, rules and methods is acquired, these are reliant on memory, suggesting what Mji (2003) terms 'limited, fragmented understanding' amongst learners that exacerbate their perceptions of 'falling behind' (Shodahl and Diers, 1984) as a set body of mathematical knowledge is passively received through demonstration by the teacher (Ernest, 2000) with some individual engagement via set tasks – but for little purpose other than passing assessments, the acquired knowledge for which can then be forgotten.

These accounts differ qualitatively in Category 3 from understanding apparent from experienced pedagogy that focuses on mathematical process as opposed to limiting mathematics to producing correct answers (Mikusa and Lewellen, 1999) with emphasis on thinking and facilitation of learners' internal relationship with mathematical learning. Recording mathematics as a way of supporting the learner's thinking processes is recognised, in addition to teachers understanding that process, and suggestion of learner autonomy in choice of mathematical strategies (Pound and Lee, 2011) – an approach facilitated by the NNS (DfEE, 1999b) and PNS (DfE, 2003). O'Sullivan et al's (2005) recommendation of teachers being aware of learners' existing understanding is apparent as difficulties are diagnosed and differentiation put in place for learners' needs with a focus on mathematical understanding using a variety of methods to aid development. Descriptions of mathematical learning are consistent with constructivist theory (Bruner, 1966) whereby mathematical knowledge is constructed through active engagement, mirroring elements of a Purist view (Ernest, 1991) where mathematical process and learners' development through construction of understanding are emphasised within a supportive environment. However, conceptual awareness of mathematical process is not in itself fully understood since accounts include anxiety regarding ITT in terms of dualist requirements for right answers, providing explanations and showing children how to do mathematics.

Non-dualist learning opportunities are presented in the highest category in the form of aspiration for pedagogy as aims are expressed for active and creative practice (Anghileri, 1995) to facilitate children's internal relationship with mathematical learning through questioning, collaboration, practical activity, use of different strategies and development of mathematical thinking. Intentions for practice are described in terms of mathematical engagement to develop relational understanding through active development with a focus on mathematical process, linking to the non-dualist notion of constructing mathematical understanding through doing (Atkinson, 1992). However, described aspirations for pedagogy do not extend to detailing mathematical process in terms of pattern seeking (Anghileri, 1995), playing, experimenting, noticing, describing, showing, articulating, testing, convincing, consolidating, developing new situations and contexts (Delaney, 2010).

Whilst different levels of mathematical understanding amongst learners are substantiated by existing theory associated with mathematics pedagogy, this study's determination of the range of variation in SPTs' perceptions at the outset of ITT with regard to mathematical understanding is unique in its facilitation of raising potential implications specific to ITT development. The variation indicates potential expectations ranging from being provided with knowledge to absorb, tasks to engage with to elicit knowledge, active engagement with learning to create personal understanding, together with associated expectations of primary mathematics teaching in this regard and the need for awareness of elements of pedagogy that aid development of mathematical understanding.

5.6 Mathematical Engagement

The lowest category in the outcome space constitutes experienced pedagogy where an external mathematical relationship is formed through rote learning of facts to be remembered in an individual learning environment, frequently with separate seating and silence, as raised in previous research by Akinsola (2008) and Lampert (1990). The exposure to an instrumental teaching environment where mathematical understanding is not achieved leads to learners believing themselves to be at fault since this external body of mathematical knowledge remains elusive. There is a tendency to 'switch off' from mathematics and an acceptance of the status quo, dovetailing existing theory of disaffection (NACCCE, 1999), with feelings of being 'written off' (Haylock, 2010) and teachers' indications of learners' lack of mathematical ability (Miller and Mitchell, 1994). The perceived 'correct' nature of mathematics gives 'no room for error' resulting in learners giving up (Skemp, 1989), disguising their situation through coping strategies (Maxwell, 1989; Cockcroft, 1982) and trying to avoid mathematics wherever possible (Brady and Bowd, 2005).

Variation in the second category constitutes awareness of lack of understanding through attempts to remember rules without knowledge of how and why they work, previously identified amongst learners by Davis (2001), and associated attempts at mathematical engagement. There is a move from memorisation of facts to memorisation of rules (Akinsola, 2008) and procedures (Kyriakides, 2009; Boaler and Greeno, 2000; Boaler, 1997) which introduce an element of internally related learning through individual engagement with tasks, but this is limited to mechanical use (Lampert, 1990) since the rules and procedures are not necessarily understood (Haylock, 2010; Grootenboer, 2008; Nunes and Bryant, 1996). There is a lack of enjoyment, interaction and use of practical resources, as described also in schools by Foss and Kleinsasser (2001). The expectation to follow the strategies of the teacher and the structure of the examples provided are regarded as unhelpful, with embarrassment, humiliation and the shame that Bibby (2002a) also observed amongst teachers. However, there is no display of the 'victim mentality' termed by Hwang (1995) since, as concurrent with Brady and Bowd's (2005) work, experienced mathematics is critiqued in relation to whether mathematical difficulties are inherent or due to teaching styles. There is clear desire to engage mathematically through attempts to question teachers, who are unapproachable, impatient and disinterested, as also evidenced in Haylock's (2010) observations of learners who were too afraid to ask questions. There is awareness of specific areas of mathematics causing difficulty and in analysis of the inadequacy of the pedagogical approach (Brown, McNamara, Hanley and Jones, 1999), limitations are identified in terms of lack of diagnosis of learning mathematics, provision for different learners' needs and teaching for understanding – with various alternative approaches to enhance mathematical engagement provided. However, this incorporates a lack of substance in terms of how mathematical understanding can be attained.

There is qualitative variation between awareness of lack of mathematical understanding and attempts to engage outlined above, and the pedagogy constituting Category 3 whereby an interactive learning environment is experienced and learners engage in mathematics through collaboration as advocated by Von Glaserfeld (1990), bringing different explanations to the learning group, suggesting the social community which communicates to reach shared meeting that Vygotsky's (1978) work espouses and implying a development of mathematical vocabulary (Wilson, 2009) as part of developing mathematical communication. Concurrent with Lee's research (2006), there is engagement via the use of visual apparatus and practical work as advocated by Brown (2000) including games that helped learning through concrete hands-on manipulation (Edwards, 1998) and puzzles to engage logic and reasoning skills. Aspects of the experienced pedagogical approach considered useful to mathematical learning are identified as elements to include in future practice, although elucidation of how these approaches aid understanding are not apparent.

This experienced pedagogy varies from Category 4's described intentions to encourage mathematical engagement and independent thinking in a supportive environment as advocated by Burton (1994), meeting children's needs through flexibility and diagnosis of understanding, supporting Nathan and Koedinger's (2000) suggestion that previous learning is built upon through making connections with existing mathematical knowledge and learners' active engagement with mathematics. Socially constructive elements of pedagogy that are identified from experience are described as approaches to be utilised in future practice, in addition to planned use of differentiated learning opportunities. Active mathematical engagement is espoused through discussion and interaction, which according to Bottle (2005) can aid mathematical understanding, and encouragement for children to question, as advocated by Pound and Lee (2011) as a crucial element of developing mathematical reasoning, with collaborative working espoused where children can describe and discuss their ideas - practice which Anghileri (1995) recommends in children making sense of both their own and others' reasoning and in developing communication skills and mathematical vocabulary. However, detailed benefits of these elements of pedagogical practice are not apparent in terms of the construction of understanding through relational engagement and the mathematical thinking that theorists recommend (Skemp, 2002; Desforges and Cockburn, 1987).

Hence, the outcome space constitutes variation in mathematical engagement ranging from an external relationship with memorisation of mathematical facts leading learners to switch off and avoid mathematical situations, internal memorisation of rules by learners aware of a lack of understanding and in need of an alternative pedagogical approach to aid mathematical understanding, experienced mathematical engagement through an internal relationship with mathematics through interaction and use of learning materials leading to mathematical understanding, and espoused practice for the latter. Whilst the elements within this range are also apparent in existing literature pertaining to mathematics education, this study identifies the existence of the range of differentiation specific to SPTs. In so doing, there is identification of a lack of detail in articulated pedagogy deemed unsatisfactory and that which apparently aids mathematical understanding, and there are hence potential implications for SPTs' increased awareness of alternative pedagogy and the use of learning materials in terms of how mathematical understanding can be aided.

5.7 Feelings About Mathematics

The initial interest in this area of research arose from practical evidence of SPTs presenting with anxiety regarding mathematics on ITT courses (Jackson, 2008; Jackson, 2007), this observation being supported by literature as negative perceptions have been observed amongst both practising (Briggs, 2009; Wilkins, 2008; Battista, 1999; Ernest, 1991) and student primary teachers (Haylock, 2010). Negative perceptions of mathematics, so clearly identified generally amongst adults (Bibby, 2002a), are also clearly part of the lowest category of the outcome space. Despite indication of mathematics itself being regarded as 'unemotional' (Paechter, 2001), assertions of learners' associations with mathematics being highly emotional (Brown, 2005; Perry, 2004; Bibby, 2002a) are upheld by this study as the fear suggested amongst adults by Akinsola (2008) and Haylock's (2010) intimation of being afraid of looking stupid in front of others and of 'not being able to do' mathematics (Buxton, 1981) clearly constitute Category 1. The imposition of mathematics is compounded by perceptions of external pressure levied by teachers who are described in the hostile and unhelpful manner that previous research has also evidenced (Brady and Bowd, 2005; Jackson and Leffingwell, 1999; Briggs and Crook, 1991) to the extent that learners are afraid of teachers, which has shown to be a factor detrimental to mathematical performance (Cockburn, 1999). Strong impressions are evoked of dislike, as also observed generally in theory (Ernest, 2000; Cherkas, 1992; Buckley and Ribordy, 1982) and anxiety (Haylock, 2010; Cornell, 1999). This anxiety is physically manifested in precisely the ways previous researchers have described learners' engagement with mathematics – tension (Akinsola, 2008), bewilderment (Buxton, 1981), confusion (Kogelman and Warren, 1978), inability to think clearly (Tobias, 1978) and panic (Buxton, 1981) whereby mathematical experience is described in this study as 'freezing in the headlights', their minds paralyzed (Morris, 1981) and ceasing to work. Although the specific physical debilitation described in theory of inability to perform on tests (Smith, 1997), illness and faintness (Smith, 1997), sweating, nausea and palpitations (Krantz, 1999), churning stomach (Maxwell, 1989) and difficulty breathing (Akinsola, 2008) is not evidenced amongst SPTs in this study, their experiences clearly present uneasiness (Smith, 1997), feelings of helplessness (Akinsola, 2008), crying whilst struggling to learn (Ambrose, 2004), and not being able to cope (Akinsola, 2008).

There is qualitative differentiation between this fear, distress and the external pressure imposed on learners described above, and the second category of variation whereby learners do not feel encouraged by teachers, and perceive learner-focused pressure constituting dissatisfaction in their ability to 'do' mathematics, evidencing comparison of their own ability with that of peers, competition to get ahead of others in the schemes of work and awareness of a societal perception of having to be numerate (McLeod, 1992). Lack of opportunities to engage with and understand mathematics is linked to feeling the frustration that Haylock (2010) also observed amongst SPTs, together with a legacy of anxiety concerning the depth of mathematical knowledge for forthcoming learning in ITT.

Feelings towards mathematics vary again in the third categorisation where, despite the more positive accounts of mathematical experience, these describe a somewhat ambivalent attitude and varying degrees of confidence. The evidence of understanding developed from an internal relationship with mathematics does not equate to fear or frustration, but neither does it evoke intrinsic enjoyment of mathematics.

The fourth category in the hierarchy, however, constitutes an enjoyment of, interest in, and liking for mathematics which correlates with confidence in mathematical ability extending to excitement at the challenge of engaging in mathematical activity, alongside fascination for the way in which mathematics allows the world's mysteries to be explored and understood.

Accounts support those of theory with regard to negative emotions associated with mathematics, but research specific to SPTs is limited, and this study specifically identifies the range of variation in mathematical emotion ranging from fear and anxiety through ambivalence to enjoyment and excitement. This study therefore not only contributes to current restricted knowledge pertaining to mathematics education, it is also the first to establish the range of variation in emotional mathematical perceptions of SPTs as they embark upon ITT. Whilst at the top end of the outcome space enjoyment, excitement and fascination is expressed, there are potentially strong implications for the identified feelings of fear and anxiety of mathematics prevalent in the lower categories with regard to SPTs' learning in ITT and their practice as teachers of mathematics.

5.8 Beliefs About The Nature Of Mathematics

In the first category of the outcome space, SPTs' accounts describe mathematics as 'something to be done', seen as separate from relational engagement and imposed upon learners from which there is no escape. The dualist perception presented is one whereby learners view mathematics as dependent on right and wrong answers (Haylock and Thangata, 2007), motivation within such a pedagogical approach being the closed questions suggested by Boaler and Greeno (2000), the correct answers to which are known already by the teacher (Lampert, 1990), that lead to concentration on the goal of giving the correct answer (Cross, 2009; Cockburn, 1999), and the behavioural reward of getting the answer right (Skinner, 1954). Mathematics is described as something learners should be good at, resulting in shame at not meeting expectations (Cockcroft, 1982), taking blame for not being able to follow what teachers present (Miller and Mitchell, 1994) and feelings of embarrassment, humiliation, stupidity and fear at not being able to produce right answers. The distress that has been demonstrated in existing research amongst learners of mathematics (Akinsola, 2008) is evident, alongside a perception of being a nuisance to teachers (Haylock, 2010), as learners get left behind, struggling to catch up to others as opposed to learning at their own level, akin to the competitive element purported by Boaler and Greeno (2000). There is a perceived need for mathematics to be carried out at speed (Bibby, 2002b) and experiences of teachers favouring the learners who do get the required right answers. Confusion is expressed at the need to present neat mathematical recording (Boaler and Greeno, 2000) linking the pedagogical approach to Ofsted's (2005) observations of teachers emphasising recording at the expense of mathematical reasoning.

The notion of mathematics constituting the requisite 'right' answer is apparent in the second category, with the qualitative difference of a degree of comfort displayed in being able to reach answers. However, these answers are produced by following teacher-given rules and procedures that are demonstrated in a short, hurried manner and not necessarily understood (Brady and Bowd, 2005), or presented in textbooks constituting received knowledge (Boaler, 2002) from the source chosen by the teacher. Despite attempted internalisation through individual working, there is expectation to use the teacher's demonstrated strategy, considered to be the correct method, as also observed in research by Bibby (2002b). The frequently single method used by the teacher is followed in the spirit of what Boaler and Greeno (2000) call 'knowing the tricks' where there is a lack of flexibility in strategies as outlined by Schuck (2002) and the danger of what Burton (1994) terms 'learner dependency' as autonomous use of a variety of methods is not encouraged.

Variation from the pursuit of right answers via reproduction of recalled facts or the following of given rules and procedures is shown in the third category whereby an internal relationship is formed between mathematics and learner through active engagement in order to reach an understanding of the process undertaken to seek answers, and an understanding therefore of how answers were reached. Learners are encouraged to make cross-curricular links, question, discuss, explore practically, work collaboratively and develop relational understanding. However, a structured view of curriculum links to a perception of mathematical facts, rules and procedures being scientifically composed and, since science is not considered to be creative, neither is mathematics.

Rather than viewing science as a structure for adherence to be used and applied, the final category depicts the creative nature of science and mathematics providing a means to understand the world, whereby learners create personal meaning (Trigwell, Prosser and Ginns, 2005). The mathematical perception is one of creative process, based on the relation between phenomena and the experiencer, the meaning being subsequently created constituting the mathematical understanding of the individual as inherent human need for 'making sense of the world'. It is thus considered a process of problem-solving that focuses on the child's understanding (Ernest, 1989; Kuhs and Ball, 1986) based on personal construction of mathematical knowledge and dependent on relational learning (Skemp, 1989). In indicating the desire to encourage children to question, there is intimation of an element of problem posing, practice advocated by Pound (2008) and considered an essential element of learning as problems are reformulated and generated in the problem solving process (Brown and Walter, 2005). Pedagogical practice is espoused that encourages exploration and investigation, with children working in a creative rather than a structured way in order to access personal meaning, indicating the development of mathematical thinking as solutions are sought to problems (Pound, 1999). The use of different strategies is advocated, an approach that can facilitate construction of meaning and understanding as connections are made, patterns spotted and relationships recognised according to Pound (2008) although this level of detail is not apparent within SPTs' accounts.

Beliefs about mathematics therefore constitute a range of variation in mathematical perceptions, separate elements of which are supported by existing research into learning mathematics. However, the variation established in this study is unique in its facilitation of the range of SPTs' perceptions to be determined and associated potential implications identified for ITT development. The perception of mathematics as an external entity to be endured in the pursuit of right answers is a limited view that could potentially be repeated in practice as well as limiting future learning, alongside the perception that right answers are produced by correct procedures, limiting the potential for relational mathematical understanding using a range of strategies and autonomous thinking. The variation extends to learners' internal relationship with mathematics formed via a mathematical process of enquiry, which is viewed as non-creative, bound by facts and rules, and therefore limited again in terms of autonomous mathematical thinking relational to the individual. In contrast, at the end of the range of variation, mathematics constitutes a creative process through which individuals make sense of phenomena by questioning, exploring and investigating. The established range is therefore of use in SPTs' reflection upon differentiated perceptions of the nature of mathematics in ascertaining personal philosophy.

5.9 Beliefs About The Nature Of Mathematicians

In the lowest category of the outcome space, accounts are consistent with learners who do not perceive themselves to be mathematicians, since mathematics is regarded as an entity to be avoided, the domain of those who 'can do it', concurring with the findings of existing research of mathematics confined to the realm of the clever (Sowder, 2001), intellectual and gifted (McVarish, 2008). It is interesting to note, however, that despite a range of research indicating a gender-related perception of mathematics (McVarish, 2008; Brady and Bowd, 2005; Furner and Duffy; 2002; Cooper and Robinson, 1989; Tobias, 1978), this is not a factor raised by SPTs in this study.

Qualitatively different from mathematicians being perceived as clever is the notion in the second category that mathematical aptitude is a result of genetics (Haylock, 2010;

Haylock and Thangata, 2007), mathematicians being blessed with logical mathematical brains (Schuck, 2002; Furner and Duffy, 2002) dependent on logic (Frank, 1990) as opposed to creativity, and an association with awareness of a lack of mathematical ability.

Despite non-dualist approaches described in the third category that constitute creative pedagogy, accounts suggest that mathematicians are not creative, since the dualist set curriculum gives an impression of mathematical content to be learnt, based on scientific principles that in themselves are not associated with creativity. Mathematics is regarded as more difficult for some learners than others, with a societal acceptance of 'not being able to do mathematics', a notion also raised amongst learners in theory (Pound and Lee, 2011; Haylock, 2010; Lockhead, 1990).

In contrast, the highest category constitutes non-dualist perception of mathematics regarded as a relational conceptualization, whereby everyone is a mathematician since everyone makes personal sense and meaning from their surroundings. Mathematics constitutes a man-made framework created for the purpose of understanding, making use of and communicating within their world. Whilst a set of mathematics facts, rules and procedures is recognised, these are regarded as the result of Marton and Booth's (1997) 'described world' as experienced by humans, not to be memorised without understanding, not to be followed without understanding, and not to be presented without learner engagement leading to understanding. The creative nature of mathematical engagement is associated with inherent pleasure, corresponding to a Purist perspective (Ernest, 1991) of the intrinsically creative roots of mathematics formed by the inventors of the past (Dawson and Trivett, 1981). The pedagogical perception in this category is that, although a body of mathematics exists in terms of what has already been discovered, its 'rediscovery' by children can be brought about, not as viewing mathematics as a known entity to be found, but as an internal relationship between the child and phenomena via creative as opposed to instrumental means as they bring about their own mathematical meaning.

Whilst differentiated elements are supported by existing mathematical research, this study's outcome space facilitates a view of the range of SPTs' perceptions of what it is to be a mathematician and the link between other key aspects of the outcome space. A lack of personal association with mathematics connects with the notion of an external relationship with mathematics that has limited development of mathematical knowledge. Perceptions of mathematical ability being dependent on genetics and the ability to be logical also associates with limited development of mathematical knowledge, with some internal relationship with mathematics. This dependency is differentiated to being based on science, science not being regarded as creative, and mathematicians therefore not being regarded as creative. Whilst an internal relationship may be formed with mathematics, a caveat is indicated that if mathematics is not understood fully, it is socially acceptable, due to the factors upon which being a mathematician is dependent. This range of perceptions of mathematicians potentially has implications for SPTs for their own learning and the message given to children if limitations are placed on mathematical learning, in contrast to everyone being considered a mathematician where mathematics is part of everyone's everyday lives and constituted of creative conceptualisation of surrounding phenomena.

5.10 Expectations For Learning About Primary Mathematics Education In ITT

In terms of future practice, no SPTs in this study made any assertion that they may prefer to teach younger children on an assumption that the mathematics required might be easier, as might be expected according to Tobias (1978) and given fears expressed about mathematical engagement and understanding. Indeed, SPTs appeared to be made of sterner stuff than might have resulted from dualist pedagogical exposure, since they have not 'dropped out' (Papert, 1981) or avoided mathematics classes (Smith, 1997) altogether, for, despite their very real fears of mathematics, they return to learning mathematics as part of ITT. However, the fear of mathematics constituting the first category of the outcome space transcends into adulthood, with the lack of mathematical confidence and self-esteem experienced by adults in theory (Pound, 2008; Akinsola, 2008) and perceived mathematical inability (Metje, Frank and Croft, 2007) manifested as ITT is approached with fear, with concerns about being asked questions to which answers are not known.

Qualitatively differentiated from the fear of the first category is the anxiety of the second whereby the nervousness that Ernest (2000) witnessed is apparent in SPTs' accounts of the desire to meet children's needs accompanied by awareness of a lack of mathematical understanding. ITT is approached with concern for providing right answers and explanations for children and to be able to 'show' children 'how to do' mathematics. However, there is also clear desire to make changes to overcome difficulties alongside indications of how this might be accomplished during ITT.

The third category constitutes awareness of mathematical process being an important element of reaching mathematical understanding, with planned actions for improvement in order to improve personal relationship with mathematics and develop understanding.

Further variation is shown in the final category of awareness of learning needs within ITT and improvements necessary, alongside acknowledgement of the stimulation inherent in challenging mathematical perceptions. Recommendations for future practice are presented in SPTs' accounts alongside recognition that although learning mathematics may not be easy, its mysterious nature need not be viewed as a frightening entity to be avoided with convictions of inability, but instead that a sense of excitement and satisfaction can arise from the challenge, in accordance with Orton and Frobisher's (1996) assertion that frustration and disappointment can live alongside pleasure and satisfaction. Changes already made are apparent within SPTs' accounts, based on recent experience, an observation previously seen in practice by Fairclough (2002). Just as mathematics itself can seen to be challenging, SPTs challenge their own thinking, as Sakshang et al (2002) suggests might be necessary, as accounts include philosophical description of pedagogical aspirations for practice. This category constitutes critical realism that espoused pedagogical practice will not be straightforward, with specific regard paid to planning for differentiation; constraints of time and curriculum content; and government pressure from the statistics, documentation and testing that Brown (2010) also raises in witnessing procedure-based pedagogy in primary mathematics education. However, despite SPTs' accounts suggesting that mathematical learning can be enjoyable, stimulating and fascinating, in line with theoretical assertions of this nature (Haylock and Thangata, 2007; Andrews and Hatch, 1999; Skemp, 1989), descriptions do not include explicit detail of how to foster this attitude in learners, although SPTs themselves portray such attitudes towards mathematics as ITT is enthusiastically anticipated.

Theory supports the elements of the differentiated perceptions described generally amongst learners of mathematics, but this study's outcome space includes the range of variation in the way that SPTs perceive their forthcoming ITT learning, ranging from fear and anxiety to recognition of necessary improvements that have been planned and begun with perceptions being challenged, alongside recognition of potential constraints to intended pedagogy.

5.11 Summary

In answer to the second research question for this study, differentiated elements of the range of variation of SPTs' mathematical perceptions at the outset of ITT are examined here with regard to their pedagogical associations, this study being the first of its kind to do so.

This exploration of the phenomenographical outcome space provides insight into varied ways in which experienced and espoused pedagogy may influence SPTs' learning in ITT and their future practice as primary teachers of mathematics, and the discussion of findings raises potential implications for SPTs' development within ITT in fulfilment of the study's aim.

In achievement of the study's intended purpose, a framework is hence provided for SPTs' reflection at the outset of ITT in determination of their mathematical perceptions and analysis of associated experienced and espoused pedagogy;

comparison with alternative perspectives; consideration of influence of mathematical perceptions on learning and teaching; awareness of potential implications for development within ITT and development of a personal philosophy of mathematics on which to base their ITT development and identify and plan for change.

The range of variation, and its links to pedagogy presented here, is taken forward in Chapter 6 where the potential implications arising are discussed in terms of how they might be addressed, so that SPTs may be enabled to take an informed position before embarking upon ITT with regard to their aspirations for practice within primary mathematics education and to identify their associated learning needs prior to their course.

Chapter 6 Potential Implications For SPTs' ITT development

6.1 Introduction

In the previous chapter, discussion of the outcome space in relation to pedagogy raises potential implications for SPTs' development in ITT. The aim of this study is to ascertain the range of variation of SPTs' perceptions of mathematics at the outset of ITT with the purpose of facilitating reflection upon personal perceptions and those of others, alongside potential pedagogical implications, in order to enable mathematical perceptions to be made explicit, awareness to be raised, personal philosophy to be generated, ITT learning needs identified and goals set for future practice as teachers of primary mathematics. In this chapter, the potential implications arising from the pedagogical exploration of the outcome space are therefore discussed and recommendations suggested for SPTs' planned development in ITT.

In doing so, relevant literature pertaining to mathematics pedagogy is integrated in substantiation of the interpretation of the outcome space with regard to potential implications for SPTs' ITT development, this being the first study of its kind to consider the variation arising from a group of SPTs at the outset of ITT in this way and to use the pragmatic facility of phenomenographical research to suggest specific actions that the findings indicate could be conducive to SPTs' development and ultimately beneficial to provision of primary mathematics education.

6.2 Potential Implications For SPTs' ITT Development In Relation To Experienced And Intended Pedagogical Approaches

Experienced and espoused pedagogical practice constituted within the outcome space of this study raises potential implications for SPTs' development in ITT with regard to potential reproduction of approaches that were associated in the lower categories with limited engagement and relationship with mathematics leading to knowledge without understanding. Recognition is needed of dualist approaches of transmission of facts to be remembered (Category 1) and given rules and procedures to be practised (Category 2), and the limitations therein for facilitation of learners' internal relationship with mathematics and construction of meaning. The findings suggest a clear need for reflection on mathematical experience and analysis of experienced pedagogy to determine its benefits and drawbacks, to be aware of alternative approaches, to compare and contrast with personal experience, to form a personal philosophy of mathematics concerning what mathematics is, how it can be learnt and how it should be taught and for setting goals for ITT learning.

The upper end of the outcome space ascertained perceptions of experienced (Category 3) and espoused (Category 4) pedagogy perceived as encouraging development of mathematical understanding through emphasis on process via active engagement and scaffolded support (Yackel and Cobb, 1996) from teachers who were aware of learning needs (O'Sullivan et al, 2005). However, the specific ways in which the described practice might aid relational understanding was not apparent and as such, indicates a need to analyse elements of non-dualist pedagogy in terms of effective practice.

Hence, consideration is needed of approaches taken to previous learning, and SPTs' aspirations for their future engagement with mathematics as learners in ITT and teachers. SPTs may be unconscious of differentiated experience and perceptions (Ambrose, 2004; Bishop, 1991) and so consideration of the variation is a good starting point for developing a broader view (Petocz and Reid, 2005) and raising awareness (Houssart, 2009). Through consideration of, and comparison with, the full range of experienced and espoused pedagogy that constitutes the outcome space, decisions can be made regarding how mathematics can be learnt and taught, with mathematical perceptions identified and made explicit for SPTs to begin to formulate a personal philosophy on which they can base their ITT development.

6.3 Potential Implications For SPTs' ITT Development In Relation To Perceived Mathematical Relevance

The variation in SPTs' mathematical perceptions ranges from mathematics being viewed as pointless (Category 1), to vague association with real life (Category 2), to its uses as applied to everyday life (Category 3), to being regarded as an essential part of everyone's lives and the aspiration for mathematics to be viewed holistically as opposed to a separate part of the primary school curriculum (Category 4).

Contrasting with perspectives of more creative practice (Briggs and Davis, 2008), the notion of mathematics lacking purpose is borne out within theory (Bottle, 2005; Romber and Kaput, 1999; Hopkins, Gifford and Pepperell, 1999) and has been shown to contribute to difficulties in learning mathematics (Pound, 2008). This study is concerned with the potentially detrimental effect such perceptions may have specifically on SPTs' learning within ITT. The findings of this study suggest there is a need for perceptions of mathematical irrelevance to be challenged by SPTs in terms of past experiences of mathematics limited to acquiring knowledge as opposed to understanding mathematics in the context of meaning relevant to the learner. From a non-dualist perspective, there can be no irrelevance placed on mathematics since it exists only as a conceptualisation brought about by human attempts to make sense of what is relevant to their desire to understand. Reflection upon such a notion is therefore worthwhile in both considering the origins of mathematics and hence the purpose for learning mathematics, and in ascertaining the value of an holistic approach to teaching mathematics, whereby it is perceived as a human creation of understanding phenomena and hence has all-encompassing relevance to other school subjects and to life, since mathematics <u>is</u> the creation of understanding that which is relevant to us.

6.3.1 'Discovery' Of Mathematics

The findings raise the issue of mathematics being 'discovered'. Discovering mathematics is contentious firstly from the criticism that children's discovery has already been pre-empted by the discovery of others before them (Papert, 1981). Such pedagogy also has its critics in that 'discovery learning' has been purported to lead to underachievement (Boaler, 1997). What is more crucial to the findings of this study is that learners demonstrating difficulty with internalising mathematics, gaining knowledge without understanding in the lower two categories of the outcome space, experienced dualist pedagogy – one which views mathematics as a separate entity and hence 'out there' waiting to be discovered. In contrast, from a non-dualist perceptive, mathematics does not exist separately for discovery, since it is a human construct that is formed through individual learners' relationship with phenomena that brings about their own relational understanding.

In practical terms, whilst social collaboration enables children to bring ideas and suggested strategies to the learning group, teachers are also part of that social construction and therefore have the opportunity to introduce alternative ideas and methods. Rather than an expectation for learners to follow procedures without understanding, methods can be introduced when relevant to the learning circumstance and to the learner. Espoused pedagogy of Category 4, although not specifically articulated, intimates one whereby learners are not instructed, nor are they expected to 'discover' things for themselves, but is a practice whereby children can engage in activities to build on and develop mathematical understanding, with opportunities for teachers to introduce ideas and strategies in context to support sense making and meaning. However, this in itself raises potential implications for SPT development since such an approach is dependent on teachers' confident mathematical knowledge and understanding. There is therefore a need for SPTs to analyse their mathematical understanding and plan to make improvements they identify as necessary.

6.3.2 Holistic Approach To Teaching Mathematics

In the midst of the variation concerning mathematical relevance, a mixed pedagogy is constituted within SPTs' accounts of experience (Category 3). In conjunction with existing theory (Mason, 2000; Hughes, Desforges and Mitchell, 2000; Hickman et al, 1998), mathematical experience has included connections being made within mathematics extending beyond number, mathematics as a means of communication and relevant to a wide spectrum of everyday life – these being pragmatic and a valuable part of learning mathematics (Ernest, 2000). However, within the outcome space (Category 3), alongside this non-dualist perception is mathematics experienced as a separate school subject (Tobias, 1993), with links made to other school curricular areas (Coles and Copeland, 2002) under question, supporting the findings of existing research whereby mathematics was shown to be taught in isolation (Hughes, Desforges and Mitchell, 2000) and difficulties experienced by children in applying mathematics learnt to new situations (Carpenter and Lehrer, 1999).

Whilst such a mixed perception of non-dualist mathematical connection between aspects of mathematics and between mathematics and real life, and the dualist notion of mathematics constituting a body of knowledge separate to other areas of learning, is observed in general practice via existing research, the concern here is the exemplification of this mixed approach within SPTs' experience and the implication for their perceptions of mathematics as holistic understanding of phenomena and their associated practice in teaching mathematics with relevance for children and their understanding of their world.

The current English statutory curriculum (DfES, 1999a) is separated into subject areas, and, whilst recent government proposals suggest schools will be at liberty to decide how to teach the awaited revised curriculum (DfE, 2010), there remains the suggestion that mathematics will constitute a separate core section of knowledge to be taught and learnt. SPTs therefore need to consider how an holistic approach can be taken to mathematics teaching and determine the logistics of incorporating a set mathematics curriculum into a wider curriculum and the teaching approaches that can be taken to children learning mathematics in an holistic way.

6.3.3 Curriculum Content And Non-Dualist Mathematical Learning

The variation of SPTs' perceptions presents a curriculum-related paradox in the descriptions (Category 3) of mathematical content, learnt in a non-dualist environment, originating from a curriculum perceived to be a structure of facts, rules and procedures (Koshy, Ernest and Casey, 2000) to be learnt in a linear form (Oxford, 1990). Although this content is not imposed upon learners in a dualist transmission mode, its origins are a dualist prescribed curriculum, also contested as such within theory by Brown, Hanley, Darby and Calder (2007) and Hughes (1999). In addition to SPTs' perceived structure of a body of knowledge to be learnt, the current statutory (DfEE, 1999a) and most recent non-statutory curriculum guidance (DfE, 2003) are presented in Piagetian (1953) stages relating to levels of achievement and age, supporting dualist notions of fixed mathematical ability (Pound and Lee, 2011; Haylock, 2010; Clemson and Clemson, 1994).

If adherence to a prescribed curriculum is unavoidable for teachers whose government provides a statutory curriculum, then there is a need for SPTs to reflect upon ways in which this content can be approached via non-dualist learning experiences that provide opportunities for development of relational understanding.

6.3.4 Use And Application Of Mathematics

This aspect of the variation (Category 3) also raises an issue concerning SPTs' perceptions of the use and application of mathematics to their everyday lives. This element of mathematics is included within the current statutory curriculum (DfEE, 1999a) and incorporates the non-dualist approaches of reasoning, problem-solving and communicating. However, this has proved an area of difficulty within primary mathematics education, evidenced by the integration of this aspect of mathematics into the revised National Curriculum (DfEE, 1999a) from its separate section of the original version, and by the recommendation of recent government guidance (DCSF, 2008a) of more emphasis on using and applying mathematics being needed.

There are therefore potential implications for SPTs in consideration of the value of using and applying mathematics – in terms of making connections, as outlined above, between aspects of mathematical learning and applying existing mathematical understanding to new situations and to the application of mathematical understanding to other areas of learning and life.

Based on the findings of this study regarding differentiated learning (Categories 1-4), there is also a need for SPTs to consider the difference between use and application of externally imposed knowledge that is recalled with a lack of understanding and children building on internally related understanding as it is applied to new phenomena.

6.3.5 Autonomy

Analysis of the range of variation of SPTs' mathematical perceptions suggests that to espouse a particular pedagogy (Category 4) is insufficient since a clear philosophy of what the approach entails is needed if SPTs are to not only to teach in the way they aspire, but also make contributions when qualified to whole-school decisions on approaches to mathematics teaching and learning, especially since colleagues may not share perceptions (Briggs, 2009). Since indications from research are that schools have a tendency to accept and conform to government guidance (Andrews, 2007), teachers having been described as curriculum deliverers (Pound and Lee, 2011), it will take confidence to critique and challenge in such an arena (Haylock, 2010). The goal of the non-dualist practice which SPTs promote will be shared by others in terms of achieving mathematical understanding, but the true nature of relational understanding is dependent on perceiving mathematics as a creative entity that is fluid and continually changing and developing (Orton in Orton and Wain, 1994; White and Gunstone, 1992). However, despite the dualist nature of a prescribed curriculum, objectives therein can be taught in a creative, non-dualist way that is not instrumental but which provides opportunity for children to question (Pound, 2008), make connections (Suggate, Davis and Goulding, 2006), construct understanding (Haylock, 2010) and pose and solve meaningful problems (Schifter, Twomey and Fosnot, 1993).

There is therefore a need for SPTs to balance acceptance of government-directed and statutory curriculum with previous intimation of the promotion of creative practice (DCSF, 2008b; QCA, 1999) and the indication from new government of "allowing" (DfE, 2010, p10) schools to make decisions regarding how to teach, in consideration of pedagogical autonomy as befits their mathematical perceptions.

6.4 Potential Implications For SPTs' ITT Development In Relation To Perceived Mathematical Understanding

Two main potential implications arise from analysis of the range of variation of SPTs' mathematical perceptions in relation to mathematical understanding. The first is that the range of variation suggests potential implications for the way in which SPTs might approach their own development of mathematical understanding within ITT as well as associated approaches to their teaching (Categories 1-4). Secondly, although there are intentions for non-dualist pedagogy to engage children in active learning to create internal relational understanding within a socially constructive learning environment (Category 4), detail is lacking with regard to how mathematical understanding is developed via the described practice. To address both of these potential implications, there is a need for a clearer understanding of the espoused pedagogy that aims to help development of mathematical understanding.

An instrumental approach, where mathematics is experienced as a teacher-led externally imposed body of knowledge (Trigwell, Prosser and Ginns, 2005), is likely to lead to a surface approach to learning (Cano, 2005). The contrasting deep approach to learning mathematics focuses on the learner (Trigwell et al, 2005) and is dependent on the non-dualist perspective of engagement with phenomena in order to create personal meaning and understanding. There is therefore a need for SPTs to consider their previous learning experiences of mathematics and identify their expectations of ITT. If an instrumental experience of being taught mathematics has led to them take a surface approach to their own learning, change will be necessary in order for deep learning to be facilitated.

Teachers' mathematical knowledge is closely linked to effectiveness of practice in the classroom (Ofsted, 2005; Mooney, Briggs, Fletcher, Hansen and McCullouch, 2009; Cockburn, 1999) yet SPTs are far from alone in their difficulties in understanding mathematics as research demonstrates international concern amongst practising teachers (Chapman, 2007). ITT courses and standards for ITT (TDA, 2007) state required mathematics subject knowledge levels, with associated tests that SPTs will meet in order to qualify for the course and qualify for teacher status, but the anxiety expressed by SPTs (Categories 1 and 2) supports suppositions in existing literature (Golding, Rowland and Barber, 2002; Mewborn, 2001; McNamara, 1994) that reaching these standards does not necessarily equate to confidence in teaching mathematics for, as also suggested in theory (Fennema and Franke, 1992), a deeper understanding is needed. Whilst this study supports the findings of previous research such as MacNab and Payne's (2003) assertion for the need for SPTs to confront the nature of their mathematical understanding, it would be flippant, based on the evidence provided by these SPTs to merely suggest they do so as this is transparently an enormous difficulty and one of which they are themselves acutely aware. It is intended, therefore, that this study proves helpful in SPTs' confrontation of the issue in its identification of a range of associated potential implications that can be reflected upon in order to develop mathematical learning aspirations.

Within the variation there is clear desire to have experienced (Category 2) and to put into practice (Categories 3 and 4) a pedagogical approach that might aid mathematical understanding via a focus on the process of and active engagement with mathematics, but there is a lack of awareness of how such approaches aid mathematical development. There is therefore a need to not only analyse experienced pedagogy and ascertain aspirations for future practice, but also to unpick the key aspects of chosen pedagogy that benefit the development of mathematical understanding.

6.5 Potential Implications For SPTs' ITT Development In Relation To Perceived Mathematical Engagement

Variation in mathematical engagement presented in the outcome space concurs with existing theory in relation to differing pedagogical approaches linked to opportunities for mathematical understanding (Categories 1-4). However, this study adds the specific identification of mathematical perceptions of SPTs as they embark upon ITT, mathematical engagement being an element to reflect upon in terms of their own learning and their future pedagogy.

6.5.1 Rule Following

The variation in mathematical perceptions relating to mathematical engagement (Categories 1 and 2) raises potential implications for SPTs to analyse experienced instrumental pedagogy in terms of its usefulness in aiding mathematical understanding. The surface approaches experienced of teacher demonstration and explanation (Askew, Brown, Johnson, Rhodes and Wiliam, 1997) follow a dualist pedagogical approach of received mathematical knowledge of facts, rules and procedures (Category 2). Learners are reliant on the teacher (Boaler, 2002) and mathematical engagement is limited to demonstrated method (Boaler and Greeno, 2000; Schuck, 2002) considered to be the right way to do mathematics (Bibby, 2002b). Although some internal relationship is involved and some satisfaction is gained from being able to follow a given procedure and set rules (Burton, 1994), in order to reach the goal of the right answer, mathematical engagement is limited alongside awareness of not necessarily understanding (Haylock, 2010; Grootenboer, 2008; Brady and Bowd, 2005; Nunes and Bryant, 1996) and being dependent on mechanically applying the rules and procedures as demonstrated (Lampert, 1990).

The findings of this study therefore support SPTs' analysis of pedagogical approaches that expect given rules and procedures to be followed, in order for the extent to which this pedagogy aids mathematical understanding to be considered, and for reflections to be related to aspirations for both their own learning in ITT and their future practice as teachers of primary mathematics.

6.5.2 Learning Materials

The pedagogical approach limiting learning to mathematical knowledge, as opposed to understanding, involves facts, rules and procedures to be memorised and used mechanically through following teachers' demonstrations and working through examples on a classroom board, worksheets, cards or in textbooks (Category 2). Although some elements of mathematical engagement are involved, this mainly constitutes completion of closed tasks (Brown and Walter, 2005) to reach right answers through reproduction of the teacher's previously demonstrated method (Desforges and Cockburn, 1987). The dualist pedagogy of transfer of a set body of knowledge comprising of rules and procedures forms a surface approach to teaching that is evidenced as resulting in surface learning of memorisation of the rules and procedures (Kyriakides, 2009; Akinsola, 2008; Boaler and Greeno, 2000; Boaler, 1997), with little idea of the how and why (Davis, 2001), a perceived reliance on logic (Frank, 1990), lack of ability (Brady and Bowd, 2005) and notion of mathematical ability being fixed (Haylock, 2010; Haylock and Thangata, 2007; Schuck, 2002; Furner and Duffy, 2002) since awareness is demonstrated of a lack of understanding

(Mji, 2003) and of relevance of the mathematical tasks in the classroom to the real world (Boaler, 1997; Nunes and Bryant, 1996; Atkinson, 1992).

The argument for SPTs to analyse their previous mathematical experience in terms of the effectiveness of their learning is therefore strengthened in terms of considering the use of learning materials in mathematics regarding the facilitation of mathematical engagement and support for understanding, alongside consideration of more open-ended learning activities and materials (Oxford and Anderson, 1995) and how they might be more conducive to developing relational understanding.

6.5.3 Use Of Resources

Throughout the latter stages of the variation (Categories 3 and 4), there is support for the use of resources, but specific benefits for encouraging active mathematical engagement are not apparent.

Children's articulation of their mathematical understanding can be promoted and developed by using resources as a basis for discussion and explanation (Bottle, 2005). There is a need for SPTs' understanding of the pedagogical underpinning of mathematical engagement in terms of the social construction of mathematical understanding through the means of discussion, explanation, sharing, evaluation and negotiation (Kamii and Lewis, 1990; Askew, 1998) alongside the nature of knowledge construction in terms of an individual's thinking (Skemp, 1989) leading to relational understanding (Skemp, 1981).

As children work in practical situations with manipulative apparatus (Foss and Kleinsasser, 2001; Brown, 2000; Edwards, 1998) they can develop the use of mathematical symbolisation and vocabulary as they refine their ability to describe what they notice in terms of pattern and create generalizations (Bottle, 2005; Anghileri, 2003; Askew and Wiliam, 1995). SPTs need to consider how mathematical language can be gradually developed from children's own representations to more formal symbolisation and vocabulary (Anghileri, 2000; Nelson-Herber, 1986).

The varied perceptions pertaining to mathematical engagement (Categories 1-4) do not specifically make reference to the pedagogical approaches that address the abstract nature of mathematics that sense making and meaning is reliant upon as learners assimilate, construct and reconstruct understanding. Mathematical engagement involves imagining, which has shown to be a source of difficulty (Pound, 1998), as has articulating those imaginings (Skemp, 1989). This is therefore a key aspect for SPTs' consideration as it has been identified as one of the main contributions to perceptions of mathematics being difficult (Orton and Frobisher, 1996).

Resources can be used to embody abstract concepts by making concrete the visual and mental images involved (Lee, 2006; Moyer, 2001). However, a non-dualist approach is one whereby abstract phenomena come to be understood by learners and originate in that which surround us and, although teachers are encouraged to set mathematical learning within meaningful situations (Atkinson, 1992) this is not always manifested in practice (Ollerton, 2010; Boaler, 2009) and it is therefore crucial that SPTs consider carefully children's practical learning opportunities in order that the link is made between their real life and their school experience (Aubrey, 1997). It is also imperative that assumptions are not made about the concrete embodiment of abstract concepts through the use of resources, since research has shown that children's understanding can be confined to the use of the concrete and not accompanied by abstract conceptual understanding (Andrews, 2007; Askew, 1998; Gravemeijer, 1997).

It is important to note that, since creation of mathematical understanding is personal, children need to make their own decisions and choices when working mathematically and the accessibility of resources for their use is therefore to be considered (Burton, 1994). Encouragement and time is also crucial for children to engage with their learning and to think mathematically in order to develop the abstract reasoning behind the use of the practical resource (Delaney, 2001; Harries and Spooner, 2000; 1992).

Determination of the variation in SPTs' mathematical perceptions relating to mathematical engagement therefore highlights aspects of experienced pedagogy to be reflected upon, and decisions to be taken regarding this aspect of their intended pedagogy in formulating their mathematical philosophy of instrumental teaching versus active mathematical engagement to promote relational understanding.

6.5.4 Non-Dualist Pedagogy

The latter part of the range of variation (Categories 3 and 4) indicates that SPTs have begun to think about pedagogical approaches conducive to learners' internal relationship with mathematics to develop understanding, but the lack of detail pertaining to how such pedagogy is helpful suggests the need for raised awareness amongst SPTs regarding the beneficial aspects of the elements of practice they espouse with regard to how mathematical learning is facilitated.

Experienced and espoused pedagogy (Categories 3 and 4) suggests that learners be encouraged to engage mathematically through socially constructing their own mathematical understanding, with mathematics viewed as subjective, to be created in the learner's consciousness, and reconstructed as learning develops. In such a learning environment, interaction enables learners to engage with opportunities to internalise mathematical ideas through action, and as part of a social community to share meaning (Vygotsky, 1978). Children are encouraged and are confident to ask questions, of the mathematics, each other and of the teacher (O'Sullivan, Harris and Sangster, 2005). Social construction is further supported as children are encouraged to collaborate – the benefits being the presentation of their different ideas, explanations and articulation of their mathematical thinking to the group (Von Glaserfeld, 1990) which enables understanding to develop in both the individual mind and the collective (Burton, 1994) in both verbal and recorded forms (Floyd, 1981) with facilitation of ideas being introduced by the teacher within the group (Burton, 1994). Active engagement in a socially supportive environment enables learners to observe, play, experiment, explore, investigate, ask questions, pose problems, seek solutions, look for patterns (Pound, 2008); and facilitates development of describing, articulating, explaining, discussing, drawing, writing, using symbols and mathematical vocabulary to express what children come to understand from abstract mathematical concepts. Vocabulary can gradually be developed and refined (Wilson, 2009a) and different strategies shared and introduced (Pound and Lee, 2011). However, as Boaler (2009) has purported, a level of mathematical understanding and confidence is needed for a teacher to be open to such questioning and interactive engagement, and this study's findings indicate there may be difficulties in this area to be overcome for SPTs' future practice.

Hence, the outcome space facilitates SPTs to reflect on a range of variation of experienced and espoused pedagogy in relation to perceptions of mathematical engagement and as such raises potential implications for them to analyse this range in terms of rule following, use of learning materials and resources; analysing the drawbacks and benefits of instrumental and constructivist pedagogy in determining their personal philosophy on which to base their learning in ITT and future practice; and identification of improvement needed in mathematical understanding and confidence.

6.6 Potential Implications For SPTs' ITT Development In Relation To Perceived Mathematics Emotion

It is recognised that teachers influence children via their attitudes (DfES, 2002) and that positive attitudes are needed for effective mathematics teaching (Akinsola, 2008; Mooney and Fletcher, 2003; Cockburn, 1999). The variation within the outcome space (Categories 1-4) ranges between the excited, enthusiastic teachers advocated by ACME (2006) and severely negative feelings about mathematics (Morris, 1981) that are evidently not constituted of the irrational phobia that Hodges (1983) mooted could be the case amongst learners, but, as Bibby (2002a) observed amongst teachers, are highly emotional to the extent of the palpable fear witnessed by Akinsola (2008) and Buxton (1981).

Haylocks's (2010) term of 'emotional baggage' is clearly apt as the mental scars described by Suggate, Davis and Goulding (2006) of past mathematical experience remain, as Houssart (2009) indicates can be the case, with SPTs in their adult life (Category 1). Whilst coping strategies have been utilized in the past (Maxwell, 1989; Cockcroft, 1982) including avoidance (Brady and Bowd, 2005), SPTs have confronted their fears to embark upon their goal of becoming primary teachers and hence engaged in learning with regard to primary mathematics education in ITT – a commendable feat - but the fear can remain (Category 1).

There is a need for ITT providers to be aware of the extremes of SPTs' emotions towards mathematics (Swars, Smith, Smith and Hart, 2009) and, as Prosser et al (1998) purport, design courses accordingly. However, the intrinsic nature of perceptions, based on individual experiences, also require direct personal involvement in terms of taking responsibility for learning (Tolhurst, 2007). The task that lies ahead is by no means to be underestimated, but theory suggests that negative perceptions can be challenged (Uusimaki and Nason, 2004) and mathematical difficulties can be overcome during ITT (Hopkins, Pope and Pepperell, 2004).

Although recognised as being potentially painful (Clarke, 1994), this study's determination of the range of variation in perceived mathematical emotion identifies a clear need for SPTs to confront negative feelings about mathematics. Since research suggests that such emotion is learned and can be unlearned (Ashcraft and Kirk, 2001; Pound and Lee, 2011), this study recommends that SPTs examine their past experience in order to identify the source of negative perceptions, and consider other aspects of mathematical perceptions within the outcome space in addressing their learning needs for ITT.

6.7 Potential Implications For SPTs' ITT Development In Relation To Perceived Nature Of Mathematics

The outcome space presents variation in a non-creative view of mathematics ranging from correct answers, through use of correct procedures, to rules and procedures being perceived as structured (Categories 1-3). In contrast, Category 4 constitutes the perception of mathematics being creative, based on a creative process. The contrast between creative and non-creative mathematical perceptions requires further examination by SPTs in terms of analysing dualist pedagogical experience and comparing with a non-dualist approach.

6.7.1 Analysing Dualist Pedagogical Experience

It is apparent within this study that non-creative perceptions of mathematics are associated with it being a difficult subject to learn, a notion that Pound (2008) and Cockcroft (1982) have shown has been the case in previous years for learners of mathematics. There is clear demonstration in this study of the problems that existing research has witnessed in learners of mathematics – fear apparently stemming from a perceived need to give the correct answers (Cross, 2009; Haylock and Thangata, 2007; Cockburn, 1999) that are expected by the teacher (Lampert, 1990), which in turn seem to stem from experience of an instrumental pedagogy (Ernest, 1989) of teaching by transmission to passive learners (Ofsted, 2008; Howell, 2002; Ernest, 2000; Desforges and Cockburn, 1987) where facts are expected to be memorised and recalled (DfE, 2003; Ambrose, 2004; Wong, 2002) to closed questions (Boaler and Greeno, 2000), written neatly (Ofsted, 2005; Boaler and Greeno, 2000) and produced quickly (Bibby, 2002b). This study facilitates the existence of a link between an experienced dualist pedagogy based on the perspective of mathematics as a structured and non-creative external, known, set body of facts to be imposed upon learners which does not lead to mathematical understanding and which lacks an internal relationship that might enable creative thinking and process.

In contrast, mathematics perceived as creative links with intentions for a non-dualist pedagogy to encourage learning through active engagement that facilitates relational understanding of phenomena through which learners make mathematical meaning, an approach which does not view mathematics as a separate entity, but as a human creation that is engaged with in making sense of and communicating meaning of phenomena, through mathematical thinking and doing. Hence understanding can only be brought about by the relationship between learner and phenomenon via active engagement in order to be able to make personal mathematical meaning.

The contrasting perceptions within the range of variation suggest a need for SPTs to confront past experiences of their own mathematical learning to analyse how they were taught and how they learnt in terms of a creative perception of the nature of mathematics brought about through relational understanding.

6.7.2 Problem Solving

Existing theory is applied to the findings of this study to analyse the variation in mathematical beliefs of SPTs in relation to associated pedagogical practice, and to identify gaps in their descriptions that indicate a desire for non-dualist pedagogy, yet do not fully ascertain how the approaches presented might aid mathematical understanding (Categories 2-4). In aspiring to use a socially constructive pedagogy for children learning mathematics (Category 4), the mysterious nature of mathematics is embraced, and in espousing learners' attempts to understand and communicate findings of phenomena, the use of problem-solving is suggested without concern about the open-endedness of such an approach (Oxford and Anderson, 1995). Instead, the challenge that working mathematically can bring is welcomed, together with the sense of satisfaction that can accompany its frustrations (Orton and Frobisher, 1996), SPTs describing their own engagement with mathematics as enjoyable, stimulating and fascinating, (Haylock and Thangata, 2007; Andrews and Hatch, 1999; Skemp, 1989). However, neither specific understanding of the benefits of problem-solving nor the difficulties inherent in the use of such an approach are specified in SPTs' descriptions.

There are difficulties inherent in a problem-solving approach for teachers who lack the mathematical self-esteem and subject knowledge to be confident enough to cope with an open-ended approach of children using various methods, not finding answers, or finding different answers (Jones, 2003). There is therefore a need for SPTs to develop awareness of the relational understanding (Skemp, 1989; Ernest, 1989; Kuhs and Ball, 1986) that can be brought about through a pedagogical approach which focuses on an active process (Hersh, 1986) involving children querying phenomena, asking questions, posing problems and reformulating understanding and further questions (Pound, 2008; Brown and Walter, 2005; Szydlik, Szydlik and Benson, 2003) and hence developing mathematical thinking through creative as opposed to structured means (Pound, 1999) with children using varied strategies and making meaning through making connections, looking for patterns and recognising relationships to make generalisations (Pound, 2008; Cockcroft, 1982).

Problem-solving is a means by which learners can learn about phenomena that are relevant to them, essential in society and which arise from their culture (Jones, 2003; Thompson, 1992; Ernest, 1991; Nunes and Bryant, 1996), and which are considered fundamental to the essence of what mathematics is all about (Ollerton, 2010). In the highest category of the outcome space (Category 4), intrinsic value in mathematical engagement that goes beyond its usefulness (Pound, 2008; Owen, 1987) is demonstrated. There is hence a need for SPTs to consider how their espoused pedagogy can not only be put into practice, but also embraced by the children they teach and their colleagues who may hold a superficial view that does not recognise the wonder of what mathematics seeks to understand and communicate from surrounding phenomena (Koshy, Ernest and Casey, 2000; Cockcroft, 1982). As Ernest (2000) purports, without such appreciation of mathematics, learners are prevented from both enjoying and understanding it, and recognising its uses.

Problem-solving is thus an approach that enables the integration of mathematics into the whole school curriculum (Sakshang et al, 2002) and, whilst SPTs support an holistic approach to mathematics (Category 4), consideration is needed regarding how this can happen in practice when teachers have been shown to worry about the time that is needed to fit problem-solving into the curriculum (Jones, 2003; Orton and Frobisher, 1996), to lack understanding of what problem-solving involves (Jones, 2003; Kelly and Lesh, 2000) and the tendency for problem-solving to be included in mathematics lessons as opposed to it being an integral approach to learning mathematics (Briggs and Davis, 2008; Fairclough, 2002) where time is allowed for children to engage in mathematical thinking and experimentation (Carpenter and Lehrer, 1999).

The differentiation in beliefs about mathematics, whilst supported by existing theory, is identified in this study as specific variation amongst SPTs at the outset of ITT, with potential implications for their analysis of past experience and formation of personal philosophy in terms of whether learning mathematics is a creative process, and whether mathematics itself is a human creation based on individuals' relational understanding. The way people feel about mathematics relates to what people believe mathematics to be (Swars, Smith, Smith and Hart, 2009), the way we teach will affect the way children learn (Ernest, 2000), the way we teach is dependent on how we view mathematics and how we learnt mathematics (Townsend and Wilton, 2003), which are in turn factors in how we subsequently learn mathematics (Hofer and Pintrich, 2002). As such, the formation of clear philosophy incorporating their beliefs about mathematics and associated pedagogical practice is crucial for SPTs in determining their future as primary mathematics teachers.

6.8 Potential Implications For SPTs' ITT Development In Relation To Beliefs About The Nature Of Mathematicians

Whilst SPTs' perceptions of mathematicians concur with various descriptions within existing literature, the outcome space of this study determines the range of variation of those perceptions, indicating a need for SPTs to confront their perceptions and compare with those of others alongside other elements of reflection recommended in this study. Limiting mathematicians to perceptions of cleverness (Category 1), logical aptitude (Category 2) and the scientific (Category 3) in turn limits the scope for recognition that, since mathematical development is the relational understanding of that which surrounds us, we all engage in mathematical thinking and process and are thereby all mathematicians. Since that process is creative, we are, by nature, creative mathematicians.

As teachers influence practice in the classroom (Cross, 2009), including through their attitudes (DfES, 2002), it is important for SPTs to clarify their perceptions of mathematicians and, in conjunction with other reflection, where necessary set about increasing their self confidence as mathematicians. Aligned with SPTs' past experience, and corresponding to theory, the lack of self-recognition as mathematicians is compounded in this study by debilitating lack of confidence (Pound, 2008) and self-esteem (Akinsola, 2008), comparison with others (Boaler and Greeno, 2000), considering mathematicians to be clever (Sowder, 2001; McVarish, 2008), feeling inadequate (Cockcroft, 1982), perceiving lack of mathematical ability (Metje, Frank and Croft, 2007) and self-blame (Miller and Mitchell, 1994) for perceived mathematical inability. Critical evaluation is therefore needed of the ways in which SPTs were encouraged to learn mathematics and whether this facilitated relational understanding and to determine the extent to which their descriptions of hostile teachers (Brady and Bowd, 2005; Jackson and Leffingwell, 1999; Briggs and Crook, 1991), teachers' judgements (Haylock, 2010; Perry, 2004; Miller and Mitchell, 1994) and humiliating incidents (Ernest, 1991) are contributory factors to their lack of mathematical confidence and self-esteem.

Since feelings of inadequacy have been shown to affect attitudes towards mathematics (Perry, 2004) it is important for SPTs to identify and address their relationship with mathematics. This will be by no means straightforward, but the first step in improving a situation can be to ensure self-awareness (Gattegno, 1971) of what is to be dealt with and plan to make changes - on the premise that mathematics anxiety is learned and as such can be unlearned (Ashcraft and Kirk, 2001). It is posited that any learner engaging in an internal relationship with phenomena to make sense and meaning and hence develop mathematical understanding unique to themselves as the experiencer, is, by definition a mathematician. Hence, we are all mathematicians since we all make meaning of that which we experience.

6.9 Potential Implications For SPTs' ITT Development In Relation To Expectations For Learning About Primary Mathematics Education In ITT

Perceptions of the expectations of ITT development are varied across the outcome space of this study and there is a need for reflection on the recommendations posed in order to address the fear (Category 1) and anxiety (Category 2) that could otherwise prove detrimental to both SPTs' ITT learning and their practice as teachers. The notion of changing one's mindset being a challenge (Category 4) is likely to be an enormous understatement for some. There is no doubt, however, throughout this study that SPTs want to do their best both for themselves and the children they will teach, as advocated by Pound and Lee (2011). Whilst theory is useful in analysing the variation amongst SPTs' perceptions of forthcoming ITT learning, this study provides identification of that range of variation for SPTs to reflect upon in the interests of evaluating their learning needs in order to develop as necessary during their course, in accordance with the formation of their personal mathematical philosophies based on their pedagogical perspectives.

6.9.1 Aspirations For Practice

In aspiring to be the best possible teachers of primary mathematics, it is worth taking a closer look at the perceptions of Category 4 and the desire for children to perceive mathematics as enjoyable and the intention to implement a socially constructive, supportive and interactive learning environment (Pound and Lee, 2011; Bottle, 2005; Billington et al, 1993) with a focus on mathematical process and where connections are made (Anghileri, 1995), independent thinking is encouraged (Burton, 1994), children's needs are acknowledged (Nathan and Koedinger, 2000) and where mathematical understanding is constructed through doing (Atkinson, 1992).

In order to pursue such an aspiration, there is a need to address the lack of detailed understanding within Category 4 of the benefits of such an approach. It is recommended that SPTs consider planning development of this aspect of mathematical learning through ITT to increase awareness of the creative nature of mathematics in noticing patterns, exploring, describing, questioning and probing (Delaney, 2010; Anghileri, 1995); socially constructed understanding via discussion, explanation, sharing, evaluation and negotiation (Askew, 1998; Kamii and Lewis, 1990); the process of mathematics and associated development of mathematical thinking within knowledge construction (Desforges and Cockburn, 1987; Skemp, 2002; Skemp, 1989; Skemp, 1981); gradual development of mathematical communication from children's own representations to formal symbols and vocabulary (Anghileri, 2000; Nelson-Herber, 1986); and autonomy based on teacher confidence, understanding and pedagogical awareness for relational mathematical understanding to be facilitated.

6.9.2 Creativity

Creativity and mathematics are not synonymous in the first three categories of the outcome space, but the highest category is creative both in the approach to teaching and the facilitation for learning. Its creative nature also allows constraints identified to be addressed.

A consideration for the issue raised concerning planning for differentiation, is problem-solving and the facilitation of personal mathematical thinking associated with an individual's mathematical development as children form conceptual understanding through a deep approach to learning (Prosser et al, 1998; Graofalo and Leicester, 1985). The issue raised of time constraints is pertinent to a creative approach that encapsulates the relationship between learner and phenomena but requires teachers to provide children with time, space and opportunity (O'Sullivan, Harris and Sangster, 2005) to take an active part in their learning, to think and reflect, make decisions and choices, ask questions and explore solutions and follow their innate curiosity and interest (McVarish, 2008), creating a balance between the perceived space in a statutory curriculum and the need for relational understanding. The holistic nature of learning mathematics, raised previously, is pertinent here in consideration of linking aspects of a statutory curriculum so that, not only do children have the opportunity to learn in the way they experience the world, precious time is saved within the timetable from separate teaching of subjects. The issue raised of government pressure in the form of statistics, documentation and testing (Brown, 2010) is one pertinent to the balance between children achieving the mathematical understanding that is a shared and ultimate goal, and the means by which that understanding is facilitated, and is closely linked to the recommendations herein for development of confidence and autonomy.

6.9.3 Mathematical Process

Analysis of the outcome space indicates a qualitative difference between learners who have been expected to produce answers, largely from recall and following set procedures and without associated understanding, to concentrating on mathematical process through active engagement with mathematics, associated with mathematical understanding. However, philosophy underpinning mathematical process is not articulated, and in contrast there is confusion regarding why some mathematics pedagogy does not focus on correct answers.

There is a need therefore for SPTs to reflect on their perceptions of mathematics seeking answers to questions, and the ultimate goal for the answers to be scientifically accepted as proved, generalised and hence 'correct'. The fundamental difference in primary school mathematics is the worth of producing a 'right' answer at the expense of not understanding how the answer was reached, nor why it is correct. Unless children can engage in mathematical activity based on their current understanding, build on it in their individual way and relate to what is going on in the way in which they understand, they are in danger of remembering what someone else has told them to be right, as opposed to developing their own relational understanding. In order for children to reach that relational understanding they need a supportive, non-threatening environment in which they can question, play, experiment, explore, investigate, observe, talk, act, notice, doodle, write, discuss, explain and hence engage in mathematical process. In other words, reaching right answers with understanding is impossible without first engaging in the process that leads to relational understanding, and mathematical process is therefore crucial.

Without engagement in such creative thinking, learners would not have mathematical understanding of surrounding phenomena, and the mathematicians, scientists and technologists of the future would not be encouraged, nor the mathematicians constituted of everyday people confident in engaging in the constant mathematical activity that is part of everyday life.

6.10 Summary

In consideration of the second research question for this study, the range of variation of perceptions of mathematics amongst student primary teachers at the outset of ITT, considered in relation to mathematics pedagogy, raises several potential implications for SPTs for which actions are hereby recommended in order to work towards establishing a philosophy of mathematics, based on reflections of experienced mathematics, and aspirations for practice. The study's purpose to provide a reflective tool in facilitation of SPTs' consideration of mathematical perceptions and associated pedagogical perspectives to create a personal philosophy on which to base their ITT development is thus enhanced by making explicit the key potential implications arising from this study, and associated recommendations - summarised in the table below:

Potential Implications For SPTs' Development Within ITT	Recommended Actions For Student Primary Teachers
Analysing experienced pedagogy	Confront past experiences of mathematics to analyse how you were taught, how you learnt and the effectiveness of the experienced pedagogical approach Consider alternative approaches Determine your own views regarding what mathematics is and how it is best learnt
Considering the relevance of learning mathematics	Evaluate past experience of learning in relation to the purpose of mathematics Consider the benefits and limitations of mathematics learnt under duress, for passing assessments, for use in and application to real life and for understanding life's phenomena Determine your own views of whether mathematics is useful, essential and/or an inherent part of understanding phenomena Reflect on how you think mathematics should form part of the wider primary school curriculum
'Discovery' learning	Reflect upon what you believe mathematics to be...is it 'out there' waiting to be discovered or is it a human conception dependent on an individual's relational understanding? Consider your role as a teacher and the difference between teaching facts, skills and procedures in isolation for children to remember, and the introduction of new ideas and strategies in context as part of social collaboration Identify the level of mathematical understanding needed for your intended pedagogy Plan your personal development accordingly
Implementing statutory curriculum	Consider your stance on the notion of mathematics being separate to other understanding of the world Consider the difference between an holistic approach to integration of mathematical learning and facilitation of relational understanding, and mathematics taught in isolation as a discrete subject Link your pedagogical aspirations to the way in which set curriculum content can be interpreted and implemented
Using and applying mathematics	Determine the difference between the use and application of relational understanding of phenomena to new mathematical situations, and the use and application of a given body of curriculum content knowledge
Autonomy	Consider the provision of opportunities for children to question, make connections, construct understanding, pose and solve meaningful problems as part of their mathematical development Establish, from the outcomes of the actions above, your personal philosophy of mathematics – what is mathematics? – how should children learn mathematics? – how do you intend to teach mathematics? What are the potential constraints to your philosophical aspirations? How do you plan to overcome those constraints?
Developing mathematical understanding	Consider past experience of learning mathematics with regard to surface and deep approaches Identify your expectations for ITT learning and evaluate the extent to which these constitute surface/deep approaches Examine your aspirations for practice and determine the extent to which your intended approach constitutes surface/deep approaches to learning Analyse your aspirations for teaching in terms of how and why your approaches will facilitate development of children's relational understanding Evaluate the depth of your mathematical understanding Ascertain your learning needs for ITT and plan ahead to address these
Analysis of mathematical engagement	Reflect upon pedagogical approaches that expect given facts, rules and procedures to be followed Consider the way you view mathematics Identify your expectations for learning mathematics in terms of - your intended practice in the future - your learning approaches within ITT Analyse your past mathematical learning experience with regard to the benefits of the use of learning materials Evaluate their use in encouraging mathematical engagement and supporting mathematical understanding

	Consider the facilitation of active engagement and open-ended learning activities in the achievement of developing relational understanding Consider the use of resources for: - facilitation of discussion and explanation - development of mathematical symbolization, vocabulary and language - development of ability to describe patterns and create generalizations - constructing and reconstructing understanding of abstract concepts - setting mathematical learning within meaningful situations - children making their own decisions and choices when working mathematically - time to engage with learning and think mathematically to develop the abstract reasoning behind the use of the practical resource Determine your views on a non-dualist pedagogical approach of active engagement in a socially supportive environment that enables learners to observe, play, experiment, explore, investigate, ask questions, pose problems, seek solutions, look for patterns and facilitates development of describing, articulating, explaining, discussing, drawing, writing, using symbols and mathematical vocabulary to express what children come to understand from abstract mathematical concepts. Consider the need for teachers' mathematical understanding and identify your personal ITT learning needs in this area
Overcoming negative mathematical emotion	Confront negative feelings about mathematics Examine past experience for the source of those feelings Consider other aspects of mathematical perceptions to identify your learning needs for ITT in overcoming negative mathematical emotion
Ascertaining beliefs about the nature of mathematics	Reflect on your experiences of learning mathematics with regard to the creativeness of approach and how relational mathematics understanding is created Consider contrasting pedagogical approaches in forming your aspirations for future practice Consider the use of problem-posing and solving - as a basis for social construction of mathematical understanding, facilitated by a teacher who interacts with learners to scaffold mathematical development in accordance with their needs - as an integral way of thinking and engaging mathematically - as part of an holistic approach to the school curriculum as opposed to a 'bolt-on'
Increasing confidence as a mathematician	Identify and address your relationship with mathematics to ensure self-awareness Plan the changes you want to make
Expectations for ITT	Identify and challenge perceptions Identify your associated learning needs in ITT Develop awareness of a creative perception of mathematics - noticing patterns, exploring, describing, questioning and probing - socially constructing mathematical understanding via discussion, explanation, sharing, evaluation and negotiation - focussing on the process of mathematics and associated development of mathematical thinking within knowledge construction instead of limitation to given recall of facts, rules and procedures to produce a correct answer that is not underpinned by understanding - gradual development of mathematical communication from children's own representations to formal symbols and vocabulary Determine your views on the importance of mathematical process
Creativity and mathematical process	Consider the notion of mathematics being a creation and a creative process Consider how a creative approach to teaching a set curriculum content can happen in practice Plan for creative practice in terms of time, space and opportunity for children to take an active part in their learning, to think and reflect, make decisions and choices, ask questions and explore solutions and follow their innate curiosity and interest to reach relational understanding

Table 6.1 – Addressing Potential Implications For ITT Development

Chapter 7 Conclusion

7.1 Introduction

It is a pertinent time to examine factors affecting primary mathematics education in Britain as the recently changed government is currently altering statutory curriculum policy for schools. In addressing the decades-old issue of mathematics being difficult to learn and teach, this study attends to the need for improvement in primary mathematics education provision by considering student primary teachers who have an important role in the future teaching of primary mathematics. This chapter summarises the key elements of the study and considers the way forward.

7.2 Conceptual Framework

The study's hierarchical phenomenographic outcome space is based on a non-dualist perspective of mathematics constituting human conceptualization, dependent on relationality whereby mathematical learning is dependent on the individual's relationship between themselves and what is learnt. In consideration of improvement needed in the learning and teaching of mathematics, explicit and conscious conceptualisation of the way in which mathematics is perceived is the essence of this study. The higher order perception presented in the outcome space is one whereby mathematics is internally constructed through the relationship between learner and learnt, and is not a position that has been hitherto explicitly reflected in British statutory curriculum policy.

7.3 Achievement Of Aims

The study achieves its aims and answers the research questions posed. The range of variation in student primary teachers' perceptions of mathematics at the outset of ITT is ascertained and related to primary mathematics pedagogy. Potential implications arising for SPTs' ITT development are thus identified and considered in terms of how they might be addressed.

It is argued that SPTs, required as they will be to implement statutory policy, must be clear of their aspirations for mathematics teaching and their own ITT learning, through a personal philosophy of mathematics, based on their mathematical perceptions. This study interprets SPTs' mathematical perceptions from a basis that these arise from experience (Ernest, 2000). A phenomenographical outcome space is presented that provides the means for reflection (Cooney and Krainer, 1996) to heighten awareness of SPTs' perceptions - effective practice as identified in existing literature (Houssart, 2009).

The outcome space constitutes four hierarchical categories of description pertaining to SPTs' qualitatively different ways of perceiving mathematics at the outset of ITT, based on reference to mathematical relationality and structured in terms of mathematical knowledge and understanding.

The variation ranges from:

- mathematics perceived as external entity imposed upon learners who would rather avoid it, having experienced limited engagement when learning and believing mathematics to be out of their reach, knowledge being confined to recall of number facts from rote learning and otherwise limited understanding

- mathematics perceived as limited knowledge of given facts, rules and procedures, followed with some anxiety in the pursuit of correct answers in attempted internal relationality to try to understand, with recognition of some relevance to life
- mathematics perceived as reaching understanding through an internal relationality via active engagement and a focus on mathematical thinking and process with curriculum content used in and applied to a range of everyday life
- mathematics perceived via espoused practice as the facilitation of children's understanding through internal relationality via active engagement and a focus on mathematical thinking and process to make sense of the mystery of the world with a sense of enjoyment, fascination and challenge

Whilst ITT providers have a part to play in SPTs' development, personal responsibility is also key to learning development. It is posited that SPTs need to be aware of their ITT learning needs and be prepared for what they identify as necessary for their own development, based on the premise that the individual student has the power and responsibility to become the teacher they aspire to be. Differentiated perceptions link to differentiated pedagogy and give rise to differentiated potential implications and associated recommendations for SPTs. The study informs the individual on the basis of the collective in order to provide a flexible means of development as advocated in theory (Smith, 2004), since it is the individual who fundamentally influences practice and it is the individual who has the power to determine what shape that practice will take (Christou, Phillipou and Menon, 2001).

This study recommends that SPTs reflect on their mathematical perceptions and consider potential implications for their future mathematical learning and teaching as they embark upon ITT. Determination of the range of variation of mathematical perceptions amongst SPTs at the outset of ITT and the accompanying analysis with regard to pedagogy and potential implications for SPTs' development in ITT forms a basis by which such reflection is facilitated.

7.4 Fulfilment Of Purpose

The study achieves its purpose in examination of a typical group (Dunkin, 2000) of SPTs prior to embarking on their ITT courses to establish their different understandings (Marton, 1986) of the phenomenon of mathematics based on their individual views (Dall'Alba, 2000) arising from their personal experience (Trigwell and Prosser, 2004) and as described by them (Tan and Prosser, 2004; Kvale, 1996) to ascertain their varied perceptions (Åkerlind, 2005a; Ambrose, 2004). Through provision of a pool of meaning (Green, 2005) from the collection of perceptions (Åkerlind, 2005b), the range of variation is ascertained and a framework of differentiated mathematical perceptions linked to pedagogical perspectives provides a basis for SPTs to consider potential implications that arise. The framework provides a reflective tool for SPTs to identify and analyse their personal perceptions of mathematics, raise awareness of a range of perceptions of mathematics, compare and contrast their personal perceptions of mathematics with the wider range, analyse pedagogical associations with their perceptions of mathematics and identify potential implications of their mathematical perceptions on ITT learning and future teaching. It facilitates potential implications to be addressed explicitly and consciously via the formation of SPTs' personal philosophy for mathematics – its nature, its learning and its teaching – upon which aspirations for practice as teachers of primary mathematics can be based, goals set for ITT learning, learning needs identified and ITT embarked upon in an informed way, with SPTs aware of, and prepared for, development to be brought about in order to become the best teachers they can be, in the interests of the children they will teach and in pursuit of potential improvement in primary mathematics education.

7.5 Choice Of Methodology

In exploration of intangible and potentially unconsciously held mathematics perceptions, the qualitative and interpretative nature of phenomenography is invaluable, especially in provision of a reflective tool via an hierarchical framework of differentiated perceptions. Perceptions are influenced by experience, interpretation of which, particularly concerning the potentially emotive subject of mathematics, is challenging, but the approach taken for this research is successful in facilitation of interpreting perceptions.

Following interview of typical SPTs due to embark upon ITT, analysis of accounts of experience enabled determination of hierarchical categories describing the range of variation of perceptions of mathematics across the collective. The relational nature of phenomenography whereby the focus is on the connection between experiencer (SPT) and experienced (mathematics), in conjunction with the researcher in interpretation of the relation, is an appropriate choice of methodology, based on the conceptual framework for this study regarding the relational nature of mathematics learning.

The categories of the outcome space do not link to any individual SPT taking part in the study (Bowden, 2000a), nor is it expected that any SPT reflecting upon the outcome space would identify themselves as confined to any one category (Barnacle, 2005) since each is comprised of key elements originating from the group (Cherry, 2005). However, the use of phenomenography in determination of the qualitatively different ways (Linder and Marshall, 2003; Dall'Alba, 2000) of perceiving mathematics and exploration of associated pedagogy achieves the study's diagnostic (Bowden, 2000a), pragmatic (Åkerlind, 2005a), practical (Green, 2005) purpose in providing a reflective tool for SPTs.

7.6 Originality

The focus of this study, relating SPTs' mathematical perceptions to concerns about their learning in British Initial Teacher Training and their future practice, is an aspect of research that is otherwise sparse.

In addressing the historical difficulty inherent in primary mathematics education and the concerns of previous research (Hofer and Pintrich, 2002; Noddings, 1992) that positive perceptions are needed for effective teaching, alongside awareness of pedagogical perspectives (Speer, 2005), this study is unique in its specific determination of SPTs' mathematical perceptions at the outset of ITT.

It is the first of its kind to ascertain a phenomenographic outcome space for SPTs in Britain at the outset of ITT with regard to mathematical perceptions, and is similarly distinctive in the specific application of pedagogical perspectives to the range of variation determined and the explicit identification of associated potential implications for SPTs' development in ITT.

Existing theory relating to mathematics learning, mathematics pedagogy, primary mathematics education and adults' attitudes towards mathematics is used to explore the elements of differentiation within this study's phenomenographic outcome space, alongside the limited existing research previously carried out specifically with British student primary teachers. Since literature is sparse in this respect, this study is of particular interest in its focus not only on SPTs, but of their position at the outset of ITT in terms of potential implications for their learning and future teaching of primary mathematics.

Given current concerns for educational provision, this is a crucial area of development since SPTs will influence primary mathematics education through their future practice as teachers (Cross, 2009) and both their ITT learning and subsequent teaching will in turn be influenced by their mathematical perceptions (Cockburn, 1999).

7.7 Original Contribution To Literature

There is a wealth of literature regarding approaches to teaching mathematics. The child-centred approach of the latter part of the twentieth century and the recommended emphasis on using and applying mathematics through methods such as problem solving of the last thirty years have not been fully utilised in practice, as demonstrated amongst participants in this study who were primary school learners of mathematics during that period. More recent plethora of government guidance has not succeeded in allaying concerns relating to mathematics and there is a clear need to change the way mathematics is taught. As future teachers of primary mathematics, SPTs' role will be crucial in such change, their learning within ITT therefore being equally crucial. Since teaching and learning are influenced by the teacher and learner's perceptions, examination of SPTs' perceptions of mathematics is essential to their ITT development, and is unique to this study's determination of their varied mathematical perceptions at the outset of British ITT.

The study is the first of its kind to explicitly consider the role of the SPT in this way for potential improvement of primary mathematics provision and the necessity for development within ITT to be approached from an informed perspective, based on SPTs' formation of personal mathematical philosophy arising from conscious awareness and reflection upon mathematical perceptions and associated pedagogy. The outcome space is hierarchical in order to represent the range of variation useful for reflection by SPTs in identifying their personal perceptions concerning differentiated elements of their experienced and intended pedagogical approaches, the relevance of mathematics, mathematical understanding, mathematical engagement, feelings about mathematics, beliefs about the nature of mathematics, beliefs about the nature of mathematicians and their expectations for learning about primary mathematics education in ITT, together with how these key elements marry with knowledge brought about by external relationship or by internal relationship with mathematics, how an internal relationship with mathematics might aid development of mathematical understanding and SPTs' aspirations for future practice as teachers.

7.8 Publication

Small-scale research from doctoral study concerning mathematics anxiety amongst SPTs (Jackson, 2008, Jackson, 2007) has been published and led to this wider study of SPTs' mathematical perceptions. It is intended that this study be used in further journal publication for the interests of the wider mathematics education community, including ITT providers. There is also editorial interest expressed for publication in book form, for the purpose of sharing the research and reflective framework with SPTs.

7.9 Limitations Of Study

A phenomenographic study such as this does not provide generalisation (Bowden, 2005), but does form a developmental purpose (Bowden, 2000a) in the shape of an educational tool (Speer, 2005; Åkerlind, 2002) in this respect. The outcome space and associated pedagogical discussion forms an holistic understanding (Petocz and Reid, 2005) of the varied ways in which SPTs perceive mathematics, reflection upon which can enable SPTs to identify their own mathematics perceptions and to consider potential implications for ITT development.

It is not a phenomenographical aim to identify reasons for perceptions (Cherry, 2005; Trigwell and Prosser, 1997) and the described experienced and espoused pedagogy is not assumed to constitute reasons for perceptions, since the categories comprise the collective and not individual SPTs.

It is similarly not a phenomenographical aim to provide ways of changing perceptions. This study does not implement change, instead providing a tool for reflection upon potential need for change through adding insight into this aspect of primary mathematics education (Åkerlind, 2004). It therefore achieves the pragmatic aim of developmental phenomenography's facility to provide an educational tool for reflection in the spirit of phenomenography's enablement of conceptual growth.

Whilst the outcome space forms an hierarchy with the aim of enabling learning development (Marton and Säljö, 1976), it is not intended as a framework to be worked through – its origins are constituted from the perceptions of a group at a given time, and just as their perceptions will change according to context, so will others'. The range of variation and associated discussion does, however, facilitate transition (Marton, 1986) and provide insight into what is needed for SPTs to move from a 'less powerful to more powerful' (Åkerlind, 2005a) relationship with mathematics.

Whilst it is recognised that research of this nature is partial (Åkerlind, 2005a), based as it is on a hypothetical range of perceptions that are qualitatively relational not only in the context of the experiencers (Marton, 2000) and the phenomenon of mathematics (Marton and Booth, 1997) but also to the researcher (Bowden, 2005; Walsh, 2000), the outcome space is a true representation of SPTs' mathematical perceptions at the outset of ITT. As befits phenomenographic study, the outcome space is a snapshot of collective meaning at a particular time, as interpreted through research. It is acknowledged that perceptions are fluid, based on continual experience and relationality, but the framework presented here is reliable in its rigorous formation and valid for purpose as a tool for SPTs to reflect upon perceptions and potential implications for learning and teaching.

7.10 Next Steps

The premise of this study is that facilitation of reflection on the hierarchical range of variation of SPTs' mathematical perceptions at the outset of ITT can enable SPTs of the future to ascertain their own perceptions, and develop a personal philosophy of the nature of mathematics, how it should be learnt and how it can be taught effectively, that will form the basis of their ITT development. Experienced pedagogy influences mathematical perceptions that may be unconscious (Ambrose, 2004). Raising awareness of what was formally implicit (Bishop, 1991) can lead to conscious, explicit perceptions and intentions for learning and teaching that can be used in the formation of mathematical philosophy, a basis for change to be made as necessary, and subsequently employed in practice for ITT learning and future teaching. This study therefore posits that attention to recommendations linked to the potential implications identified and reference to the hierarchical framework, can prove useful in SPTs' development of self-awareness through critically evaluating mathematical experience, determining perceptions of mathematics, analysing pedagogical perspectives, and clarifying personal philosophy for learning and teaching mathematics.

By ascertaining their personal mathematics philosophy, SPTs will be in a position to plan for their learning in ITT. A circle of development is hence formed as SPTs embark upon ITT and continual experience leads to ongoing reflection during and beyond ITT, illustrated in the figure below:

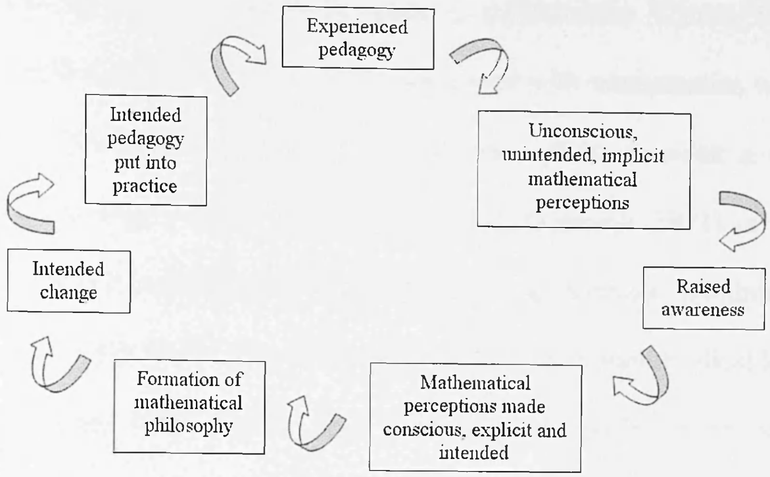

Figure 7.1

Reflective Cycle For ITT And Beyond, Based On Mathematical Experience

It is crucial that SPTs themselves engage in such a reflective cycle for, as Ernest (2000) purports, perceptions cannot be taught. SPTs may feel secure with their mathematical perceptions and aspirations and see no need to change, but it is nevertheless a worthwhile activity to expand awareness and gain understanding of the perspectives of colleagues they will work alongside (Valderrama, 2008). For those SPTs who do identify a need for change, this study's framework of hierarchical perceptions and associated potential implications and actions provides a means to engage with a process of change, starting with personally challenging assumptions and beliefs (Edwards, 1996; Orton and Frobisher, 1996) and considering their potential implications (Schuck, 2002).

Whilst it is acknowledged that confronting mathematical perceptions may prove challenging (Edwards, 1996), it is nonetheless necessary for SPTs to examine their perceptions and consider potential implications (Schuck, 2002) for their future learning and practice. Changing perceptions is difficult (Cross, 2009; Liljedahl, 2005;

Skemp, 1978), particularly with regard to mathematics (Clarke, 1994), as is clearly exemplified in the severe emotions associated with mathematics within the outcome space. However, in raising awareness, this study provides a starting point for potential change (Cherry, 2005; Green, 2005; Gattegno, 1971) in making conscious and explicit perceptions that might otherwise lay dormant (Ambrose, 2004; Bishop, 1991), in order for SPTs to take control of their own mathematical learning (Tolhurst, 2007; Biggs, 1987) and be in a position to set goals for learning prior to ITT, particularly in light of the increasing reduction of time spent on primary mathematics education on ITT courses (Brown, 2010).

7.11 Areas For Future Research

It is clear from the outcome space that SPTs' mathematical perceptions vary, and, in recognising that these may be deeply ingrained from years of experience (Liljedahl, 2005) challenging and changing perceptions, where SPTs deem necessary, will be undoubtedly difficult (Cross, 2009; Skemp, 1978). However, regardless of experience, there is unanimous desire apparent in participants of this study to become the best teachers they can be, alongside awareness of the influence their perceptions might have on their future practice (Hofer and Pintrich, 2002). There is therefore scope for further research into SPTs' reflection on mathematical perceptions, their identification of the need for change and the ways in which this is addressed.

Despite research suggesting the urgent need for change in mathematics education (Hogden and Askew, 2007), difficulties in primary mathematics education will not be solved overnight, but the findings of this study provide both a starting point for change amongst SPTs with responsibility for the primary mathematics teaching of the future, and a contribution to the lack of research concerning SPTs' primary mathematics philosophy and practitioners' questioning of their mathematical perceptions (Bibby, 2002b; De Corte, Op 't Eynde and Verschaffel, 2002). Further research is warranted into SPTs' development of personal philosophy for learning and teaching mathematics and the extent to which this is supported by pedagogical approaches presented within ITT provision for their learning and in the primary school environment for their teaching.

Concerns about primary mathematics provision extend to practising teachers and research supports the need for change in teachers' perceptions if practice is to develop (Ernest, 1989). Reflection on perceptions and their potential implications is a continuous process, as is the need for research, and future study is justified of pedagogical approaches employed by SPTs beyond ITT and by experienced practising teachers in terms of mathematical perceptions and philosophy, particularly in light of expected changes to the British statutory curriculum.

7.12 Concluding Reflection

This research hinges on the valuable insight provided by the participants of the study and must culminate with an interview transcript excerpt that sums up the essence of its enquiry and findings, for mathematics has:

> *that sense of mystery to me – that I don't really understand it all*

The phenomena of life <u>are</u> a *mystery* and <u>nobody</u> can claim to *understand it all.* If only mathematics could be universally accepted as the means by which we attempt to understand according to our <u>own</u> conceptualisation and relationality, without pressure or expectation, instead of the imposition of others' ways of understanding…the mysterious nature of mathematics might be more widely recognised as synonymous with the mysterious nature of the phenomena that surround us and embraced as a source of enjoyment, stimulation and challenge – its undoubted frustrations, complexities and sometimes sheer impossibility welcomed with confident wonder.

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