Complex exponential Smoothing

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Introduction

- Exponential Smoothing methods performed very well in many competitions:
  - M-Competitions in 1982 and 2000,
  - Competition on telecommunication data in 1998 and 2008,
  - Tourism forecasting competition in 2011.

- In practice forecasters usually use:
  - SES for the level time series,
  - Holt's method for trend time series,
  - Holt-Winters method for a trend-seasonal data.

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Introduction

- Holt's method is not performing consistently. Examples:
  - M-Competitions;
  - Taylor, 2008;
  - Gardner & Diaz-Saiz, 2008;
  - Acar & Gardner, 2012.

- Holt's method is still very popular in publications:
  - Gelper et. al, 2010;
  - Maia & de Carvalho, 2011.

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Introduction

- Several modifications for different types of trends were proposed over the years:
  - Multiplicative trend (Pegels, 1969);
  - Damped trend (Gardner & McKenzie, 1985);
  - Damped multiplicative trend (Taylor, 2003);
  - Prior data transformation using cross-validation (Bermudez et. al., 2009).

- Model selection procedure based on IC is usually used.
- Is the selected model always appropriate?

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Objectives

- Propose a model overcoming limitations of Holt's method and SES;
- Study properties of the model;
- Carry out a competition on different data sets.

Theoretical framework

- Simple exponential smoothing:

$$\hat{y}_{t+1} = \alpha y_t + (1-\alpha)\hat{y}_t$$

- Principle of CES: smooth level and combine it with "correction", using complex variables (Svetunkov, 2012).
- Basic form of CES:

$$\hat{y}_{t+1} + i\hat{x}_{t+1} = (\alpha_0 + i\alpha_1)(y_t + i\varsigma_t) + (1-\alpha_0 + i - i\alpha_1)(\hat{y}_t + i\hat{x}_t)$$

$$i^2 = -1$$

$$\varsigma_t = y_t - \hat{y}_t \qquad \varsigma_t = y_{t-s} - \hat{y}_{t-s} \qquad \varsigma_t = \Delta y_t \qquad \varsigma_t = f(x_{1,t}, x_{2,t}, \ldots)$$

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Theoretical framework

$$\hat{y}_{t+1} + i\hat{x}_{t+1} = (\alpha_0 + i\alpha_1)(y_t + i\varsigma_t) + (1 - \alpha_0 + i - i\alpha_1)(\hat{y}_t + i\hat{x}_t)$$

- Complex variables -> system of real variables:

$$\begin{cases} \hat{y}_{t+1} = (\alpha_0 y_t + (1-\alpha_0)\hat{y}_t) - (\alpha_1 \varsigma_t + (1-\alpha_1)\hat{x}_t) \\ \hat{x}_{t+1} = (\alpha_0 \varsigma_t + (1-\alpha_0)\hat{x}_t) + (\alpha_1 y_t + (1-\alpha_1)\hat{y}_t) \end{cases}$$

- Final forecast of CES consists of two parts:
  - level,
  - "correction".

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Theoretical framework

- ARMA(N,N):

$$\begin{cases} \left(1-\sum_{j=1}^{N} a_j B^j\right) y_t = \left(1-\sum_{j=1}^{N} b_j B^j\right) \varepsilon_t \\ \hat{x}_t = \sum_{j=1}^{N} a_j \varepsilon_{t-j} + \sum_{j=1}^{N} b_j y_{t-j} \end{cases}$$

- The order depends on the complex smoothing parameter value:
  - if $\alpha_0 + i\alpha_1 \rightarrow 1 + i$ then $N \rightarrow 0$
  - otherwise $N \rightarrow \infty$

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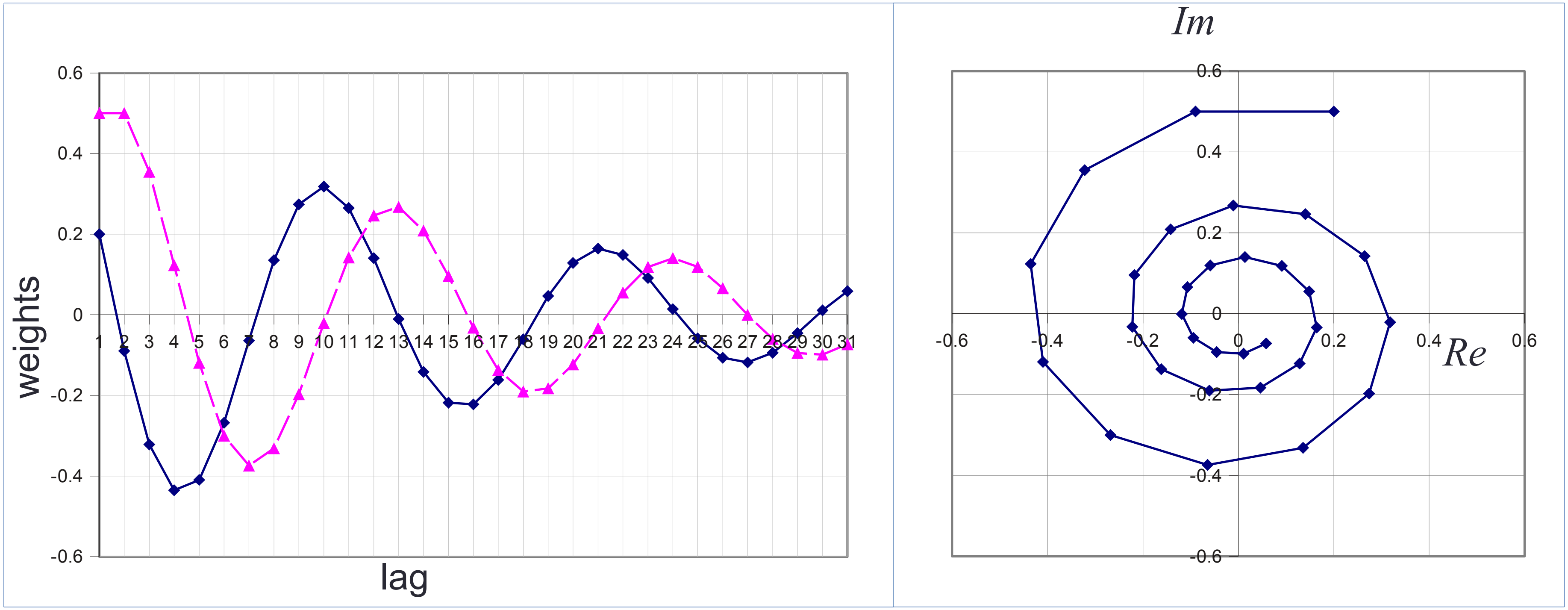
Theoretical framework

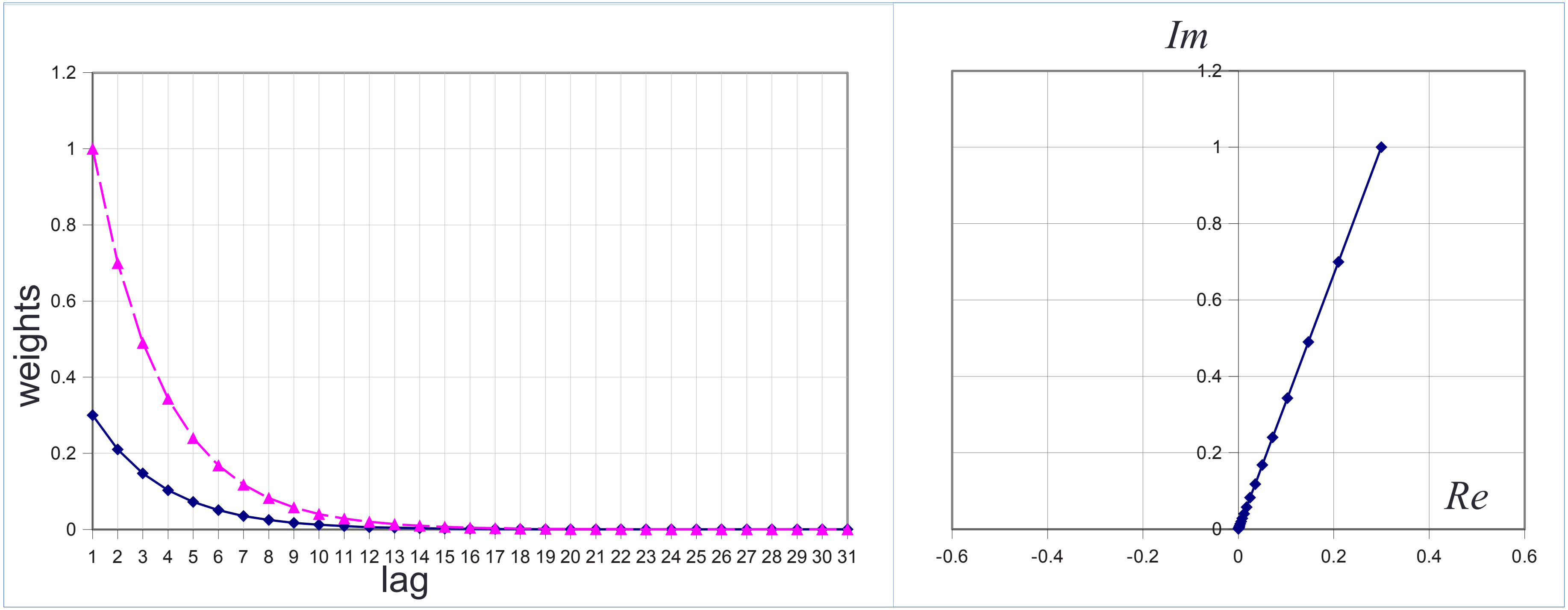
- Weights distribution in time



$$\alpha_0 + i\alpha_1 = 0.2 + i0.5$$

Theoretical framework

- Weights distribution in time



$$\alpha_0 + i\alpha_1 = 0.3 + i$$

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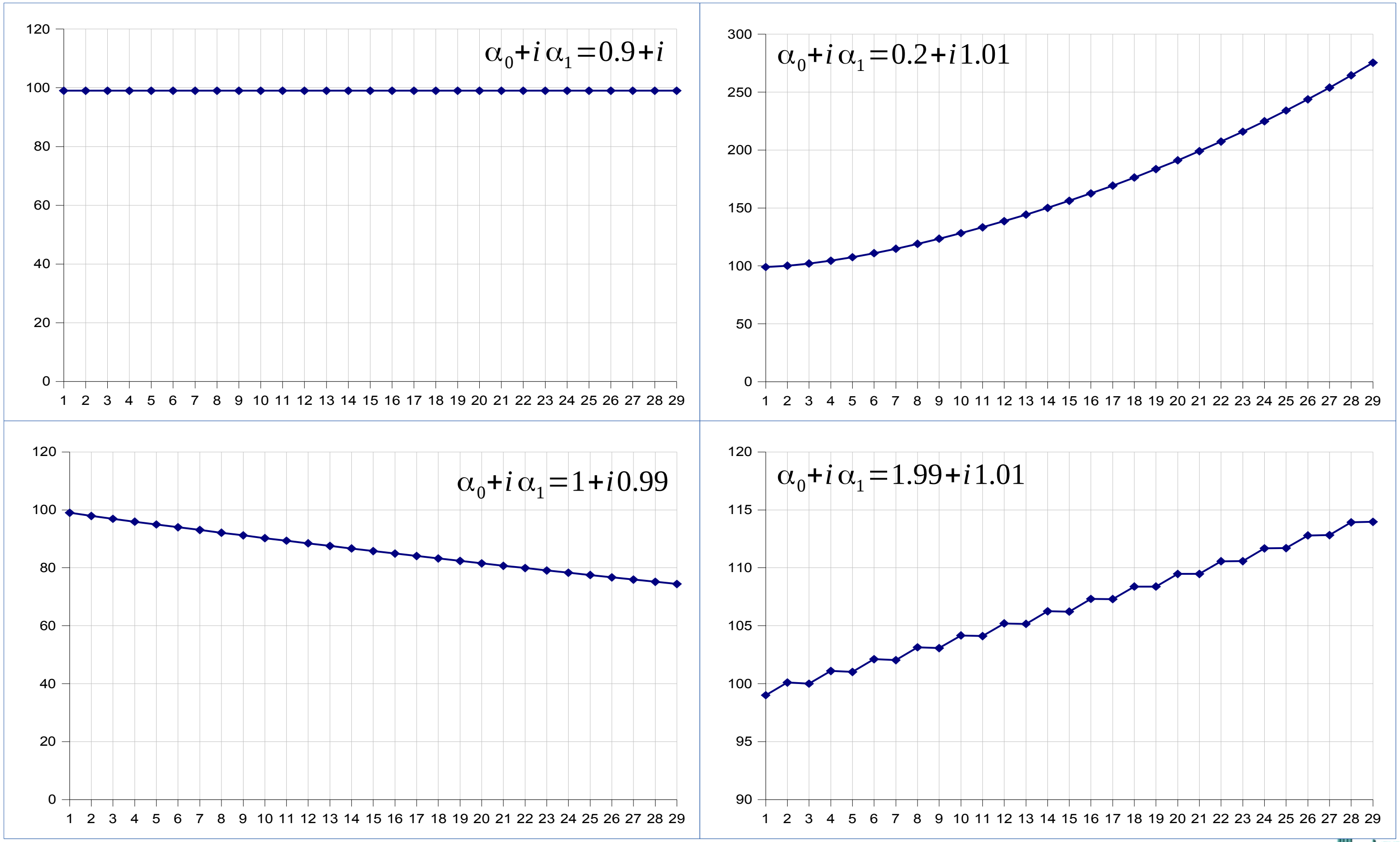
Theoretical framework

- Forecasting trajectories



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Empirical results: setup

- M3-Competition data. 3003 time series.
- Rolling origin.
- Automated ETS was used to split data into categories:
  - level non-seasonal,
  - level seasonal,
  - trend non-seasonal,
  - trend seasonal.

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Empirical results: setup

- M3-Competition data. 3003 time series.
- Rolling origin.
- Automated ETS was used to split data into categories.

Series type	Number of series		Overall	Forecasting horizon	Rolling origin horizon
	Level series	Trend series			
year	255	390	645	6	12
quart	306	450	756	8	16
month	686	742	1428	18	24
other	61	113	174	8	16
Overall	1308	1695	3003		

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Empirical results: competitors

1. Naive (Naive),
2. Simple exponential smoothing (SES),
3. Holt's additive trend (AAN),
4. Pegels' multiplicative trend (MMN),
5. State-space ETS with AICc model selection (ZZN),
6. Gardner's Damped trend (AAdN),
7. Taylor's Damped multiplicative trend (MMdN),
8. Theta using Hyndman & Billah, 2003 (Theta),
9. Hyndman & Khandakar 2008 Auto ARIMA (ARIMA),
10. **Complex exponential smoothing** (CES).

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Empirical results

- MASE was calculated for each of the horizons from each of the origins,
- Nemenyi test was conducted to compare methods for each of the series type.
- General results for CES:
  - at least as good as SES on level series,
  - outperforms MMN and AAN on level series,
  - at least as good as MMN and AAN on trend series,
  - outperforms all the methods on monthly trend series.

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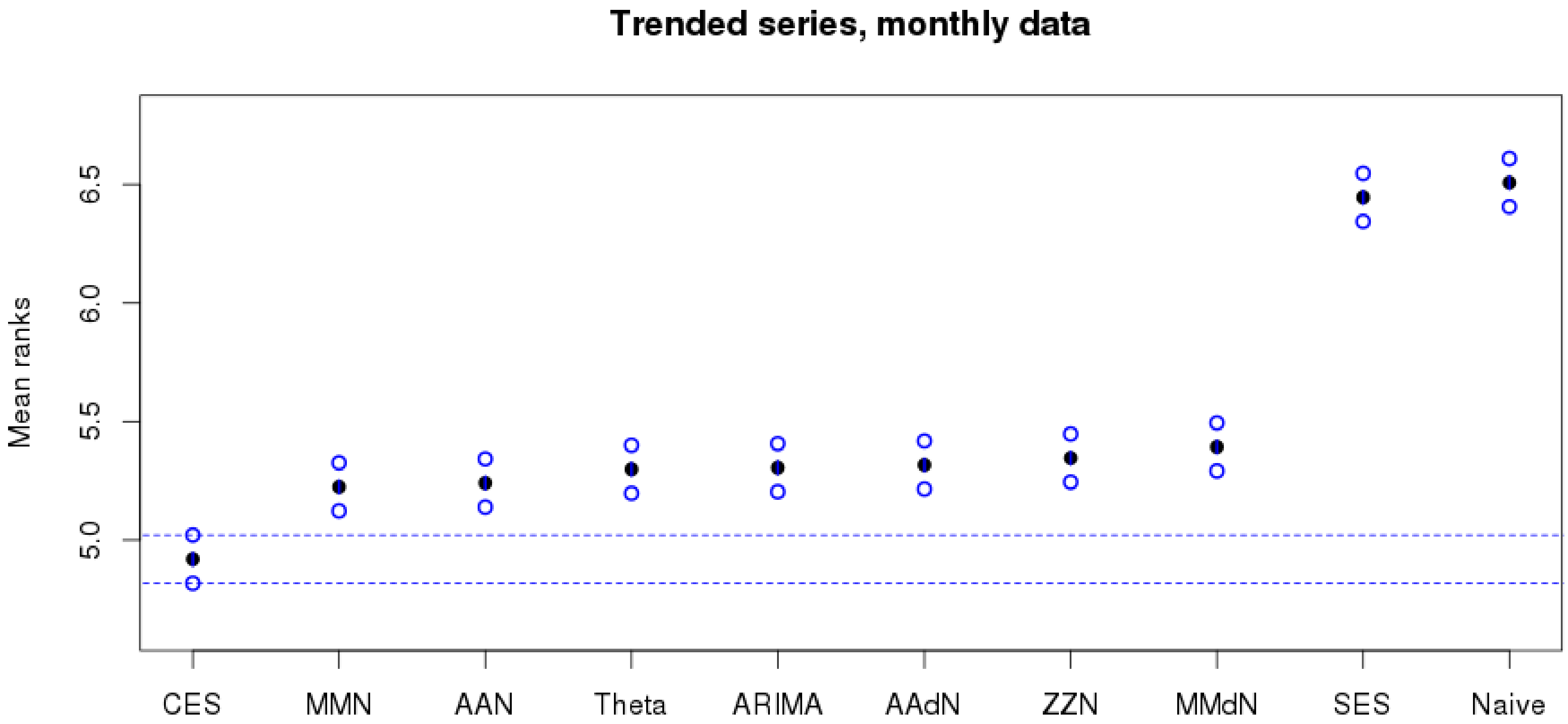
Empirical results. Nemenyi test



Conclusions

- CES
  - is flexible,
  - is able to identify trends and levels,
  - does it better than Holt and Pegels,
  - has an underlying varying-order ARMA(N,N),
  - outperforms all the other methods on monthly data,
  - is at least as good as SES.

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Future works

- Study the influence of the number of observations on CES accuracy;
- Derive state-space form of CES;
- Derive variance and likelihood function;
- Implement seasonal time series forecasting using CES;
- Implement exogenous variables in CES;
- Use other forms of correction parameter.

Thank you!

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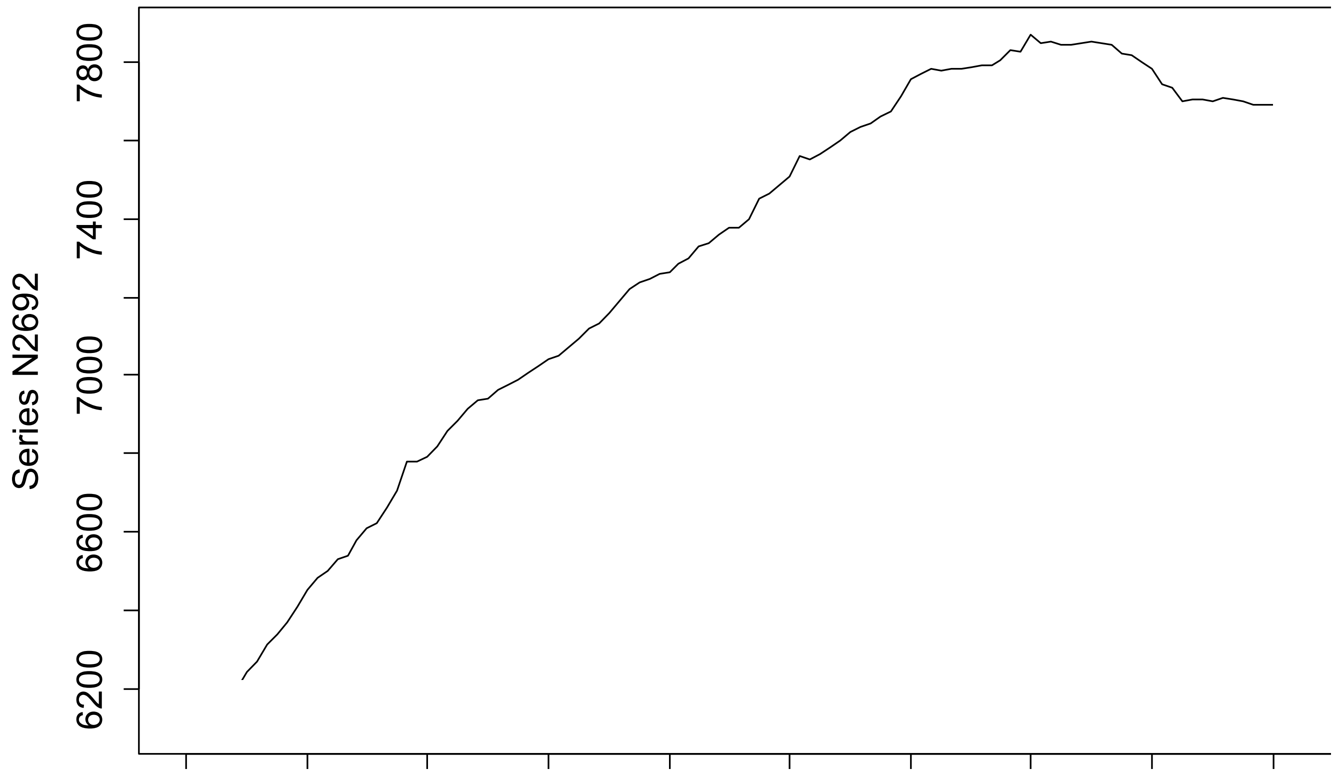
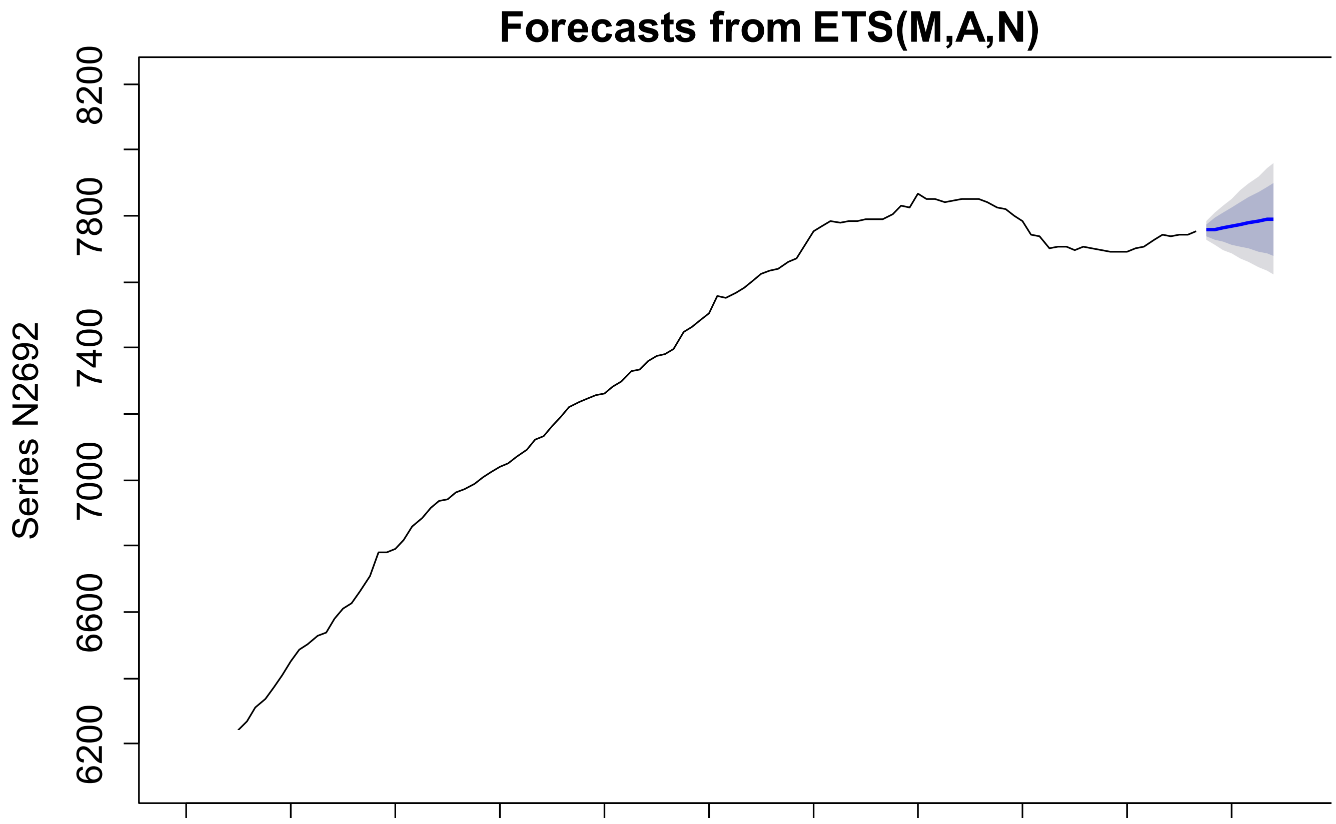
Example. Trended series

- Series N2692 from M3



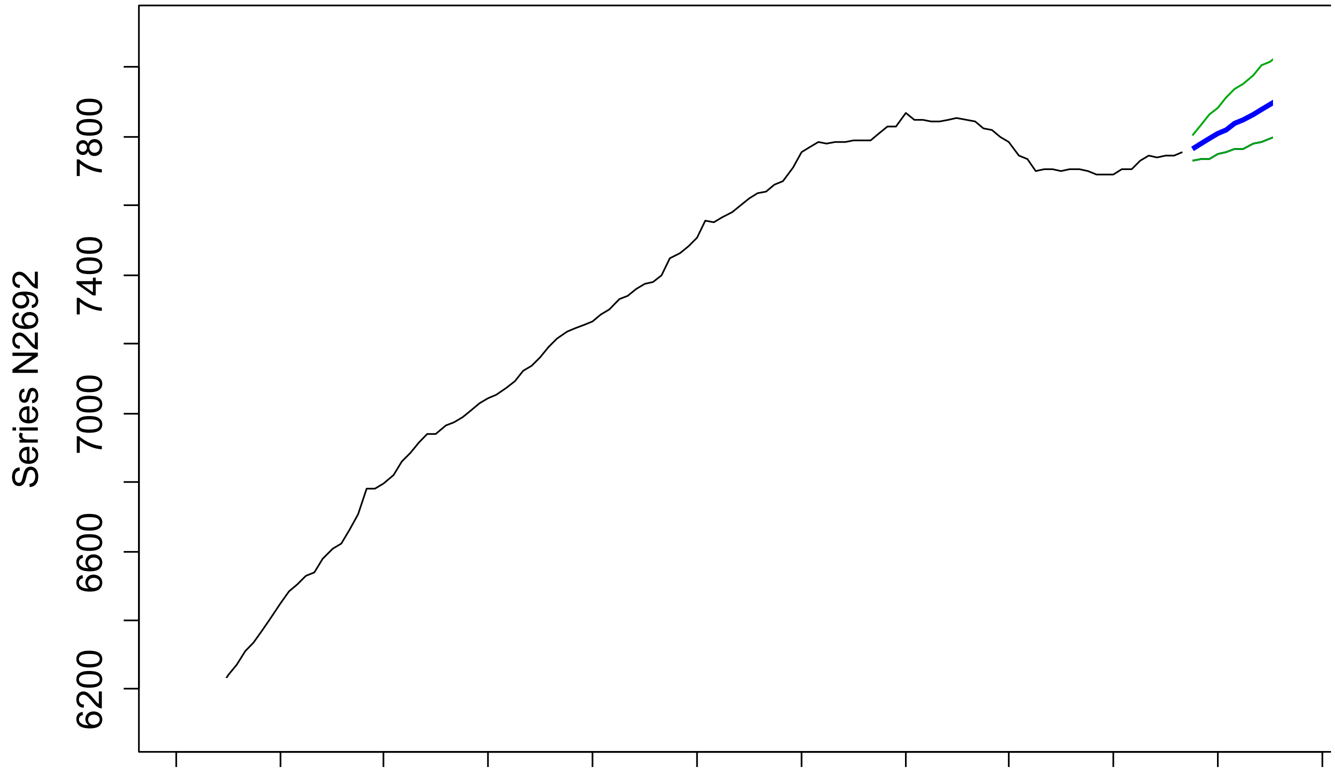
Example. Trended series

- ETS(M,A,N)



Example. Trended series

- CES



$$\alpha_0 + i\alpha_1 = 1.999993 + 1.003635i$$

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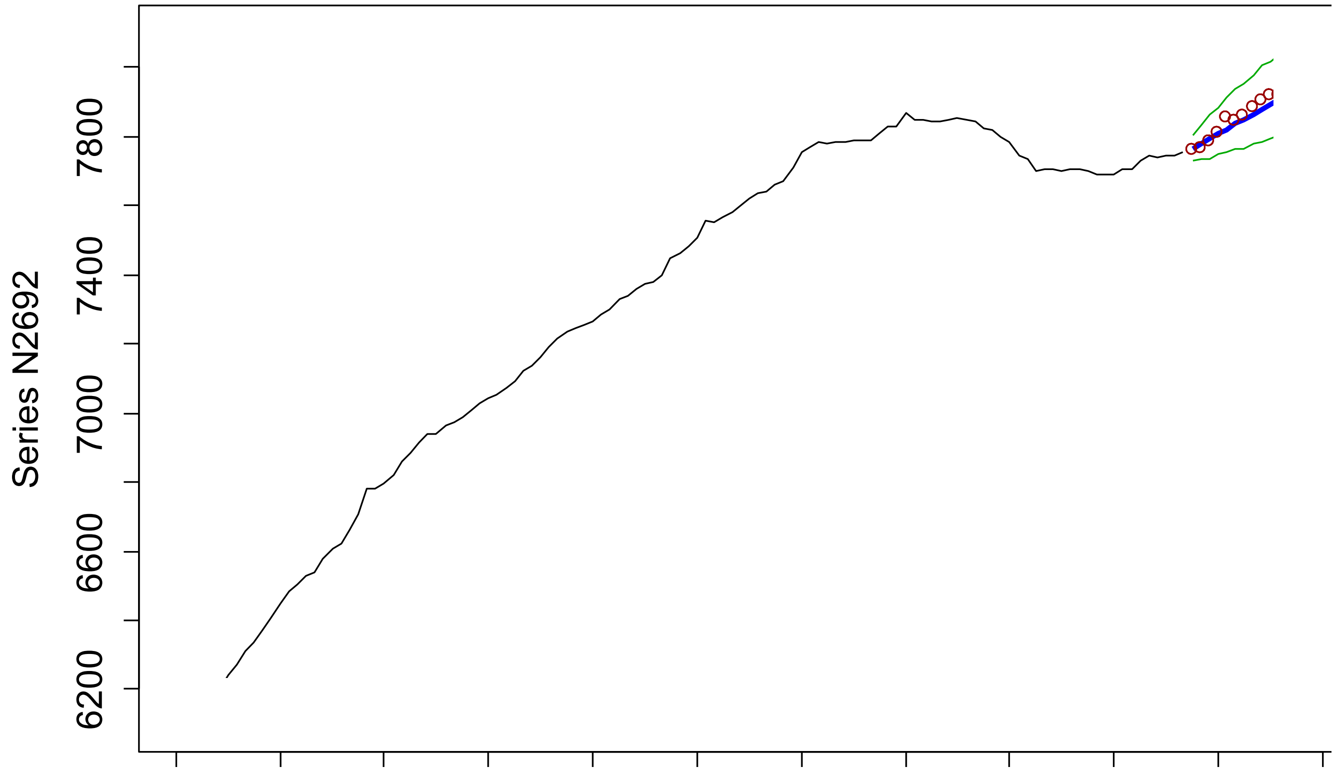
Example. Trended series

- CES



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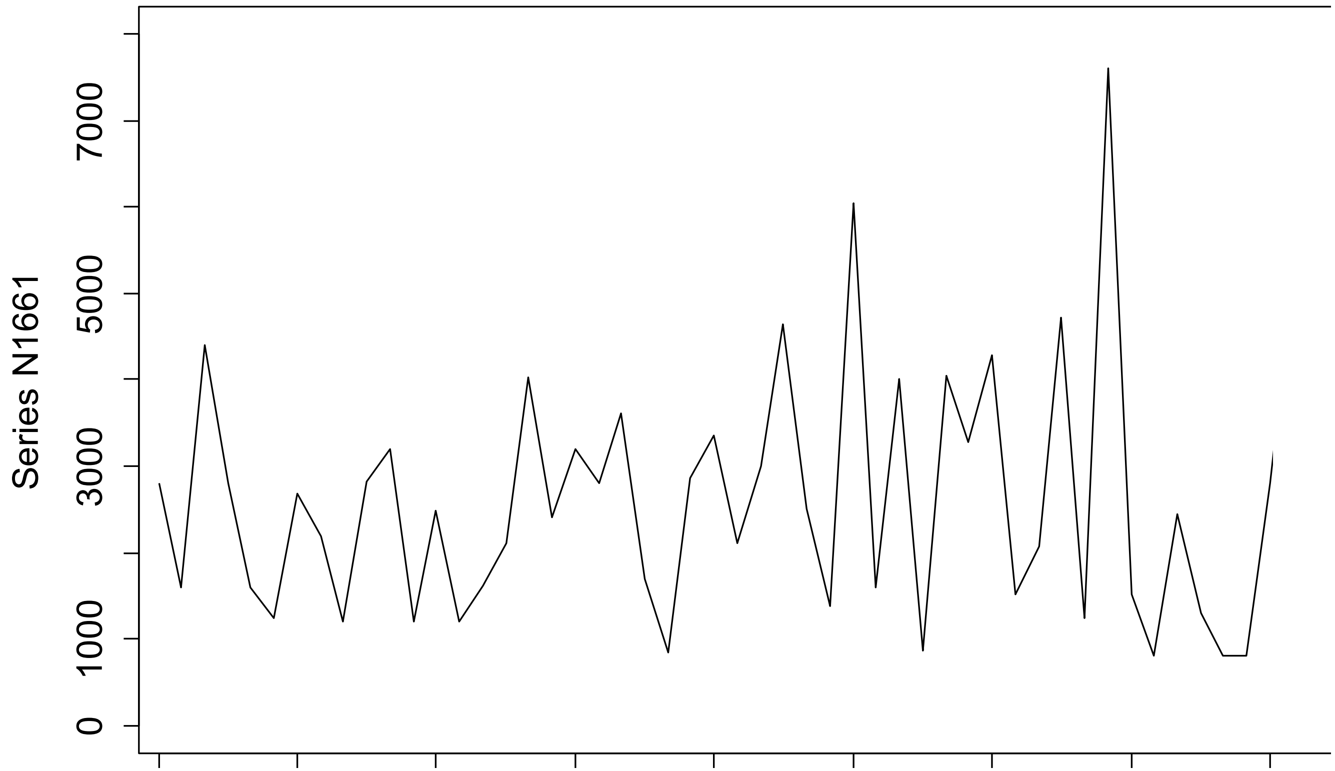
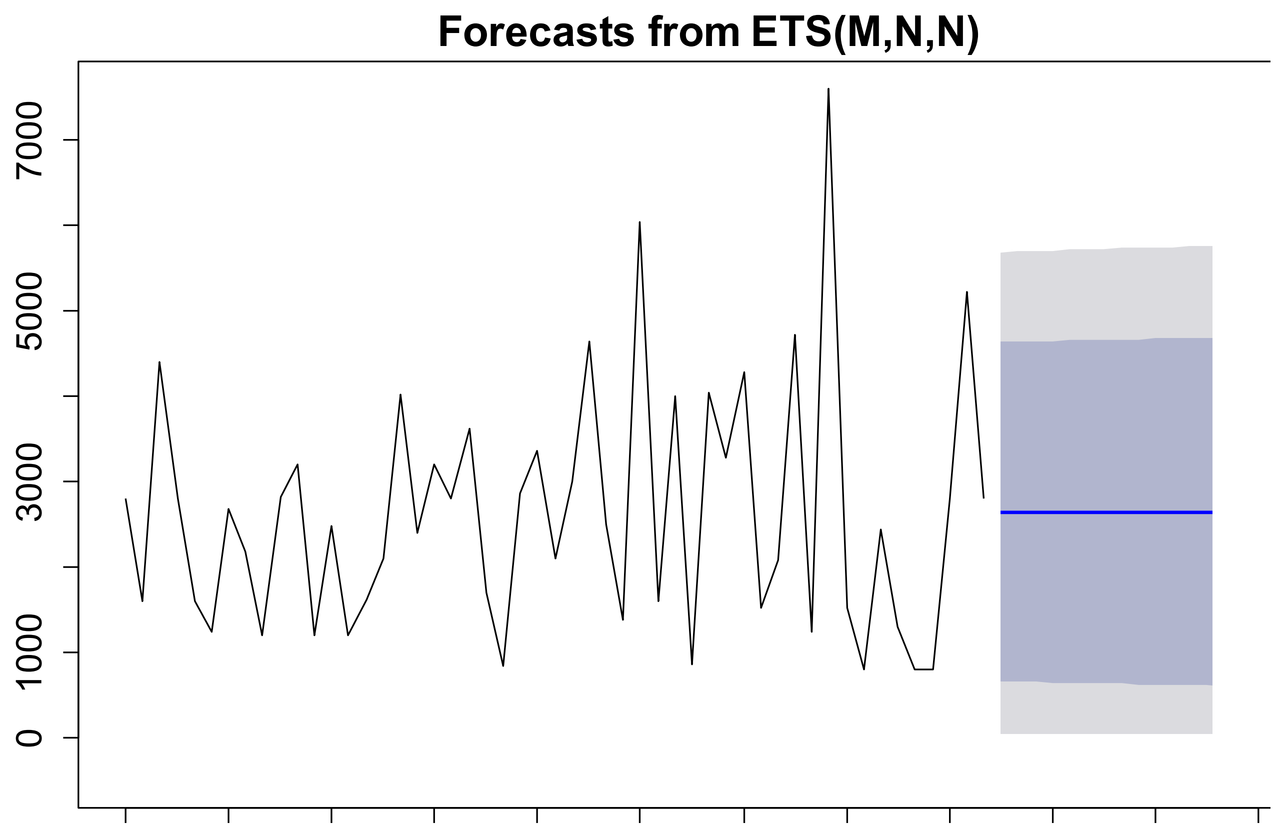
Example. Stationary series

- Series N1661 in M3



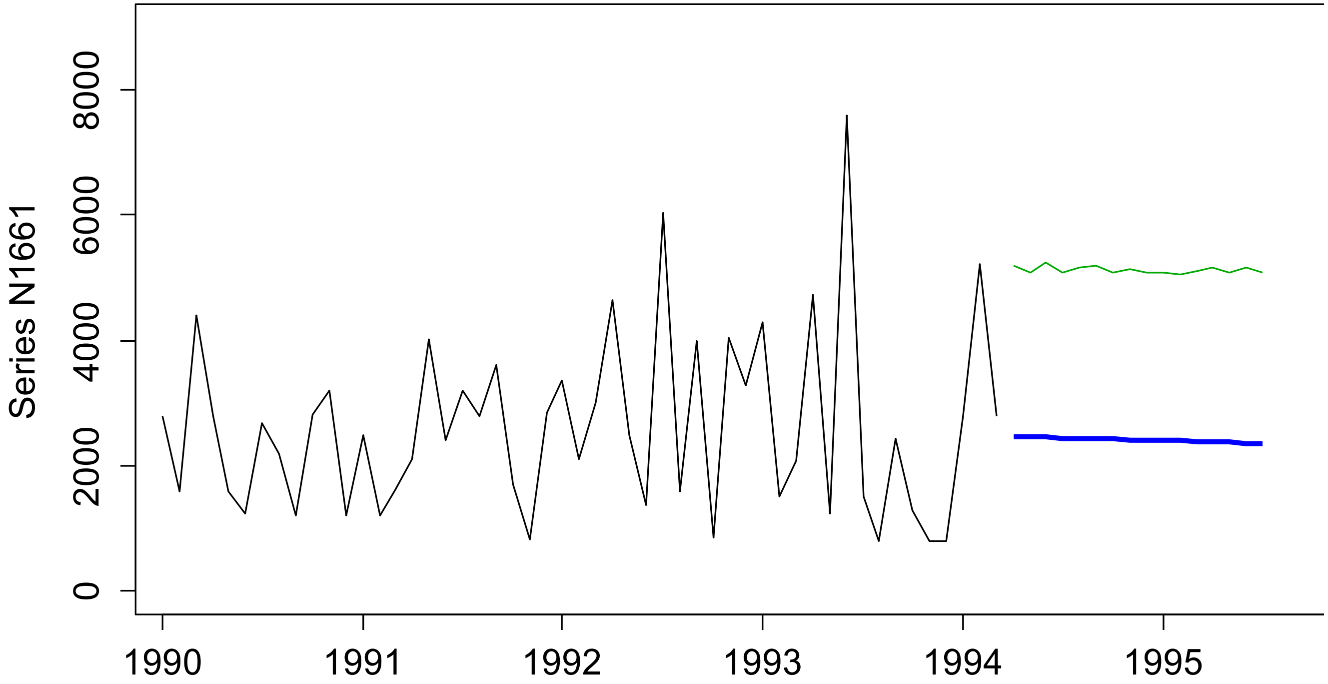
Example. Stationary series

- ETS(M,N,N)



Example. Stationary series

- CES



$$\alpha_0 + i\alpha_1 = 0.9673464 + 0.9970947i$$

Example. Stationary series

- CES