# One for all: forecasting intermittent and non-intermittent demand using one model

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# Introduction

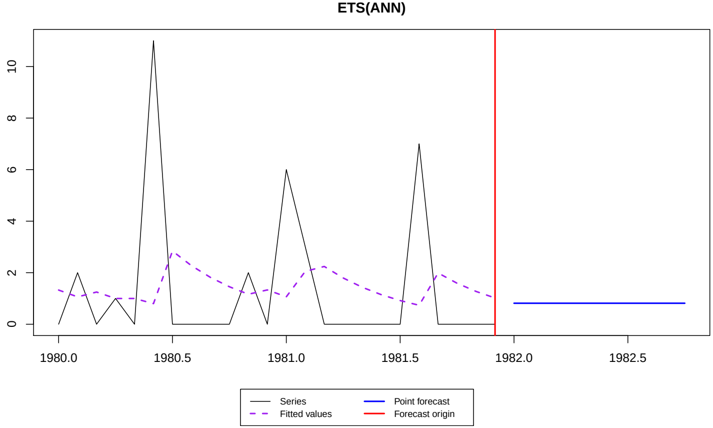
Typical task in supply chain is to produce forecasts for many products.

Demand on each of the products may have its own characteristics and in general can be:

- non-intermittent;
- intermittent.

# Introduction

Simple Exponential Smoothing applied to the intermittent data.



# Introduction

Intermittent data is considered as a separate case.

It is identified and then forecasted, usually using Croston (1972):

$$\begin{aligned} \hat{y}_t &= \frac{1}{\hat{q}_t} \hat{z}_t \\ \hat{z}_t &= \alpha_z z_{t-1} + (1 - \alpha_z) \hat{z}_{t-1}, \\ \hat{q}_t &= \alpha_q q_{t-1} + (1 - \alpha_q) \hat{q}_{t-1} \end{aligned} \quad (1)$$

where $z_t$ is the demand size, $q_t$ is the demand interval,

$\alpha_z$ and $\alpha_q$ are the smoothing parameters.

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# Introduction

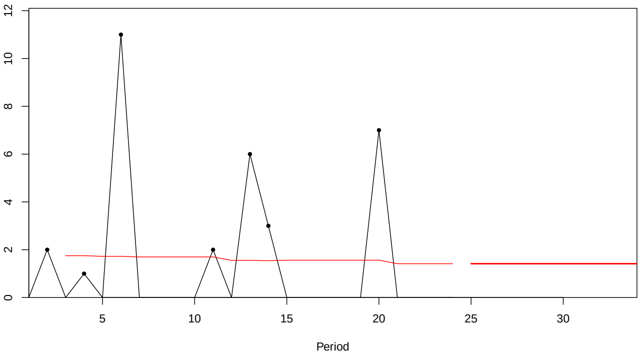



Figure: Intermittent data and Croston's forecast.

# Introduction

We also have SBA (Syntetos and Boylan, 2005), TSB (Teunter et al., 2011), HES (Prestwich et al., 2014), INARMA etc.

All of them are separated from ETS / ARIMA / regression / etc.

# Introduction

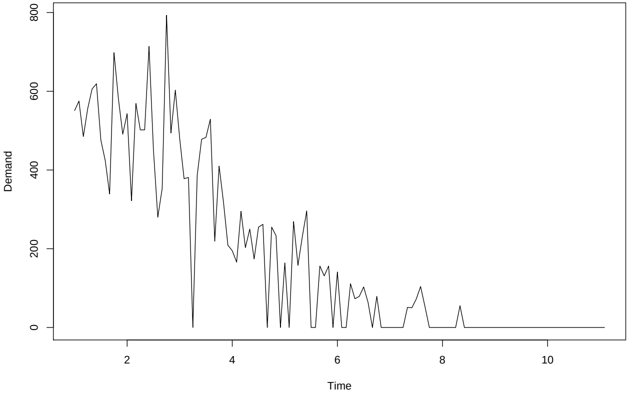
How to categorise the data?

Johnston and Boylan (1996), Syntetos et al. (2005), Petropoulos and Kourentzes (2015)

BUT!

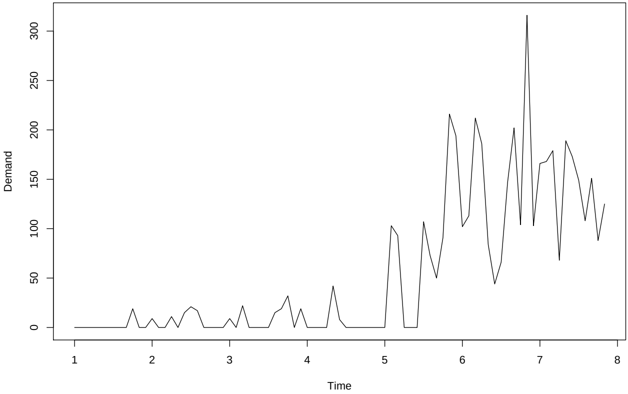
# Introduction

Demand on a fast moving product may become obsolete...



# Introduction

...or demand is just building up.



# Problems

- Products can change their characteristics over time;
- Methods for intermittent demand work with level only.

Overall we need a model that:

1. can switch between intermittent / non-intermittent regimes;
2. can deal with trend and / or seasonality;
3. can be applied to a wide variety of data.

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# Intermittent state-space model (iSS)

# Intermittent state-space model

The model is based on the original idea of Croston (1972):

$$y_t = o_t z_t, \tag{2}$$

where $o_t \sim \text{Bernoulli}(p_t)$ and $z_t$ is a **statistical model** of our choice.

$z_t$ can be **ETS**, ARIMA, regression, diffusion model, etc.

$o_t = 1$ means that there is a sale. $o_t = 0$ means no sale today.

If $o_t = 1,$ for all $t$, then this is non-intermittent model.

# Intermittent state-space model

Multiplicative models are preferred (paper submitted to IJF):

$$y_t = o_t(T \times S \times E) \tag{3}$$

Example. iETS(M,N,N) with time varying probability:

$$\begin{aligned} y_t &= o_t z_t \\ z_t &= l_{t-1}(1+\epsilon_t) \\ l_t &= l_{t-1}(1+\alpha\epsilon_t) \end{aligned} , \tag{4}$$

$1+\epsilon_t \sim \text{logN}(0,\sigma^2)$, which means that $z_t$ is always positive.

States are updated on every observation (potential demand).

But sales happen only when $o_t = 1$.

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# How to model the probability?

$p_t$ has a statistical model of its own.

So far we have developed three models for $p_t$:

- Fixed probability model;
  $p_t = p$ for all $t$.

- Croston's model;
  $p_t = \frac{1}{1+q_t}$, where $q_t$ is ETS(M,N,N).

- TSB model.
  $p_t \sim \text{Beta}(a_t, b_t)$, where $a_t$ and $b_t$ are ETS(M,N,N).

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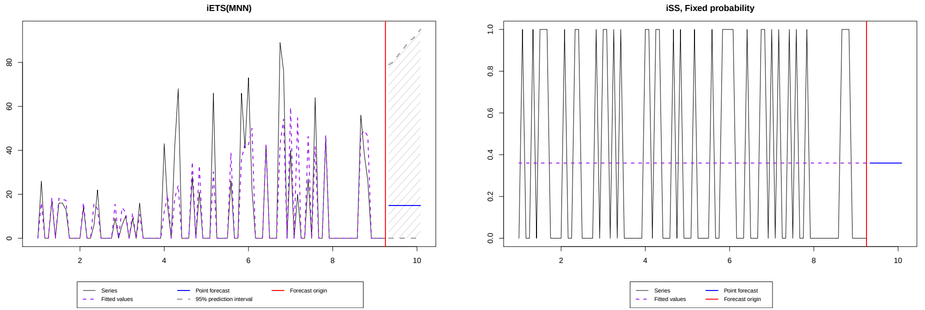
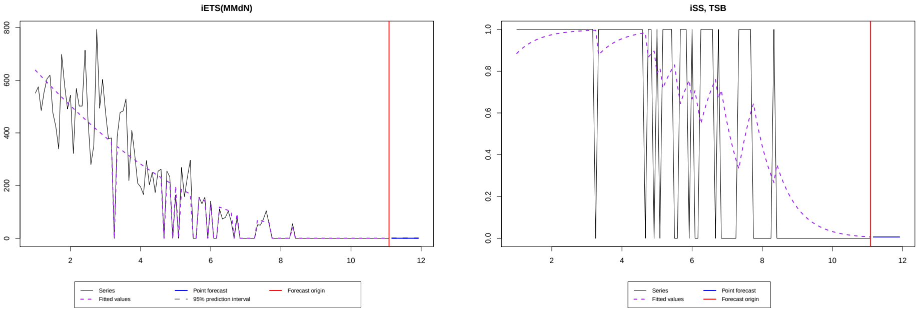
# Examples

iETS(M,N,N) with fixed probability...



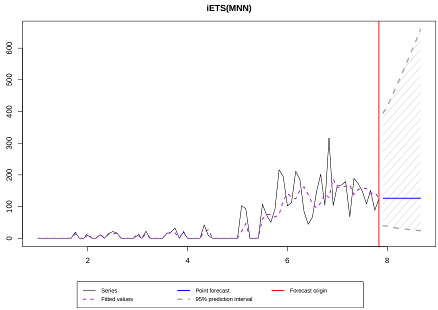
# Examples

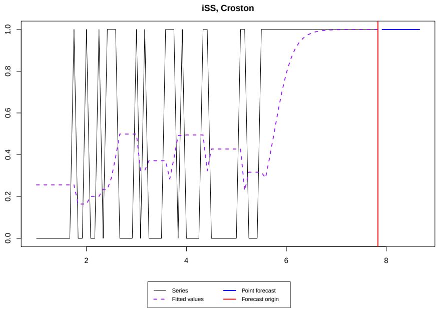
iETS(M,Md,N) with TSB and demand becoming obsolete



# Examples

iETS(M,N,N) with Croston and building up level of demand...



# Model selection

Selection can be done in several directions:

1. $z_t$ – select the best ETS model (error / trend / seasonality);
2. $p_t$ – select the best model between Fixed / Croston / TSB;
3. $p_t$ – select the best ETS model for Croston / TSB.

Here we only discuss (1) and (2), restricted with non-seasonal data.

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# Model selection

Concentrated log-ikelihood function for iETS model:

$$\ell(\theta, \hat{\sigma}_z^2 | \mathbf{Y}) = -\frac{T_1}{2} \left( \log(2\pi e) + \log(\hat{\sigma}_z^2) \right) - \sum_{o_t=1} \log(z_t) + \sum_{o_t=1} \log(\hat{p}_t) + \sum_{o_t=0} \log(1 - \hat{p}_t) , \quad (5)$$

$\theta$ is the vector of the parameters, $\sigma_z^2$ is the variance of the residuals of demand sizes, $\mathbf{Y}$ is the vector of actual values, $T_1$ is the number of observations of non-zero demand.

The selection can be done using AIC, AICc, BIC etc.

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# Experiments

# Data

- WF Wholesale data (Johnston et al., 1999);
- Daily data with working days only;
- One year – 248 observations;
- 120 branches, around 600 SKUs;
- Some series have negative values;
- Excluded series with less than 5 non-zero observations;
- Excluded data with no variability;
- Aggregated SKU for all branches to have non-intermittent data;
- Overall – 10221 time series.

# Contestants

- iETS(Z,Z,N);
- ETS(A,N,N);
- Croston;
- TSB;
- Naive;
- Zeroes.

`es()` function from `smooth` package for R (from CRAN) for all.

# Error measures

- sMSE - scaled Mean Squared Error;
- sPIS - Periods-in-stock;
- sCE - Cumulative Error;
- PLS - Prediction Likelihood Score;
- Prediction intervals coverage (distance from 95%).

Other settings

- Horizon of 20 days (one month);
- Fixed origin.

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# Results

| Model | sMSE | sPIS | sCE | PLS | PI |
|---|---|---|---|---|---|
| iETS(ZZN) | 0.550 | -5.014 | -0.535 | **-15.621** | 0.070 |
| ETS(ANN) | **0.547** | **-2.141** | **-0.263** | -114.620 | **0.040** |
| Croston | 0.556 | 8.616 | 0.761 | -19.627 | 0.072 |
| TSB | **0.547** | -2.502 | -0.298 | -18.033 | 0.120 |
| Naive | 0.761 | -2.853 | -0.331 | -95.841 | 0.049 |
| Zeroes | 0.578 | -21.746 | -2.131 | -113.343 | 0.040 |

Table: Mean Error measures.

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# Results

| Model | sMSE | sPIS | sCE | PLS | PI |
|---|---|---|---|---|---|
| iETS(ZZN) | 0.018 | 3.343 | **0.241** | **-7.338** | 0.050 |
| ETS(ANN) | 0.020 | 5.373 | 0.478 | -42.811 | 0.050 |
| Croston | 0.031 | 11.410 | 1.026 | -8.120 | 0.050 |
| TSB | 0.019 | 5.018 | 0.466 | -7.713 | 0.050 |
| Naive | 0.020 | **-2.131** | -0.345 | -50.179 | 0.050 |
| Zeroes | **0.015** | -4.100 | -0.571 | -43.038 | 0.050 |

Table: Median Error measures.

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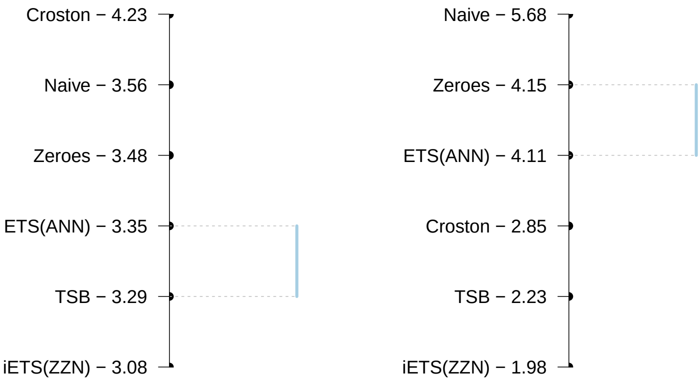
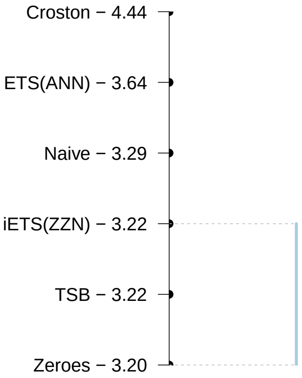
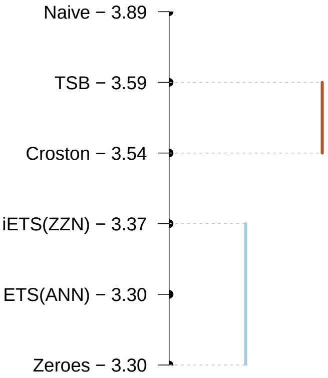
# Nemenyi test (Demšar, 2006) on sMSE and PLS



# Nemenyi test on absolute sPIS and Coverage



# Conclusions

# Conclusions

- Connection between intermittent and conventional models;
- We can use one model for wide variety of series;
- Categorisation based on modelling approach;
- Good results on real data.

# Future experiments

- iETS applied to seasonal data;
- Include inventory simulations;
- Another dataset (more heterogeneous).

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# Thank you for your attention!

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# Integer models

Option 1: Poisson model (similar to Hyndman et al., 2008):

$$\begin{aligned} & y_t = o_t z_t, \\ & z_t \sim \text{Poisson}(\lambda_t) \\ & \lambda_t = l_{t-1}(1 + \epsilon_t) \\ & l_t = l_{t-1}(1 + \alpha \epsilon_t) \end{aligned}$$

$\lambda_t$ is not observable!

Option 2: iETS with rounding up:

$$y_t = o_t \lceil z_t \rceil,$$

where $\lceil \cdot \rceil$ is ceiling function.

# Results on the same data

| Model | sMSE | sPIS | sCE | PLS | PI |
|---|---|---|---|---|---|
| iETS(ZZN) | 0.550 | -5.014 | -0.535 | **-15.621** | 0.070 |
| ETS(ANN) | **0.547** | **-2.141** | **-0.263** | -114.620 | 0.040 |
| Croston | 0.556 | 8.616 | 0.761 | -19.627 | 0.072 |
| TSB | **0.547** | -2.502 | -0.298 | -18.033 | 0.120 |
| Naive | 0.761 | -2.853 | -0.331 | -95.841 | 0.049 |
| Zeroes | 0.578 | -21.746 | -2.131 | -113.343 | 0.040 |
| Int iETS | 1.615 | 18.021 | 4.083 | -26.559 | **0.030** |

Table: Mean Error measures.

# Results on the same data

| Model | sMSE | sPIS | sCE | PLS | PI |
|---|---|---|---|---|---|
| iETS(ZZN) | 0.018 | 3.343 | **0.241** | **-7.338** | 0.050 |
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| Croston | 0.031 | 11.410 | 1.026 | -8.120 | 0.050 |
| TSB | 0.019 | 5.018 | 0.466 | -7.713 | 0.050 |
| Naive | 0.020 | **-2.131** | -0.345 | -50.179 | 0.050 |
| Zeroes | **0.015** | -4.100 | -0.571 | -43.038 | 0.050 |
| Int iETS | 0.038 | 5.886 | 0.521 | -8.751 | 0.050 |

Table: Median Error measures.

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