# Fiscal policy and parental trade-offs

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# Research question and method of analysis

- Study the influence of the government support for education and fertility on the long-run development of the economy
- Analyse the effects of the subsidy rates for education and fertility which reduce the costs for education provision and childbearing decisions
- Utilise the model of overlapping generations introduced by de la Croix and Doepke (AER, 2003)
- The model features endogenous choices of heterogeneous adult households for education provision for their children and number of children to have, which form the human capital and population size of the future generations
- Introduce the government sector into the model economy
- Perform the analysis for the deterministic framework with perfect foresight, and for stochastic framework with uncertainty in the human capital accumulation

# Literature

Analysis for the deterministic environment that uses OLG framework:

- Subsidy for education:
  - Long-run increase in number of children and abilities of adults (Fanti and Gori, 2011; and Chen, 2015)
  - Long-run welfare improvement (Chen, 2015)
- Subsidy for children (child allowance):
  - Long-run decrease in level of human capital, and long-run increase in fertility (Chen, 2015)
  - Long-run decrease in level of human capital, however, no effect on fertility (Fanti and Gori, 2011)
  - Long-run decrease in fertility when parents spend small amount of time on raising children
  - Long-run increase in fertility when parents spend large amount of time on raising children (Momota, 2000)

# Literature (cont.)

Analysis for the stochastic environment:

- Kogan and Walker (2007), real option theory:
  - An environment with higher risks (in the labour market) creates the incentives for higher education attainment which increases human capital.
  - Additionally, with increase in the labour income tax, individuals decide to stay in education longer.
- Akyol and Athreya (2004), dynamic heterogeneous agent model:
  - Subsidy for tertiary education increases participation in the higher education and produces welfare improvement when there is uncertainty for returns on the education investment

# Summary for the results

Deterministic version of the analysis:

- Subsidy for education results in increase of education attainment, level of human capital, welfare of the economy and level of output per capita produced
- It, however, decreases the optimal choice for children and population size
- Subsidy for fertility is found to produce the opposite

Stochastic version of the analysis:

- At the average level, the results are consistent with outcomes from the deterministic environment
- For the individual ability groups, however, subsidy for education was not effective for ones who optimally provide their children with zero education
- Subsidy for fertility is found to improve utility of bottom four ability groups, whereas the rest of the population experiences a decline of utility
- Level of inequality in distribution of the human capital decreases with the subsidy for education, and it increases with subsidy for fertility

# Model economy – Households

- Three generations households: young, adult and elderly
- Households are heterogeneous in the human capital
- Youth: receive education which forms the human capital
- Additionally, in the stochastic version of the analysis, young households receive idiosyncratic shocks while human capital is accumulating
- Adults: maximise personal utility, participate in the (perfectly competitive) labour market, earn labour income, consume, save, pay taxes, receive subsidies, make decision for level of education to provide for their children and number of children to have
- Adult households experience the trade-offs in their decisions for the number of children to have and level of education to provide their children with
- Elderly: receive return on investment, consume, pay taxes

# Model economy – Households (cont.)

For $i \in [1, I]$ and $t \in [1, T]$

$$\max_{<c_t^i, s_t^i, e_t^i, n_t^i>} E_t[u_t^i] = E_t\left[\ln c_t^i + \beta \ln d_{t+1}^i + \gamma \ln n_t^i h_{t+1}^i\right]$$

$$(1+\tau_t^c)c_t^i + s_t^i + e_t^i n_t^i w_t \bar{h}_t = (1-\tau_t^{inc}) w_t h_t^i (1-\phi n_t^i) + n_t^i w_t \bar{h}_t (e_t^i sub_t^{ed} + \bar{e}_t sub_t^{chil})$$

$$E_t[(1+\tau_{t+1}^c) d_{t+1}^i] = E_t\left[\frac{1}{(1+\rho)}\left(1 + r_{t+1}(1-\tau_{t+1}^k)\right) s_t^i\right]$$

$$E_t[h_{t+1}^i] = E_t\left[\frac{1}{(1+\rho)} B(\theta + e_t^i)^\eta (h_t^i)^\pi (\bar{h}_t)^\kappa \times \exp^\varepsilon\right]$$

# Model Economy – Producer

- Production sector: single firm maximises its profit by optimally choosing its factor inputs
- It produces output using physical capital and effective labour force

$$\max_{<K_t, L_t>} \Pi_t = Y_t - w_t L_t - (r_t + \delta) K_t$$

$$Y_t = A K_t^{\alpha} L_t^{1-\alpha}$$

- Both factor inputs are supplied by households

# Model economy – Government

- Authority that provides adult households with subsidy for education and fertility
- It taxes consumption, labour income and capital income to finance its budget
- At any given point, the government runs the balanced budget

$$\tau_t^c c_t + \tau_t^c d_t + r_t \tau_t^k s_t + \tau_t^{inc} w_t h_t (1 - \phi n_t) = n_t w_t \bar{h}_t (e_t sub_t^{ed} + \bar{e}_t sub_t^{chil})$$

# Parametrization

- Closely follow original calibration of de la Croix and Doepke (2003)
- (Note: one period of analysis = one generation = 30 years)
- We, however, normalise values of $A$, $B$ and $\gamma$ to obtain $w_1 = 1$, $\bar{h}_1 = 1$ and $\bar{n}_1 = 1$

# Algorithm – Deterministic version of the analysis

1. Obtain the steady state for the representative household (i.e. homogeneous case) without the presence of the government ($t = 1$)
2. Divide the population into two groups – one with 10% larger level of human capital than at the initial steady state, and other with 10% lower ($t = 2$)
3. Introduce the government which provides the subsidy and collects the tax ($t = 2$)
4. Calculate the transition path to the second steady state with the government presence ($t \in [2, T)$)
5. Calculate the second steady state with the government policy in place ($t = T$)

# Results – Deterministic environment – Second steady-state – Deviation from original steady-state – $sub^{ed}$

| variable | $\tau^c = 0.53\%$ | $\tau^{inc} = 0.63\%$ | $\tau^k = 1.02\%$ |
|---|---|---|---|
| $e_i$ | +16.28% | +15.25% | +16.28% |
| $h_i$ | +9.27% | +8.69% | +9.27% |
| $n_i$ | -1.85% | -1.12% | -1.85% |
| $K$ | +10.66% | +8.67% | +10.66% |
| $L$ | +6.53% | +6.69% | +6.53% |
| $r$ | -3.36% | -1.63% | -3.36% |
| $w$ | +1.27% | +0.61% | +1.27% |
| $c_i$ | +10.08% | +8.67% | +10.66% |
| $d_i$ | +7.32% | +7.35% | +7.07% |
| $s_i$ | +10.66% | +8.67% | +10.66% |
| $Y$ | +7.89% | +7.35% | +7.89% |
| $u_i$ | +20.20% | +18.28% | +20.92% |

- Subsidy for education results in increase of education attainment, level of human capital, welfare of the economy and level of output per capita produced.
- It, however, decreases the optimal choice for children and population size

# Results – Deterministic environment – Second steady-state – Deviation from original steady-state – $sub^{chil}$

| variable | $\tau^c = 0.46\%$ | $\tau^{inc} = 0.55\%$ | $\tau^k = 0.78\%$ |
|---|---|---|---|
| $e_i$ | -9.09% | -9.82% | -9.09% |
| $h_i$ | -5.33% | -5.76% | -5.33% |
| $n_i$ | +7.69% | +8.66% | +7.69% |
| $K$ | -8.62% | -10.05% | -8.62% |
| $L$ | +1.63% | +1.76% | +1.63% |
| $r$ | +9.85% | +11.49% | +9.85% |
| $w$ | -3.48% | -4.03% | -3.48% |
| $c_i$ | -9.04% | -10.05% | -8.62% |
| $d_i$ | -2.36% | -2.34% | -2.49% |
| $s_i$ | -8.62% | -10.05% | -8.62% |
| $Y$ | -1.91% | -2.34% | -1.91% |
| $u_i$ | -15.16% | -16.83% | -14.52% |

- Subsidy for fertility is found to produce the opposite

# Algorithm – Stochastic version of the analysis

1. Discretise the population into 15 ability groups with each group being defined by its marginal level of human capital. Before uncertainty is introduced, there is uniform distribution of human capital across the population
2. Discretise the random variable, which is log-normally distributed, into 5 nodes using Binomial probability to calculate probability of occurrence of each node
3. Introduce idiosyncratic shocks into the human capital accumulation process, and obtain the stable distribution for the original steady state without government presence ($t = 1$)
4. Introduce the government policy and calculate the transition path to the second steady state ($t \in [2, T)$)
5. Reach the second steady state with government presence, and obtain the stable distribution ($t = T$)

# Results – Stochastic environment – Second steady-state – Deviation from original steady-state – $sub^{ed}$ – $\tau^c$

| | $Y$ | $K$ | $L$ | $r$ | $w$ | $\tau^c$ | | |
|---|---|---|---|---|---|---|---|---|
| | +7.38% | +11.09% | +6.62% | -3.68% | +1.39% | 0.51% | | |
| | $\bar{h}$ | tot.pop. | $\bar{c}$ | $\bar{s}$ | $\bar{n}$ | $\bar{e}$ | $\bar{d}$ | $\bar{u}$ |
| | +9.59% | -71.28% | +7.13% | +11.11% | -3.18% | +16.21% | +7.54% | +20.51% |
| $i$ | $h_i$ | pop.share | $c_i$ | $s_i$ | $n_i$ | $e_i$ | $d_i$ | $u_i$ |
| 1 | 0.2019 | 0.0000 | +0.73% | +1.47% | 0% | 0 | -1.87% | +0.10% |
| 2 | 0.2466 | 0.0001 | +0.90% | +1.40% | 0% | 0 | -1.82% | +0.11% |
| 3 | 0.3012 | 0.0007 | +0.88% | +1.31% | 0% | 0 | -1.88% | +0.13% |
| 4 | 0.3679 | 0.0054 | +0.84% | +1.34% | -1.03% | +10.53% | -1.86% | +0.05% |
| 5 | 0.4493 | 0.0207 | +0.89% | +1.42% | -0.75% | +5.05% | -1.63% | +0.09% |
| 6 | 0.5488 | 0.0508 | +0.90% | +1.44% | -0.55% | +2.87% | -1.89% | +0.13% |
| 7 | 0.6703 | 0.0974 | +0.88% | +1.40% | -0.43% | +2.64% | -1.86% | +0.18% |
| 8 | 0.8187 | 0.1562 | +0.87% | +1.38% | -0.33% | +2.39% | -1.87% | +0.25% |
| 9 | 1.0000 | 0.2092 | +0.87% | +1.38% | -0.26% | +1.95% | -1.86% | +0.38% |
| 10 | 1.2214 | 0.1965 | +0.87% | +1.42% | -0.21% | +1.92% | -1.87% | +0.71% |
| 11 | 1.4918 | 0.1940 | +0.87% | +1.39% | -0.16% | +1.70% | -1.86% | +3.62% |
| 12 | 1.8221 | 0.0671 | +0.87% | +1.38% | -0.14% | +1.59% | -1.88% | +1.38% |
| 13 | 2.2255 | 0.0053 | +0.87% | +1.38% | -0.10% | +1.59% | -1.87% | +0.55% |
| 14 | 2.7182 | -0.0034 | +0.87% | +1.38% | -0.08% | +1.55% | -1.87% | +0.35% |
| 15 | 3.3201 | -0.0000 | +0.77% | +1.38% | -0.07% | +1.55% | -1.87% | +0.26% |

- At the average level, the results are consistent with outcomes from the deterministic environment
- For the individual ability groups, however, subsidy for education is not effective for ones who optimally provide their children with zero education

# Results – Stochastic environment – Second steady-state – Deviation from original steady-state – $sub^{chil} - \tau^c$

| | $Y$ | $K$ | $L$ | $r$ | $w$ | $\tau^c$ | | |
|---|---|---|---|---|---|---|---|---|
| | -2.56% | -10.64% | +1.73% | +12.23% | -4.22% | 0.48% | | |
| | $\bar{h}$ | tot.pop. | $\bar{c}$ | $\bar{s}$ | $\bar{n}$ | $\bar{e}$ | $\bar{d}$ | $\bar{u}$ |
| | -6.67% | +237.78% | -11.04% | -10.62% | +15.69% | -8.79% | -3.04% | -20.98% |
| $i$ | $h_i$ | pop.share | $c_i$ | $s_i$ | $n_i$ | $e_i$ | $d_i$ | $u_i$ |
| 1 | 0.2019 | 0.0001 | -4.69% | -4.16% | +40.44% | 0 | +3.85% | +0.83% |
| 2 | 0.2466 | 0.0015 | -4.68% | -4.21% | +30.85% | 0 | +3.92% | +0.44% |
| 3 | 0.3012 | 0.0068 | -4.66% | -4.26% | +23.92% | 0 | +3.91% | +0.09% |
| 4 | 0.3679 | 0.0176 | -4.70% | -4.16% | +29.41% | -71.05% | +3.91% | +0.47% |
| 5 | 0.4493 | 0.0368 | -4.67% | -4.18% | +19.56% | -23.23% | +3.89% | -0.25% |
| 6 | 0.5488 | 0.0722 | -4.66% | -4.14% | +13.88% | -10.34% | +3.91% | -0.87% |
| 7 | 0.6703 | 0.1264 | -4.68% | -4.20% | +10.24% | -4.15% | +3.94% | -1.54% |
| 8 | 0.8187 | 0.1858 | -4.68% | -4.22% | +7.76% | -0.80% | +3.91% | -2.44% |
| 9 | 1.0000 | 0.2166 | -4.67% | -4.20% | +5.98% | +1.37% | +3.91% | -3.97% |
| 10 | 1.2214 | 0.1500 | -4.67% | -4.21% | +4.68% | +2.80% | +3.92% | -7.72% |
| 11 | 1.4918 | 0.1387 | -4.68% | -4.20% | +3.70% | +3.75% | +3.89% | -40% |
| 12 | 1.8221 | 0.0375 | -4.67% | -4.23% | +2.94% | +4.52% | +3.91% | -14.35% |
| 13 | 2.2255 | 0.0087 | -4.68% | -4.22% | +2.36% | +5.10% | +3.91% | -6.31% |
| 14 | 2.7182 | 0.0014 | -4.67% | -4.23% | +1.89% | +5.50% | +3.91% | -4.12% |
| 15 | 3.3201 | -0.0000 | -4.68% | -4.22% | +1.52% | +5.82% | +3.91% | -3.07% |

- Subsidy for fertility is found to improve utility of bottom four ability groups, whereas the rest of the population experiences a decline of utility
- Simultaneous increase in fertility and education provision for higher ability groups

# Results – Stochastic environment – Inequality in distribution of the human capital

| steady state | Gini coefficient | Coefficient of variation | Mean absolute deviation | Kuznets ratios $\leqslant 40\%$ | Kuznets ratios $\geqslant 60\%$ | The range |
|---|---|---|---|---|---|---|
| original | 0.2062 | 0.3745 | 0.2905 | 0.1349 | 0.3362 | 3.1180 |
| $sub^{ed} + \tau^{c}$ | 0.1835↓ | 0.32306↓ | 0.2748↓ | 0.0777↓ | 0.4595↓ | 2.8452↓ |
| $sub^{ed} + \tau^{inc}$ | 0.1838↓ | 0.32309↓ | 0.2752↓ | 0.0780↓ | 0.4540↓ | 2.8594↓ |
| $sub^{ed} + \tau^{k}$ | 0.1835↓ | 0.32306↓ | 0.2748↓ | 0.0777↓ | 0.4595↓ | 2.8452↓ |
| $sub^{chil} + \tau^{c}$ | 0.2400↑ | 0.4184↑ | 0.3557↑ | 0.2406↑ | 0.3234↑ | 3.3410↑ |
| $sub^{chil} + \tau^{inc}$ | 0.2404↑ | 0.4189↑ | 0.3567↑ | 0.2440↑ | 0.3193↑ | 3.3577↑ |
| $sub^{chil} + \tau^{k}$ | 0.2400↑ | 0.4184↑ | 0.3557↑ | 0.2406↑ | 0.3234↑ | 3.3410↑ |

- Level of inequality in distribution of the human capital decreases with the subsidy for education, and it increases with subsidy for fertility

# Ranking of the policies

From best to worst on the basis of change in welfare:

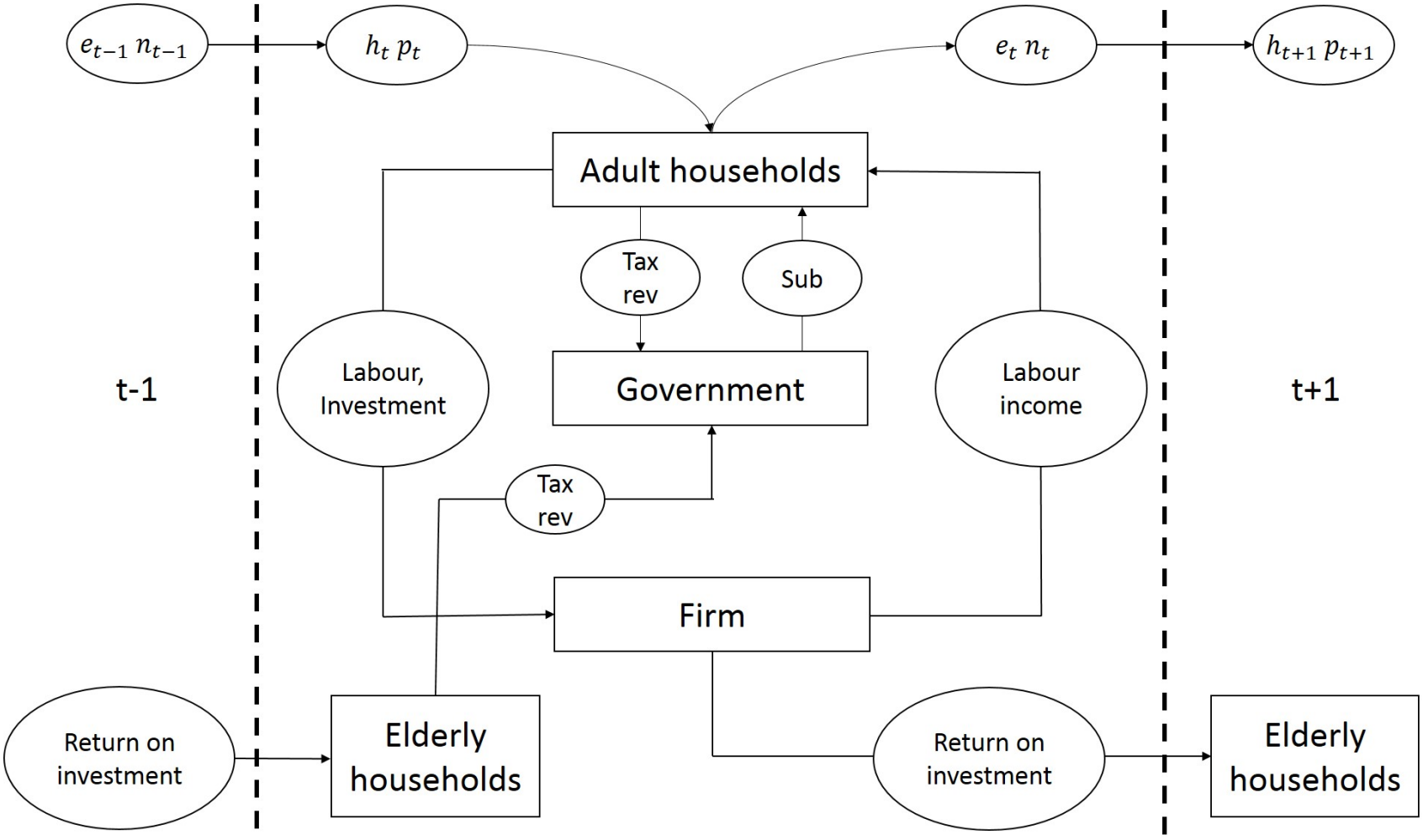
1. Subsidy for education financed with capital income tax
2. Subsidy for education financed with tax on consumption
3. Subsidy for education financed with labour income tax
4. Absence of government intervention
5. Subsidy for fertility financed with capital income tax
6. Subsidy for fertility financed with tax on consumption
7. Subsidy for fertility financed with labour income tax

# Conclusion

- Utilise the model of the overlapping generations of de la Croix and Doepke (AER, 2003)
- Introduce the government sector and uncertainty into human capital accumulation
- Study the influence of the government support for education and fertility on the long-run development of the economy which consists of adult households with trade-offs between education provision for their children and number of children to have
- Perform the analysis for the deterministic and stochastic environment
- Find subsidy for education to improve education attainment, human capital, welfare; but it reduces fertility choices and population size.
- Subsidy for fertility creates the opposite

# Appendix

# A1 – Model economy – Interactions between the sectors



# A2 – Closed form analytical solution – Households

$$e_t^i = \frac{\eta\phi(1-\tau_t^{inc})x_t^i - \theta(1-sub_t^{ed}) - \eta\bar{e}_t sub_t^{chil}}{(1-sub_t^{ed})(1-\eta)}$$

$$n_t^i = \frac{\gamma}{(1+\beta+\gamma)} x_t^i \left[(1-\tau_t^{inc})\phi x_t^i + e_t^i(1-sub_t^{ed}) - \bar{e}_t sub_t^{chil}\right]^{-1}$$

where $x_t^i = h_t^i/\bar{h}_t$

$$c_t^i = \frac{(1-\tau_t^{inc})}{(1+\tau_t^c)} \frac{1}{(1+\beta+\gamma)} w_t h_t^i$$

$$s_t^i = (1-\tau_t^{inc}) \frac{\beta}{(1+\beta+\gamma)} w_t h_t^i$$

$$d_{t+1}^i = \frac{1}{(1+\rho)} \frac{(1-\tau_t^{inc})}{(1+\tau_{t+1}^c)} \frac{\beta}{(1+\beta+\gamma)} \left(1 + r_{t+1}(1-\tau_{t+1}^k)\right) w_t h_t^i$$

# A3 – Closed form analytical solution – Producer

$$r_t = \alpha A {K_t}^{\alpha-1} L_t^{1-\alpha} - \delta$$

$$w_t = (1-\alpha) A K_t^{\alpha} L_t^{-\alpha}$$

$$L_t = \frac{\sum_{i=1}^{I} p_t^i \left[ h_t^i (1 - \phi n_t^i) - n_t^i e_t^i \bar{h}_t \right]}{\sum_{i=1}^{I} p_t^i}$$

$$K_t = \frac{1}{(1+\rho)} \left( \frac{\sum_{i=1}^{I} p_{t-1}^i s_{t-1}^i}{\sum_{i=1}^{I} p_t^i} + (1-\delta) K_{t-1} \right)$$

$$p_t^i = p_{t-1}^i n_{t-1}^i$$

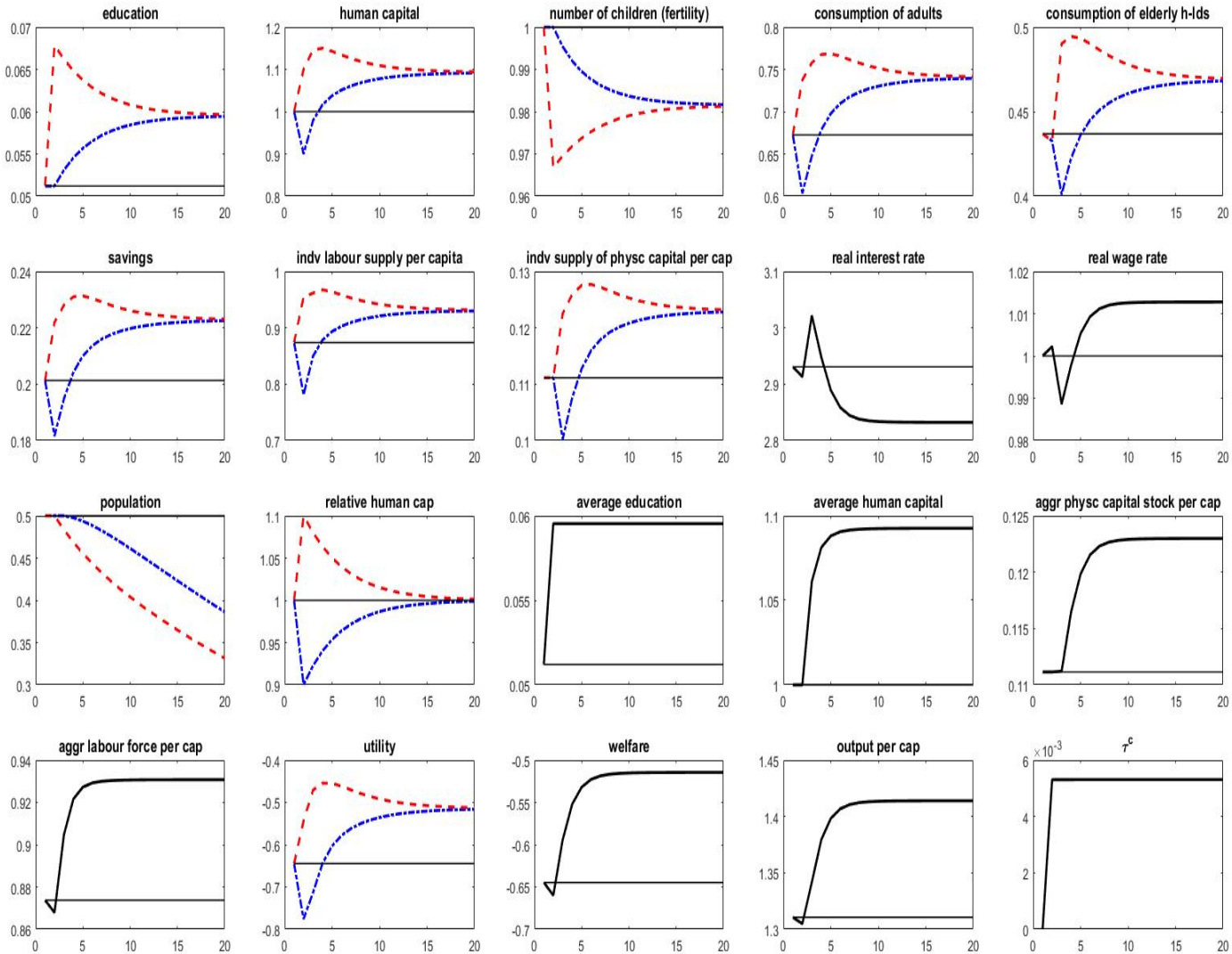
# A4 – Calibration – Deterministic version of the analysis

| parameter | value | interpretation |
|---|---|---|
| $A$ | 2.9826 | productivity level |
| $B$ | 7.2107 | efficiency of human capital accumulation |
| $\alpha$ | $1/3$ | elasticity of physical capital |
| $\beta$ | $0.99^{120}$ | psychological discount factor |
| $\gamma$ | 0.18766 | altruism factor |
| $\delta$ | 1 | depreciation rate of physical capital stock |
| $\eta$ | 0.5 | relative significance of education for human capital |
| $\theta$ | 0.0119 | instrument parameter for non-zero human capital of children when education provision is zero |
| $\pi$ | 0.2 | relative significance of human capital of parents for human capital of children |
| $\kappa$ | 0.1 | relative significance of quality of education for human capital |
| $\rho$ | $1.02^{30} - 1$ | growth rate of human capital over generations |
| $\phi$ | 0.075 | time-cost parameter to raise children |

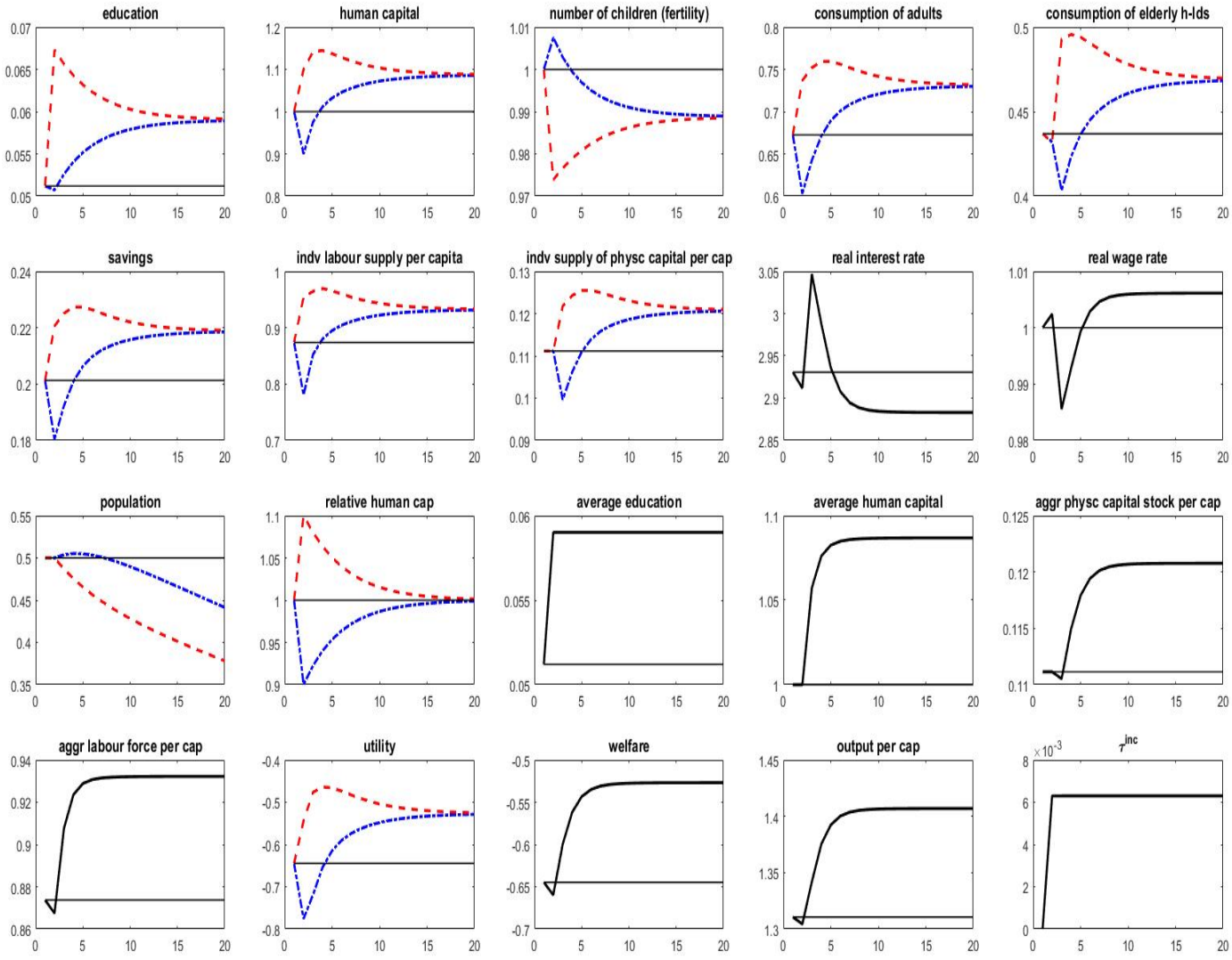
# A5 – Deterministic environment – Initial steady state

| variable | value | interpretation |
|---|---|---|
| $e_i$ | 0.0512 | education attainment/provision |
| $h_i$ | 1 | human capital |
| $n_i$ | 1 | number of children/adults/old-age households |
| $K$ | 0.1111 | physical capital stock per capita |
| $L$ | 0.8737 | effective labour force per capita |
| $r$ | 2.9308 | real interest rate |
| $w$ | 1 | real wage rate |
| $c_i$ | 0.6725 | consumption of adults |
| $d_i$ | 0.4369 | consumption of old-age households |
| $s_i$ | 0.2013 | savings |
| $Y$ | 1.3106 | real output per capita |
| $u_i$ | -0.6448 | utility of households/welfare per capita |

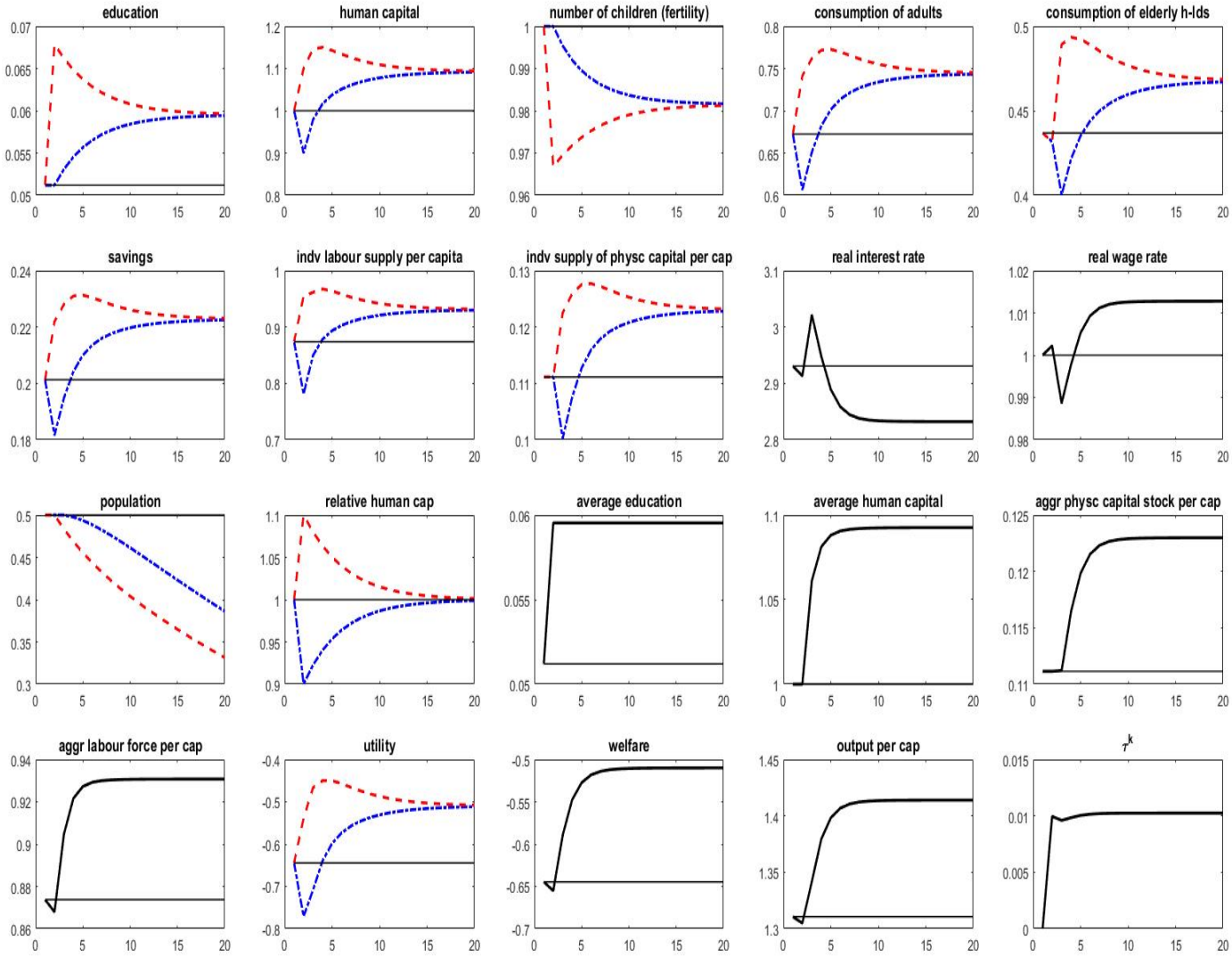
# A6 – Simulation – Deterministic environment – Heterogeneous households – $sub^{ed}$ – $\tau^c$



# A7 – Simulation – Deterministic environment – Heterogeneous households – $sub^{ed}$ – $\tau^{inc}$



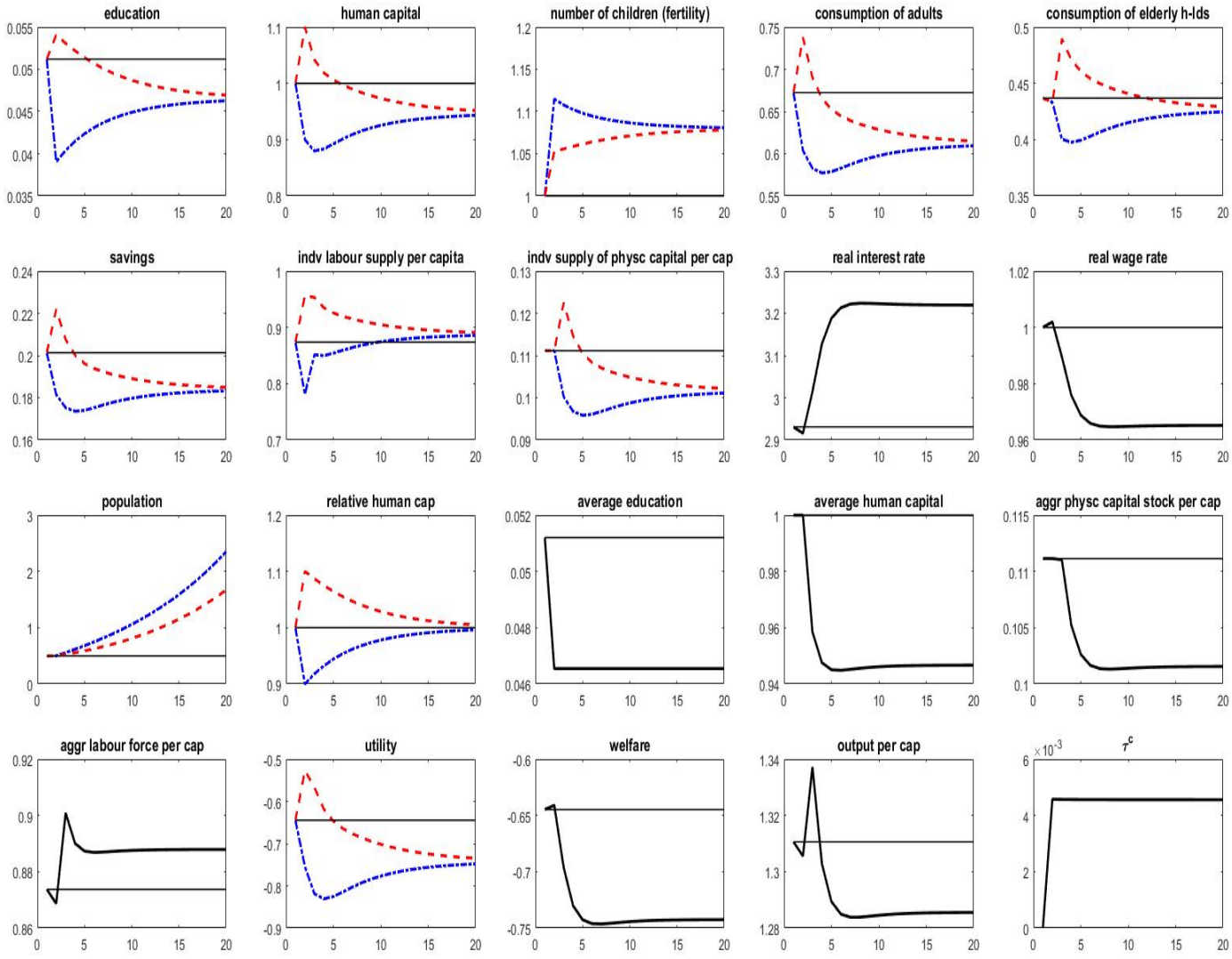
# A8 – Simulation – Deterministic environment – Heterogeneous households – $sub^{ed}$ – $\tau^k$



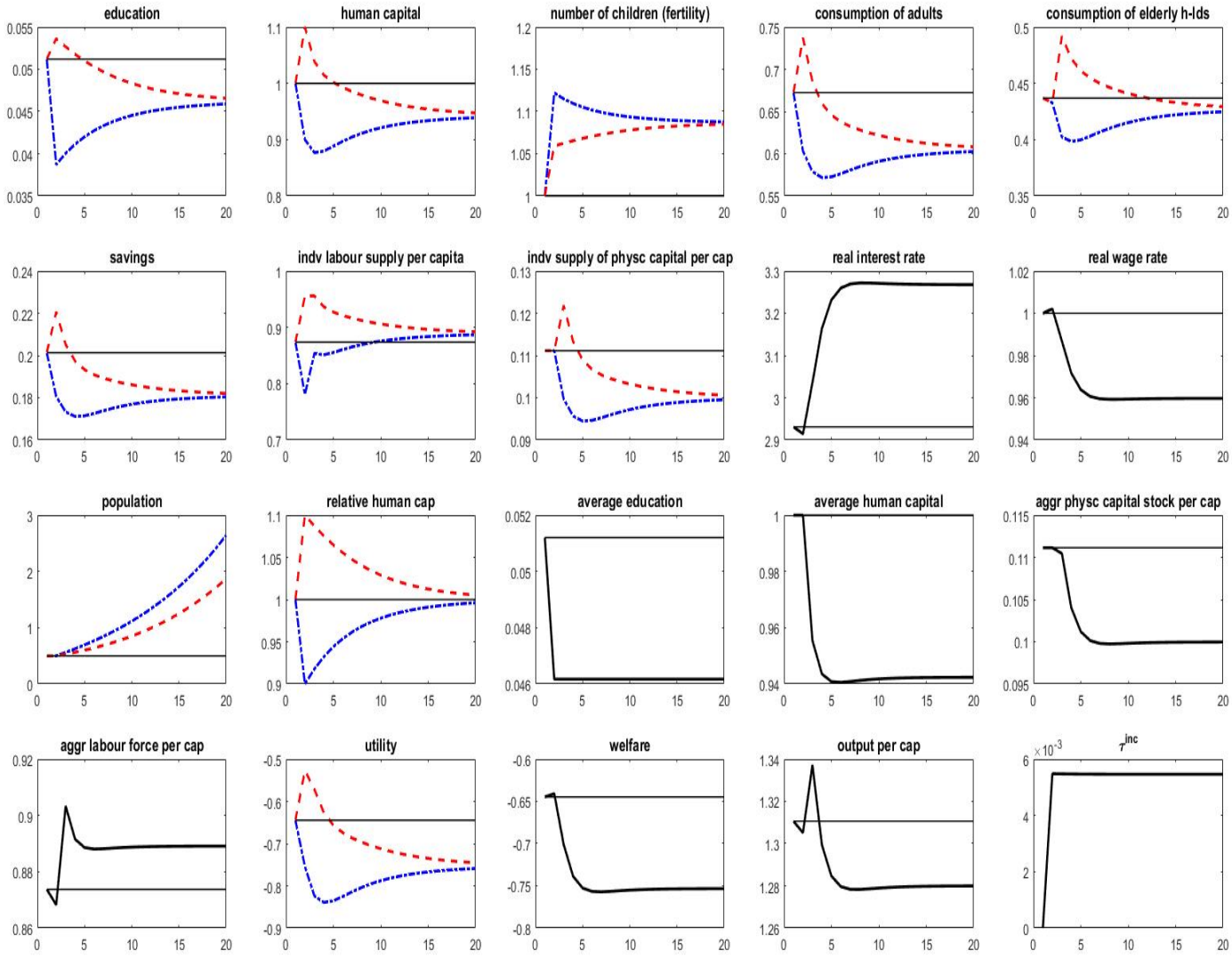
# A9 – Deterministic environment – Second steady-state – Deviation from original steady-state – *sub*$^{ed}$

| variable | $\tau^c = 0.53\%$ | $\tau^{inc} = 0.63\%$ | $\tau^k = 1.02\%$ |
|---|---|---|---|
| $e_i$ | +16.28% | +15.25% | +16.28% |
| $h_i$ | +9.27% | +8.69% | +9.27% |
| $n_i$ | -1.85% | -1.12% | -1.85% |
| $K$ | +10.66% | +8.67% | +10.66% |
| $L$ | +6.53% | +6.69% | +6.53% |
| $r$ | -3.36% | -1.63% | -3.36% |
| $w$ | +1.27% | +0.61% | +1.27% |
| $c_i$ | +10.08% | +8.67% | +10.66% |
| $d_i$ | +7.32% | +7.35% | +7.07% |
| $s_i$ | +10.66% | +8.67% | +10.66% |
| $Y$ | +7.89% | +7.35% | +7.89% |
| $u_i$ | +20.20% | +18.28% | +20.92% |

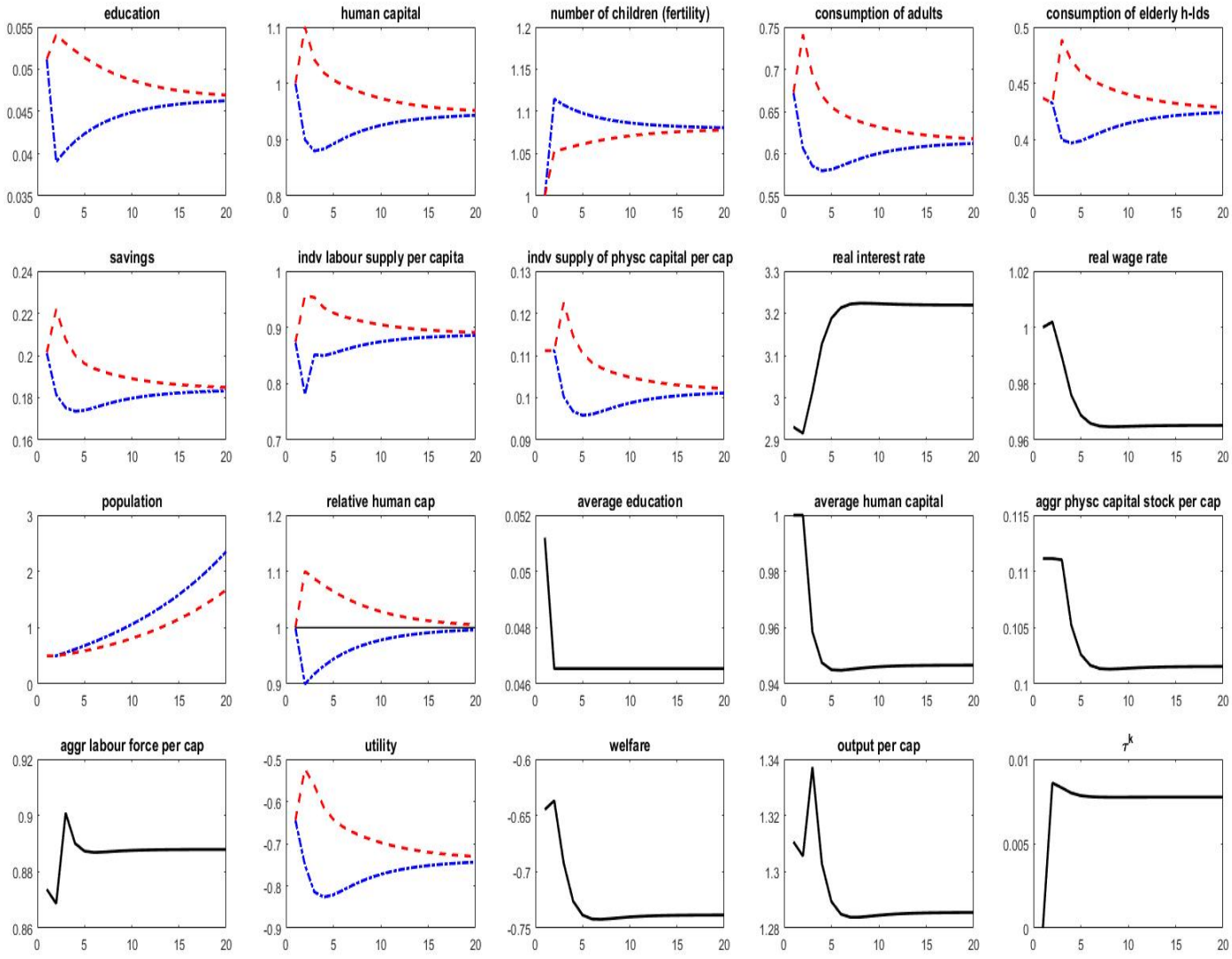
# A10 – Simulation – Deterministic environment – Heterogeneous households – $sub^{chil}$ – $\tau^c$



# A11 – Simulation – Deterministic environment – Heterogeneous households – $sub^{chil}$ – $\tau^{inc}$



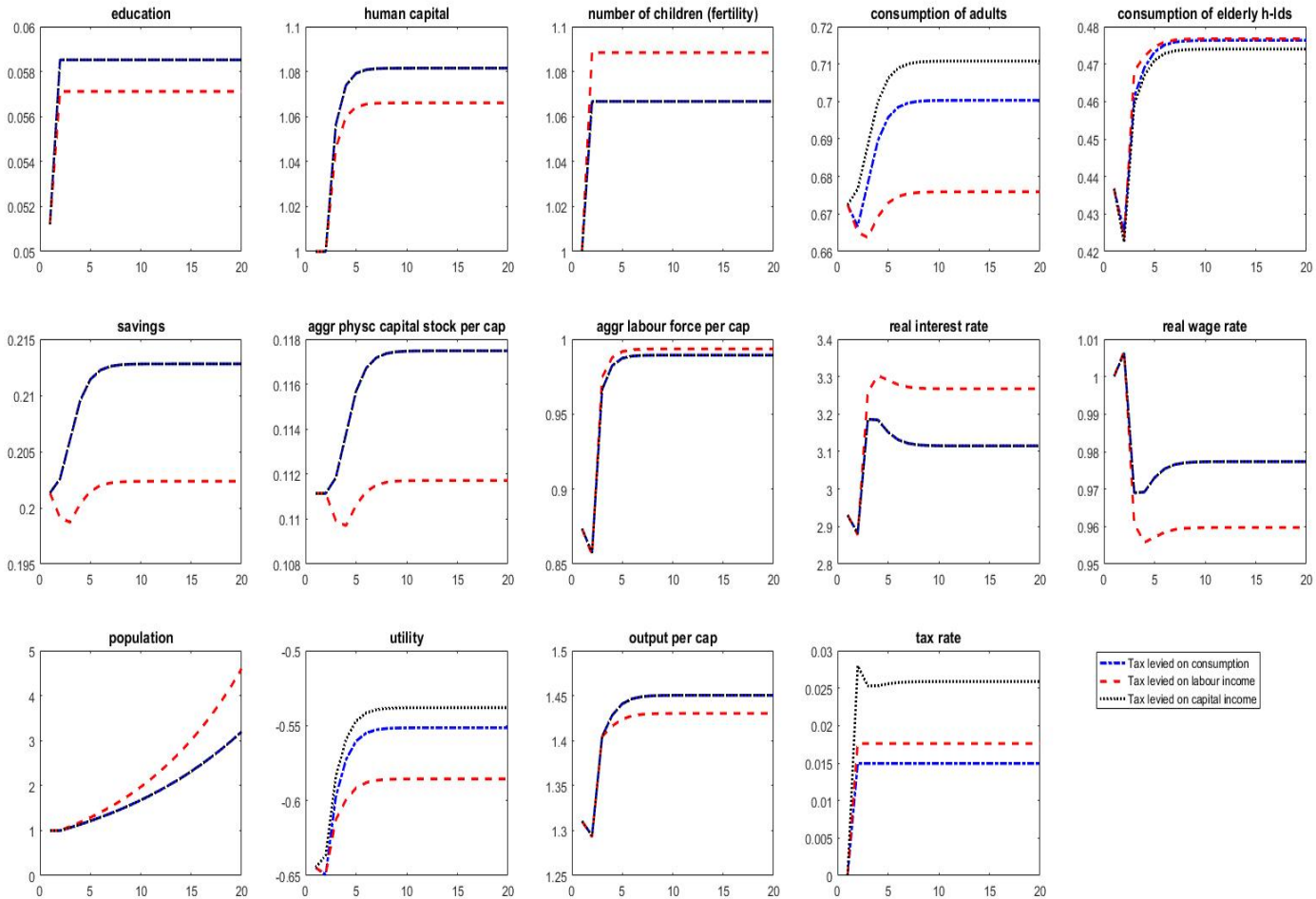
# A12 – Simulation – Deterministic environment – Heterogeneous households – $sub^{chil}$ – $\tau^k$



# A13 – Deterministic environment – Second steady-state – Deviation from original steady-state – $sub^{chil}$

| variable | $\tau^c = 0.46\%$ | $\tau^{inc} = 0.55\%$ | $\tau^k = 0.78\%$ |
|---|---|---|---|
| $e_i$ | -9.09% | -9.82% | -9.09% |
| $h_i$ | -5.33% | -5.76% | -5.33% |
| $n_i$ | +7.69% | +8.66% | +7.69% |
| $K$ | -8.62% | -10.05% | +10.66% |
| $L$ | +1.63% | +1.76% | +1.63% |
| $r$ | +9.85% | +11.49% | +9.85% |
| $w$ | -3.48% | -4.03% | -3.48% |
| $c_i$ | -9.04% | -10.05% | -8.62% |
| $d_i$ | -2.36% | -2.34% | -2.49% |
| $s_i$ | -8.62% | -10.05% | -8.62% |
| $Y$ | -1.91% | -2.34% | -1.91% |
| $u_i$ | -15.16% | -16.83% | -14.52% |

# A14 – Deterministic environment – Homogeneous households – Simultaneous subsidy for education and fertility – $1.6sub^{ed} \approx sub^{chil}$



# A15 – Calibration – Stochasic version of the analysis

| parameter | value | interpretation |
|---|---|---|
| $A$ | 2.9480 | productivity level |
| $B$ | 7.2856 | efficiency of human capital accumulation |
| $\alpha$ | $1/3$ | elasticity of physical capital |
| $\beta$ | $0.99^{120}$ | psychological discount factor |
| $\gamma$ | 0.1790 | altruism factor |
| $\delta$ | 1 | depreciation rate of physical capital stock |
| $\eta$ | 0.5 | relative significance of education for human capital |
| $\theta$ | 0.0119 | instrument parameter for non-zero human capital of children when education provision is zero |
| $\pi$ | 0.2 | relative significance of human capital of parents for human capital of children |
| $\kappa$ | 0.1 | relative significance of quality of education for human capital |
| $\rho$ | $1.02^{30} - 1$ | growth rate of human capital over generations |
| $\sigma$ | 0.2 | st. deviation of the idiosyncratic shocks to the human capital |
| $\phi$ | 0.075 | time-cost parameter to raise children |

# A16 – Discretisation of $\varepsilon$

$$E_t[h^i_{t+1}] = E_t\left[\frac{1}{(1+\rho)} B(\theta + e^i_t)^{\eta}(h^i_t)^{\pi}(\bar{h}_t)^{\kappa} \times \exp^{\varepsilon}\right]$$

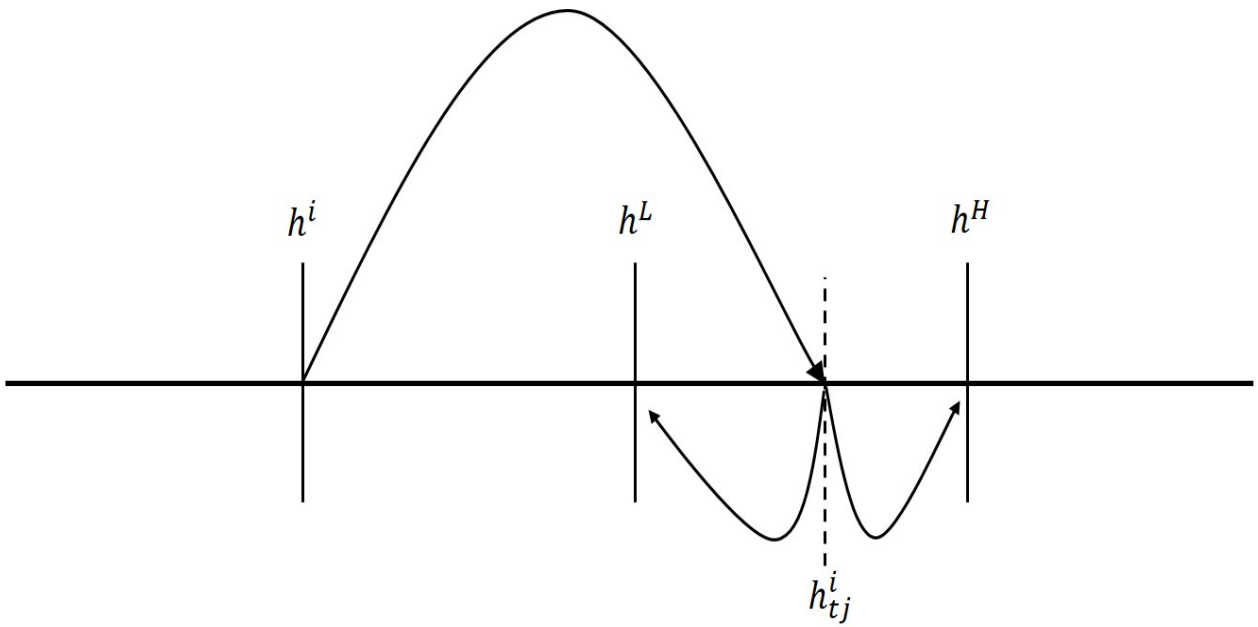
$$\ln(\varepsilon) \sim N(\mu, \sigma^2)$$

| $j$th node | $z_j$ | $j-1$ | $J-1$ | ${}_{J-1}C_{j-1}$ | $b_j$ |
|---|---|---|---|---|---|
| 1 | -2 | 0 | 4 | 1 | $\frac{1}{16}$ |
| 2 | -1 | 1 | 4 | 4 | $\frac{4}{16}$ |
| 3 | 0 | 2 | 4 | 6 | $\frac{6}{16}$ |
| 4 | 1 | 3 | 4 | 4 | $\frac{4}{16}$ |
| 5 | 2 | 4 | 4 | 1 | $\frac{1}{16}$ |

# A17 – Extrapolation and interpolation

$$p_t^L = \frac{(h^H - h_{t,j}^i)}{(h^H - h^L)} p_{t-1}^i b_j^i + p_t^L$$

$$p_t^H = \frac{(h_{t,j}^i - h^L)}{(h^H - h^L)} p_{t-1}^i b_j^i + p_t^L$$



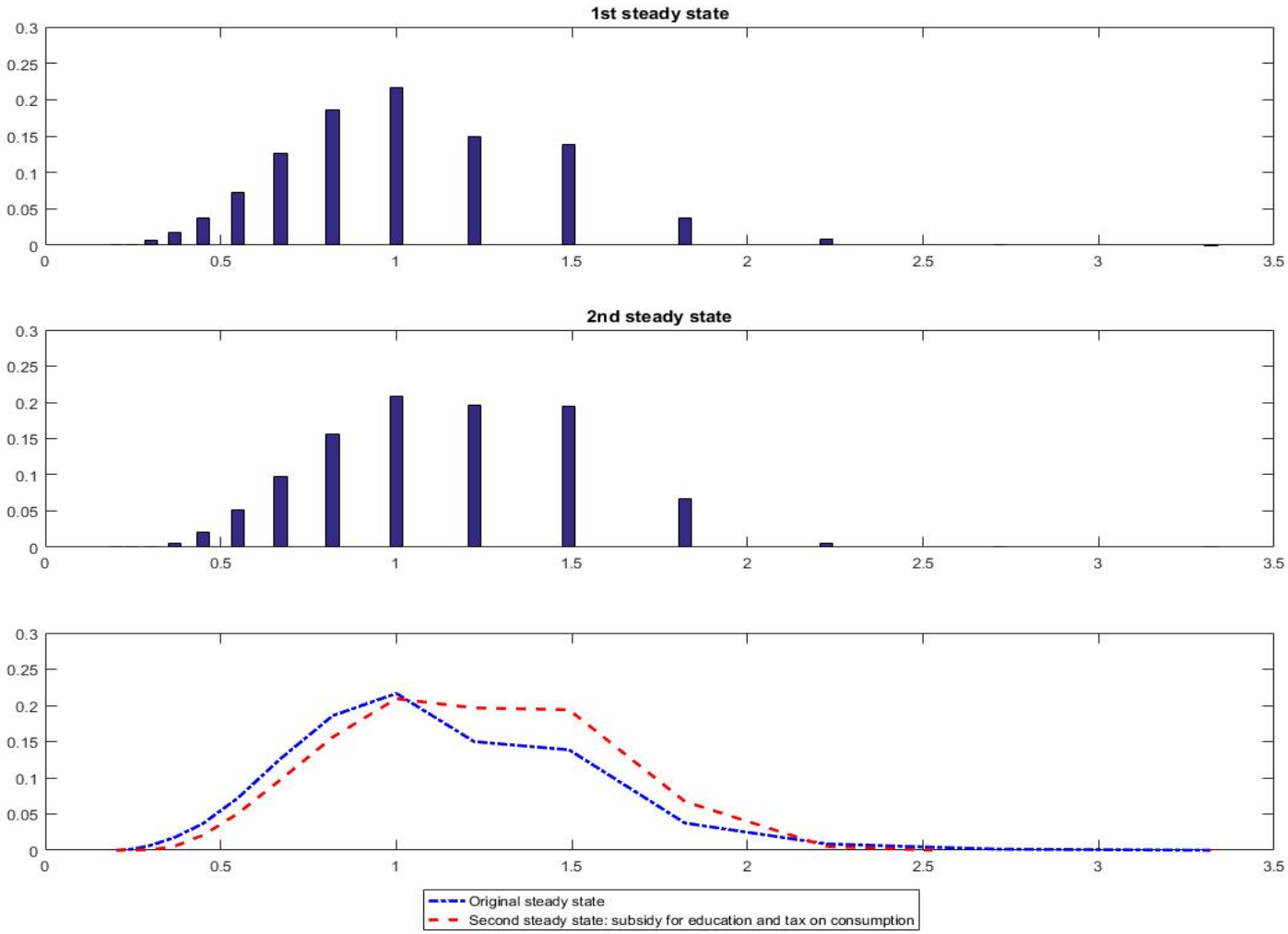
# A18 – Stochastic environment – Initial steady state

| | $Y$ | $K$ | $L$ | $r$ | $w$ | | | |
|---|---|---|---|---|---|---|---|---|
| | 1.2731 | 0.1118 | 0.8487 | 2.7955 | 1.0000 | | | |
| | $\bar{h}$ | tot.pop. | $\bar{c}$ | $\bar{s}$ | $\bar{n}$ | $\bar{e}$ | $\bar{d}$ | $\bar{u}$ |
| | 1.0000 | 1.0000 | 0.6765 | 0.2025 | 1.0000 | 0.0512 | 0.4244 | -0.7551 |
| $i$ | $h_i$ | pop.share | $c_i$ | $s_i$ | $n_i$ | $e_i$ | $d_i$ | $u_i$ |
| 1 | 0.2019 | 0.0001 | 0.1366 | 0.0409 | 1.6141 | 0 | 0.0857 | -2.9272 |
| 2 | 0.2466 | 0.0015 | 0.1668 | 0.0499 | 1.6141 | 0 | 0.1046 | -2.6315 |
| 3 | 0.3012 | 0.0068 | 0.2037 | 0.0610 | 1.6141 | 0 | 0.1278 | -2.3359 |
| 4 | 0.3679 | 0.0176 | 0.2489 | 0.0745 | 1.4192 | 0.0038 | 0.1561 | -2.0632 |
| 5 | 0.4493 | 0.0368 | 0.3039 | 0.0910 | 1.2477 | 0.0099 | 0.1907 | -1.7906 |
| 6 | 0.5488 | 0.0722 | 0.3712 | 0.1111 | 1.1353 | 0.0174 | 0.2329 | -1.5118 |
| 7 | 0.6703 | 0.1264 | 0.4534 | 0.1357 | 1.0574 | 0.0265 | 0.2844 | -1.2289 |
| 8 | 0.8187 | 0.1858 | 0.5538 | 0.1658 | 1.0011 | 0.0376 | 0.3474 | -0.9430 |
| 9 | 1.0000 | 0.2166 | 0.6764 | 0.2025 | 0.9593 | 0.0512 | 0.4243 | -0.6550 |
| 10 | 1.2214 | 0.1500 | 0.8262 | 0.2473 | 0.9276 | 0.0678 | 0.5183 | -0.3653 |
| 11 | 1.4918 | 0.1387 | 1.0091 | 0.3021 | 0.9031 | 0.0881 | 0.6330 | -0.0745 |
| 12 | 1.8221 | 0.0375 | 1.2325 | 0.3690 | 0.8841 | 0.1129 | 0.7732 | 0.2174 |
| 13 | 2.2255 | 0.0087 | 1.5054 | 0.4507 | 0.8690 | 0.1431 | 0.9444 | 0.5100 |
| 14 | 2.7182 | 0.0014 | 1.8387 | 0.5505 | 0.8571 | 0.1801 | 1.1535 | 0.8032 |
| 15 | 3.3201 | -0.0000 | 2.2458 | 0.6724 | 0.8476 | 0.2252 | 1.4088 | 1.0969 |

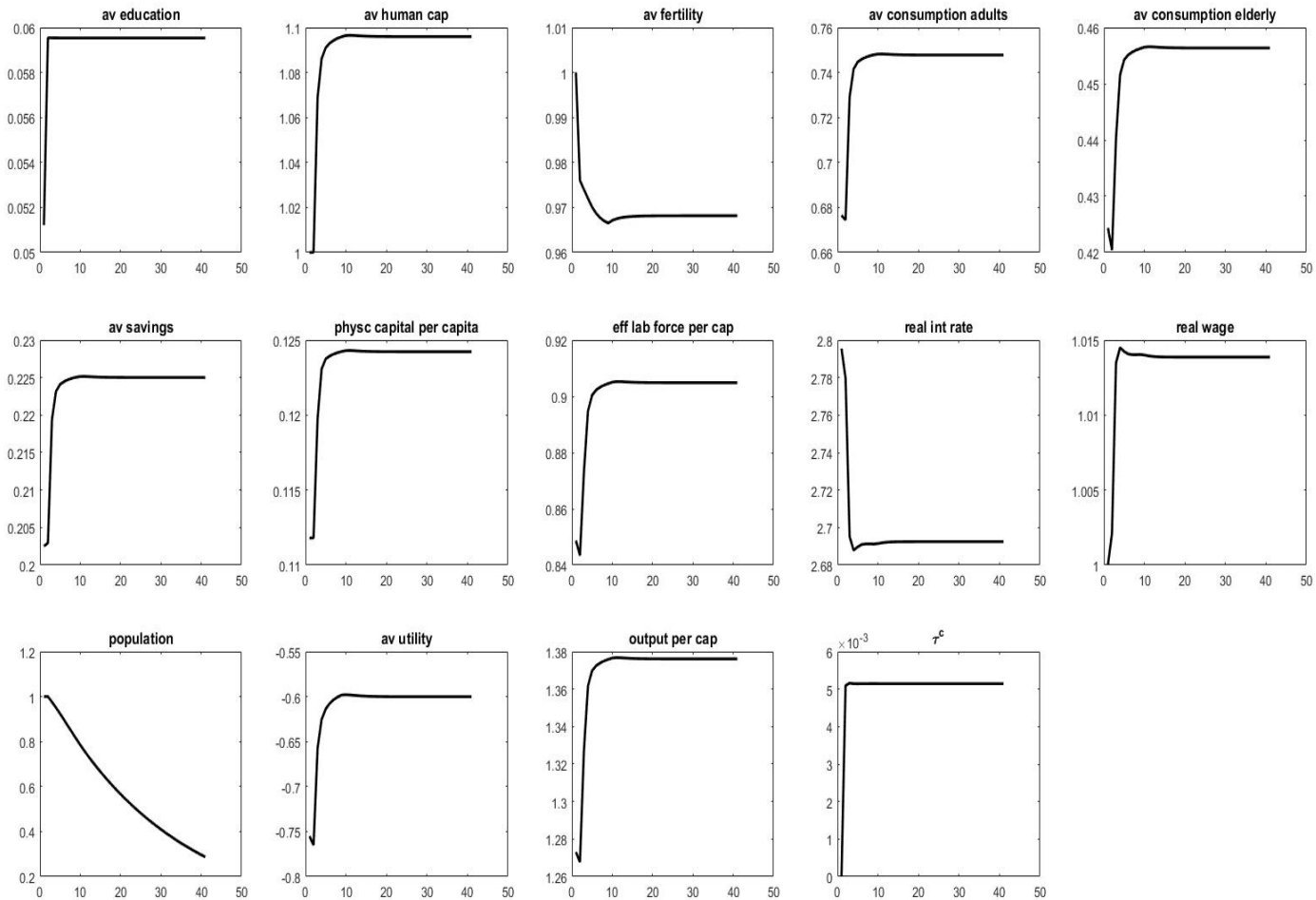
# A19 – Stochastic environment – Second steady state – Deviation from original steady-state – $sub^{ed}$ – $\tau^{c}$

| | $Y$ | $K$ | $L$ | $r$ | $w$ | $\tau^{c}$ | | |
|---|---|---|---|---|---|---|---|---|
| | +7.38% | +11.09% | +6.62% | -3.68% | +1.39% | 0.51% | | |
| | $\bar{h}$ | tot.pop. | $\bar{c}$ | $\bar{s}$ | $\bar{n}$ | $\bar{e}$ | $\bar{d}$ | $\bar{u}$ |
| | +9.59% | -71.28% | +7.13% | +11.11% | -3.18% | +16.21% | +7.54% | +20.51% |
| $i$ | $h_i$ | pop.share | $c_i$ | $s_i$ | $n_i$ | $e_i$ | $d_i$ | $u_i$ |
| 1 | 0.2019 | 0.0000 | +0.73% | +1.47% | 0% | 0 | -1.87% | +0.10% |
| 2 | 0.2466 | 0.0001 | +0.90% | +1.40% | 0% | 0 | -1.82% | +0.11% |
| 3 | 0.3012 | 0.0007 | +0.88% | +1.31% | 0% | 0 | -1.88% | +0.13% |
| 4 | 0.3679 | 0.0054 | +0.84% | +1.34% | -1.03% | +10.53% | -1.86% | +0.05% |
| 5 | 0.4493 | 0.0207 | +0.89% | +1.42% | -0.75% | +5.05% | -1.63% | +0.09% |
| 6 | 0.5488 | 0.0508 | +0.90% | +1.44% | -0.55% | +2.87% | -1.89% | +0.13% |
| 7 | 0.6703 | 0.0974 | +0.88% | +1.40% | -0.43% | +2.64% | -1.86% | +0.18% |
| 8 | 0.8187 | 0.1562 | +0.87% | +1.38% | -0.33% | +2.39% | -1.87% | +0.25% |
| 9 | 1.0000 | 0.2092 | +0.87% | +1.38% | -0.26% | +1.95% | -1.86% | +0.38% |
| 10 | 1.2214 | 0.1965 | +0.87% | +1.42% | -0.21% | +1.92% | -1.87% | +0.71% |
| 11 | 1.4918 | 0.1940 | +0.87% | +1.39% | -0.16% | +1.70% | -1.86% | +3.62% |
| 12 | 1.8221 | 0.0671 | +0.87% | +1.38% | -0.14% | +1.59% | -1.88% | +1.38% |
| 13 | 2.2255 | 0.0053 | +0.87% | +1.38% | -0.10% | +1.59% | -1.87% | +0.55% |
| 14 | 2.7182 | -0.0034 | +0.87% | +1.38% | -0.08% | +1.55% | -1.87% | +0.35% |
| 15 | 3.3201 | -0.0000 | +0.77% | +1.38% | -0.07% | +1.55% | -1.87% | +0.26% |

# A20 – Stochastic environment – Distribution of the human capital – $sub^{ed} - \tau^c$



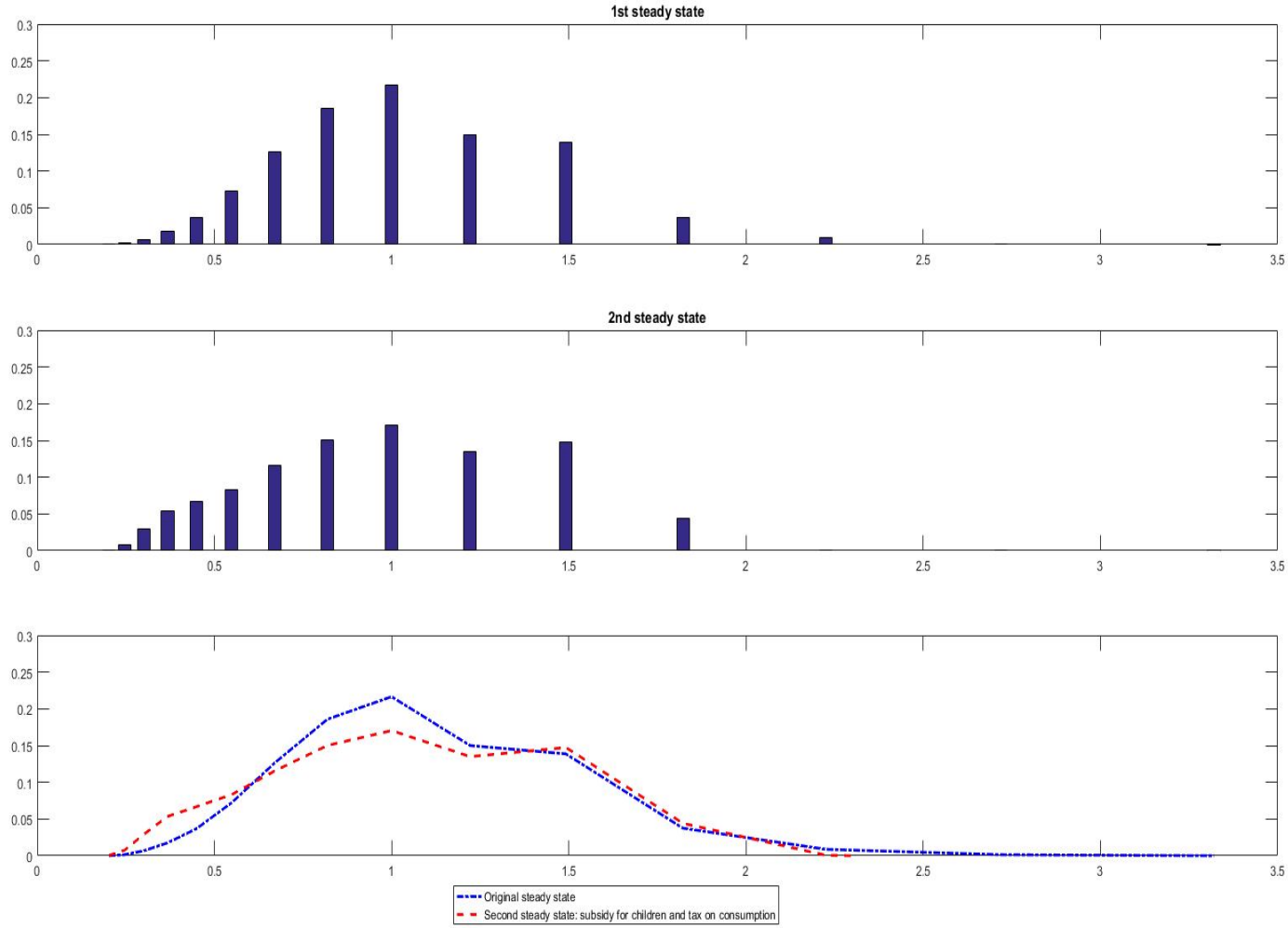
# A21 – Stochastic environment – Transition to the second steady state – Representative household – $sub^{ed}$ – $\tau^c$



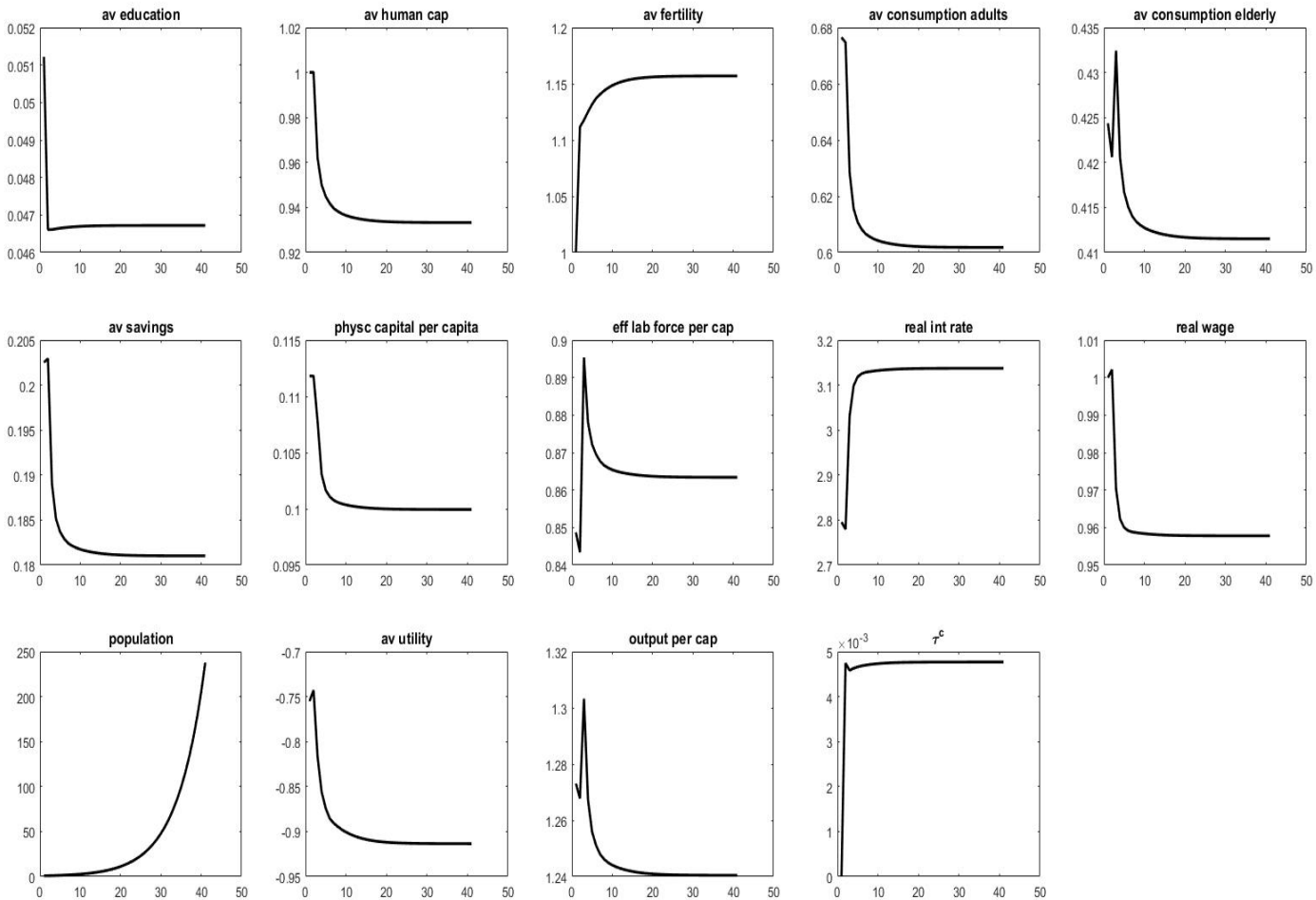
# A22 – Stochastic environment – Second steady state – Deviation from original steady-state – $sub^{chil} - \tau^c$

| | $Y$ | $K$ | $L$ | $r$ | $w$ | $\tau^c$ | | |
|---|---|---|---|---|---|---|---|---|
| | -2.56% | -10.64% | +1.73% | +12.23% | -4.22% | 0.48% | | |
| | $\bar{h}$ | tot.pop. | $\bar{c}$ | $\bar{s}$ | $\bar{n}$ | $\bar{e}$ | $\bar{d}$ | $\bar{u}$ |
| | -6.67% | +237.78% | -11.04% | -10.62% | +15.69% | -8.79% | -3.04% | -20.98% |
| $i$ | $h_i$ | pop.share | $c_i$ | $s_i$ | $n_i$ | $e_i$ | $d_i$ | $u_i$ |
| 1 | 0.2019 | 0.0001 | -4.69% | -4.16% | +40.44% | 0 | +3.85% | +0.83% |
| 2 | 0.2466 | 0.0015 | -4.68% | -4.21% | +30.85% | 0 | +3.92% | +0.44% |
| 3 | 0.3012 | 0.0068 | -4.66% | -4.26% | +23.92% | 0 | +3.91% | +0.09% |
| 4 | 0.3679 | 0.0176 | -4.70% | -4.16% | +29.41% | -71.05% | +3.91% | +0.47% |
| 5 | 0.4493 | 0.0368 | -4.67% | -4.18% | +19.56% | -23.23% | +3.89% | -0.25% |
| 6 | 0.5488 | 0.0722 | -4.66% | -4.14% | +13.88% | -10.34% | +3.91% | -0.87% |
| 7 | 0.6703 | 0.1264 | -4.68% | -4.20% | +10.24% | -4.15% | +3.94% | -1.54% |
| 8 | 0.8187 | 0.1858 | -4.68% | -4.22% | +7.76% | -0.80% | +3.91% | -2.44% |
| 9 | 1.0000 | 0.2166 | -4.67% | -4.20% | +5.98% | +1.37% | +3.91% | -3.97% |
| 10 | 1.2214 | 0.1500 | -4.67% | -4.21% | +4.68% | +2.80% | +3.92% | -7.72% |
| 11 | 1.4918 | 0.1387 | -4.68% | -4.20% | +3.70% | +3.75% | +3.89% | -40% |
| 12 | 1.8221 | 0.0375 | -4.67% | -4.23% | +2.94% | +4.52% | +3.91% | -14.35% |
| 13 | 2.2255 | 0.0087 | -4.68% | -4.22% | +2.36% | +5.10% | +3.91% | -6.31% |
| 14 | 2.7182 | 0.0014 | -4.67% | -4.23% | +1.89% | +5.50% | +3.91% | -4.12% |
| 15 | 3.3201 | -0.0000 | -4.68% | -4.22% | +1.52% | +5.82% | +3.91% | -3.07% |

# A23 – Stochastic environment – Distribution of the human capital – $sub^{chil} - \tau^c$



# A24 – Stochastic environment – Transition to the second steady state – Representative household – $sub^{chil}$ – $\tau^c$



# A25 – Stochastic environment – Inequality in distribution of the human capital

| steady state | Gini coefficient | Coefficient of variation | Mean absolute deviation | Kuznets ratios $\leqslant 40\%$ | Kuznets ratios $\geqslant 60\%$ | The range |
|---|---|---|---|---|---|---|
| original | 0.2062 | 0.3745 | 0.2905 | 0.1349 | 0.3362 | 3.1180 |
| $sub^{ed} + \tau^{c}$ | 0.1835↓ | 0.32306↓ | 0.2748↓ | 0.0777↓ | 0.4595↓ | 2.8452↓ |
| $sub^{ed} + \tau^{inc}$ | 0.1838↓ | 0.32309↓ | 0.2752↓ | 0.0780↓ | 0.4540↓ | 2.8594↓ |
| $sub^{ed} + \tau^{k}$ | 0.1835↓ | 0.32306↓ | 0.2748↓ | 0.0777↓ | 0.4595↓ | 2.8452↓ |
| $sub^{chil} + \tau^{c}$ | 0.2400↑ | 0.4184↑ | 0.3557↑ | 0.2406↑ | 0.3234↑ | 3.3410↑ |
| $sub^{chil} + \tau^{inc}$ | 0.2404↑ | 0.4189↑ | 0.3567↑ | 0.2440↑ | 0.3193↑ | 3.3577↑ |
| $sub^{chil} + \tau^{k}$ | 0.2400↑ | 0.4184↑ | 0.3557↑ | 0.2406↑ | 0.3234↑ | 3.3410↑ |

# A26 – Summary for the results

Deterministic version of the analysis

- Subsidy for education results in increase of education attainment, level of human capital, welfare of the economy and level of output per capita produced.
- It, however, decreases the optimal choice for children and population size
- Subsidy for fertility is found to produce the opposite

Stochastic version of the analysis

- At the average level, the results are consistent with outcomes from the deterministic environment
- For the individual ability groups, however, subsidy for education was not effective for ones who optimally provide their children with zero education
- Subsidy for fertility is found to improve utility of bottom four ability groups, whereas the rest of the population experiences a decline of utility

# A27 – Summary for the results (cont.)

Stochastic version of analysis (cont.)

- Level of inequality in distribution of the human capital decreases with the subsidy for education, and it increases with subsidy for fertility

Ranking of the outcomes (from best to worst) on the basis of welfare:

1. Subsidy for education financed with capital income tax
2. Subsidy for education financed with tax on consumption
3. Subsidy for education financed with labour income tax
4. Absence of government intervention
5. Subsidy for fertility financed with capital income tax
6. Subsidy for fertility financed with tax on consumption
7. Subsidy for fertility financed with labour income tax