Subsidy for education and fertility: long–run evolution of the economy with heterogeneous households and idiosyncratic shocks to the human capital

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Abstract

We extend original model of *de la* Croix and Doepke (2003) with the government sector and assumption that human capital accumulation contains uncertainty. In line with our previous results[1], we find that at the average level, subsidy for education creates positive effect for education attainment, which, however, diminishes fertility. Furthermore, it improves welfare and level of output produced per adult member of population; but, the population decreases in its size. With subsidy for fertility, at the average level we find the opposite, and overall size of population increases. Unlike in deterministic analysis, however, we find that neither of these policies lead to the outcome with full equality of population in human capital level. Instead, we find that subsidy for education reduces the level of inequality in the distribution of the human capital compared to the original steady-state, whereas subsidy for fertility creates the opposite. Furthermore, even though subsidy for education produces improvement in individual utility levels across all ability groups when implemented with tax on consumption or capital income, it is found to be ineffective to stimulate the education provision by low ability households. In turn, subsidy for fertility leads to welfare improvement of lower ability groups only, who experience largest increase in fertility and optimally decide to drastically reduce education provision. Finally, we obtain unexpected results for the high ability households when subsidy for fertility is in place: these households increase their fertility and education provision simultaneously, which resolves the parental 'quality-quantity' trade-off for them.

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[1]Kalinyak, A., Role of education, fertility and government policies in deterministic overlapping generations model. *First chapter of PhD thesis.*
The first paragraph of the Introduction contains summary for these previous results

1 Introduction

We continue the discussion for the long-run effects of the government support for education and fertility in this paper. In our previous analysis we utilised the model of overlapping generations of *de la* Croix and Doepke (2003) which we extended with the government sector. We introduced the government in form of authority that can provide subsidy for education and fertility which can be financed with the tax on consumption, labour income and capital income. According to our results, in the deterministic environment, subsidy for education and fertility both led to equality in the human capital for entire population at the second steady-state. We found, however, that resulted population from these two subsidy programs are completely different. With subsidy provided for education, we observed increase in the optimal education provision across the population, which improved the human capital levels in comparison to the original steady-state. This improvement, however, has been obtained at a cost of decrease in fertility decisions which negatively influenced the population size. With subsidy for fertility however, our results indicated the opposite: improvement in population size but deprivation of human capital, which arose from parental 'quality-quantity' trade-off for the children. In the present paper, we extend the model economy that has been utilised for our deterministic case before, and we perform our analysis for a stochastic environment.

For the current version of the analysis we place an assumption that during the human capital formation process young households receive idiosyncratic shocks. This assumption allows us to analyse the effect of peer groups, effort of the individuals, non-cognitive abilities and other unobserved variations that take place when the human capital is forming. Furthermore, it allows to analyse the way in which incentive programs for education and fertility and various means of financing them can affect the long-run economic development. In the current version of analysis, however, we focus more on a latter than on former.

Based on the previous results presented in the literature, the research focuses on analysing the education decisions in a stochastic environments caused by the risks that take place in the labour markets primarily, however, which makes it difficult to compare the present paper to existent literature. For instance, Kogan and Walker (2007) use the real option theory techniques to study the impact of public policy on the choices for education under uncertainty. According to the authors, the environment with higher risks (in the labour market, for example) creates additional incentives to stay in education longer which causes greater accumulation of human capital. Furthermore, authors conclude that in the proposed environment, the

increase in the labour income tax reduces the benefits of staying in school, however the benefit of leaving the education depreciates by larger amount which makes individuals to stay in the education longer. Slightly more closely related to the current research can be viewed the study of Akyol and Athreya (2004) where the impact of subsidy for tertiary education is studied in the environment with uncertainty for returns on the education investment. According to the results that authors obtain from the analysis of the dynamic heterogeneous-agent model that they develop, the subsidy for education is found to increase the participation in the higher education and is suggested to produce the welfare improvement. Unfortunately, based on the overview of the literature so far, the impact of the subsidy for fertility in the environment where the uncertainty exists during human capital accumulation has not been considered.

According to our results, the government program which provides the subsidy for education increases the average education attainment which increases the average human capital and helps to reduce the impacts of negative shocks during the human capital formation. Due to the parental 'quality-quantity' trade-off, however, the subsidy for education reduces individual choices of adult households for fertility which causes a reduction in the population size. With the government support for education, however, the model population enjoys higher levels of welfare and output per adult household, which results from improvement in abilities of adult households and their children. Upon visual examination of the distribution functions for the human capital combined with calculation for inequality measures, our results suggest that the subsidy for education could possibly decreases the long-run level of inequality. Reduction in the cost for education attainment through this subsidy program improves parental incentives for education provision which helps to accumulate further human capital especially to the lower ability groups[2]. This additional level of human capital that children obtain helps them to leave lower ability groups and become a part of higher ability ones when they become adults. This brings population closer together and helps to reduce inequality in distribution of the human capital. Additionally, as mentioned above, improvement of education provision helps to minimise the impact of negative shocks when human capital is forming, which assists in reduction of inequality in distribution of human capital further. Due to stochastic nature of human capital, however, the population at the second steady-state does not reach complete equality in human capital, and therefore, is not presented

[2]With exception for the households from the lowest ability groups who provided their children with zero education at the original steady-state, and continue to do so when economy reaches the second steady-state

by a single ability group with single human capital level. Furthermore, any policy instrument that creates disincentive for education provision is found to cause a higher level of inequality in distribution of human capital. This is the reason why with the labour income tax independently from subsidy of choice, and with subsidy for fertility independently from tax option, the second steady-states have higher levels of inequality in distribution of human capital. Additionally, with the subsidy for fertility, due to parental 'quality-quantity' trade-off for children, the economy reaches the second steady-state that is the opposite to one with subsidy for education.

Lastly, if we would rank the policies that we consider in this paper on the basis of improvement in level of welfare, we find that the best outcome is reached when government provides subsidy for education and finances it with the capital income tax. The worst outcome in terms of welfare is reached when subsidy for fertility financed with labour income tax, however. On the other hand, if the ranking criteria is based on ability to reduce the inequality in the distribution of the human capital, the best policy is the subsidy for education that is financed with either tax on consumption or capital income, since both of these tax options affect education, and, therefore, the human capital accumulation, in the same way. Oppositely the worst policy in term of inequality in distribution of human capital is the subsidy for fertility that is financed with labour income tax. As we have discussed above, any policy instrument which creates additional disincentives for education attainment prevents equality in human capital across members of the population.

The rest of the paper has the following structure. In the second part, we formally state the problem of each sector of the model economy for the stochastic framework and introduce the conditions that must be satisfied to reach the market equilibrium and the steady-states – first without the government presence, and then with various policy options. In the third part, we explain the way in which the random component within the human capital formation function is discretised, and then, in the fourth part of the paper we present the calibration for the model parameters. Next, we perform the policy experiments for variety of government instruments that we consider. First, we discuss the differences that appear between original steady-state and the resulted steady-states, and then we examine the transition paths of the model economy. In our discussion, we focus both on the individual decisions of the households and on the average outcomes for the whole population. Lastly, before conclusion is made, we compare the resulted levels of inequalities from the different policy options using standard inequality measures as the range, coefficient of variation, Gini coefficient and others.

2 The model

We utilize the model of overlapping generations presented by *de la* Croix and Doepke (2003) to describe the behaviour of the households and the representative producer. In the present paper, however, we assume that young households (i.e. children) receive idiosyncratic shocks while the accumulation of human capital takes place. We introduce the government sector which supports the adult households – who are heterogeneous in the human capital – by reducing the cost of education provision and fertility decisions. In the upcoming analysis, we examine the role of government support for education and fertility while uncertainty takes place in formation of the human capital. As a result of this analysis, we are able to form conclusions for the long-run effects that these policies have on the evolution of the model population and model economy overall.

2.1 Household sector

In line with *de la* Croix and Doepke (2003), at any period of time, the model population consists of three generations of households – young households (i.e. children), adult households (i.e. parents), and elderly households (i.e. grandparents); however, only adult households make economically relevant decisions. All households are heterogeneous in their endowment of human capital. Young households receive the level of education e_t^i optimally chosen by their parents[3]. In the deterministic version of the model, this optimal choice of parents for their children primary determines the human capital of the future adult households. For the stochastic version of the model, which is considered by this paper, however, the young households receive idiosyncratic shocks during their human capital accumulation. This stochastic process is introduced into the analysis to formalise the observation that human capital is influenced by both observed (e.g. level of education received) and unobserved (e.g. effort, non-cognitive abilities, effects of peer groups, etc.) characteristics of an individual.

Therefore, we modify the original human capital formation function outlined by *de la* Croix and Doepke (2003) to account for the presence of uncertainty; and the growth adjusted expected level of human capital for the

[3]Who are members of ability group i from continuum of ability groups I with the human capital h_t^i

future adult households[4] from the i^{th} ability group is defined as following:

$$E_t[h^i_{t+1}] = E_t\left[\frac{1}{(1+\rho)}B(\theta + e^i_t)^\eta (h^i_t)^\pi (\bar{h}_t)^\kappa \times \exp^\varepsilon\right] \tag{1}$$

where $\frac{1}{(1+\rho)}B(\theta + e^i_t)^\eta (h^i_t)^\pi (\bar{h}_t)^\kappa$ comes from the deterministic version of the human capital formation function and $\exp^\varepsilon$ is the component that depicts uncertainty. From the deterministic part, ρ is the long-run growth rate of the human capital, B is the efficiency of human capital accumulation, θ is the instrument parameter that ensures non-zero level of human capital for children who optimally receive zero level of education, η is the parameter that shows relative significance of education for human capital formation, π is the parameter that shows relative significance of human capital of parents for human capital of children, and κ is the parameter that shows relative significance of quality of education for human capital attainment. Therefore, according to (1), the human capital of children h^i_{t+1} is increasing with the parental choice for education e^i_t, human capital of parents h^i_t and quality of education which is approximated by the human capital of teachers that is given as the average level of human capital $\bar{h}_t$. From the stochastic component, ε is a random variable which introduces uncertainty in the human capital formation process.

Given the endowment of the human capital h^i_t, which is obtained during the youth of the households, the adult households face the utility maximisation problem. Adults maximise their expected utility function (2) subject to the resource constraints (3) and (4), and subject to the endowment of the human capital of children given by (1).

$$\max_{c^i_t, s^i_t, e^i_t, n^i_t} E_t[u^i_t] = E_t\big[\ln c^i_t + \beta \ln d^i_{t+1} + \gamma \ln n^i_t h^i_{t+1}\big] \tag{2}$$

$$(1+\tau^c_t)c^i_t + s^i_t + e^i_t n^i_t w_t \bar{h}_t = (1-\tau^l_t)w_t h^i_t (1-\phi n^i_t) + n^i_t w_t \bar{h}_t (e^i_t sub^e_t + \bar{e}_t sub^n_t) \tag{3}$$

$$E_t[(1+\tau^c_{t+1})d^i_{t+1}] = E_t\left[\frac{1}{(1+\rho)}\big(1 + r_{t+1}(1-\tau^k_{t+1})\big)s^i_t\right] \tag{4}$$

where E_t is an expectation operator, which indicates expectation of the adult households, given the information at time t; c^i_t is the growth adjusted level of consumption for adult households at time t given the endowment of human capital h^i_t; d^i_t is the growth adjusted level of consumption for elderly

[4]I.e.: present period young households who receive education today and enter the labour market in the next period of analysis

households; s_t^i is the growth adjusted level of savings that adult households make to finance their consumption at the elderly stage of existence; e_t^i is the level of education that adult households optimally choose and young households receive at time t; $\bar{e}_t$ is the average level of education in the economy at time t; n_t^i is the number of children chosen by the adult households to have; u_t^i is the level of utility for adult households from the ability type i which increases with consumption during adulthood and at the old age, and with the 'quality and quantity' of children. The coefficient β is the discount factor of adult households, γ is altruism factor of adult and elderly households toward children, ρ is a long-run growth rate of the human capital, and ϕ is a time-cost of parents to raise a single child.

According to the budget constraint (3), the total expenditure of adult households is presented by the left hand side of the expression and consists of their consumption and savings, and expenditure on the education provision which is a product of the optimal choice for education, the number of children that adult household has and provides education for, the real wage w_t (which in this case is the wage that teachers receive), and the human capital of teachers. The right hand side of (3) contains expression for the total income of the adult household, which is the income from participation in the labour market plus amount of subsidies that household receives from the government. According to the original formulation of *de la* Croix and Doepke (2003), the labour income is given as the product of the real wage, human capital, and time that adult household has away from taking care for the children. Given the government policy instruments that appear in (3), sub_t^e and sub_t^n indicate the subsidy rates for education and fertility; and τ_t^c and τ_t^l are the tax rates on consumption and labour income, respectively.

Based on the budget constraint for the elderly households (4), the expected growth adjusted consumption at the old age depends on the expected real return on financial investment that households make during adulthood, which depends of the real interest rate r, savings made during adulthood, tax rate on consumption and tax rate on the capital income τ^k.

Given this utility maximisation problem, the adult households choose their level of consumption and savings, together with number of children to have and level of education to provide for their children. These choices, however, are now made in the stochastic environment, where children are subjected to the idiosyncratic shocks ε while human capital is accumulated.

2.2 Production sector

Following *de la* Croix and Doepke (2003), the production sector is presented by a single firm which produces consumable goods by employing effective labour force and the physical capital stock. This firm maximises its own profit (5) subject to the technological constraint (6), which forms the demand for both of the factor inputs.

$$\max_{K_t, L_t} \Pi_t = Y_t - w_t L_t - (r_t + \delta) K_t \tag{5}$$

$$Y_t = A K_t^{\alpha} L_t^{1-\alpha} \tag{6}$$

In this optimisation problem Π_t is the real profit of the firm, Y_t is the real output of the firm per adult household, w_t is the real wage paid to the unit of the effective labour force, L_t is the effective labour force per adult household, r_t is the real interest rate (real rental rate of physical capital stock) paid for the unit of the physical capital stock, K_t is the physical capital stock per adult household, δ is the depreciation rate of the physical capital stock, A is a productivity level, and α is the share of the capital income.

Next, the supply of effective labour force per adult household is depicted by (7).

$$L_t = \frac{\sum_{i=1}^{I} p_t^i \left[h_t^i (1 - \phi n_t^i) - n_t^i e_t^i \bar{h}_t \right]}{\sum_{i=1}^{I} p_t^i} \tag{7}$$

Effective labour force per adult household is the aggregate number of adult households in the model population who are not involved in teaching, with time $(1 - \phi n_t^i)$ than can be contributed to the labour market, divided by the total population size of adult households, where p_t^i is the size of population of adult households with h_t^i level of human capital.

Lastly, the evolution of the physical capital stock per adult household is given by (8).

$$K_t = \frac{1}{(1+\rho)} \left(\frac{\sum_{i=1}^{I} p_{t-1}^i s_{t-1}^i}{\sum_{i=1}^{I} p_t^i} + (1 - \delta) K_{t-1} \right) \tag{8}$$

According to (8), the present level physical capital stock per adult household adjusted to the long-run growth rate of the human capital is determined by the past level of physical capital stock per adult household, depreciation rate of the physical capital stock δ, present population size of adult households, and past period aggregate level of investment of adult households from different ability group. Since we assume that the financial market does not have any rigidities or uncertainties, the savings of adult households are equal to the level of investment in this model economy.

2.3 Government

We introduce the government in form of policy maker who provides subsidies for education and fertility to the household sector. To provide these means of support, we consider that the government can finance its budget with taxes on consumption, labour income and capital income that government collects from the households. These tax rates are optimally chosen by the government in order to satisfy the budget constraint (9) given the optimal choices of the households and producer with government policy instruments in place. Therefore, for our analysis we assume that budget constraint of the government is always balanced. The left hand side of (9) depicts the total income of the government given the tax option that government implements, and the right hand side of (9) presents the total expenditure of the government given the subsidy program in place.

$$\tau_t^c c_t + \tau_t^c d_t + r_t \tau_t^k s_t + \tau_t^l w_t h_t (1 - \phi n_t) = n_t w_t \bar{h}_t (e_t sub_t^e + \bar{e}_t sub_t^n) \quad (9)$$

In the budget constraint of the government, c_t depicts the aggregate consumption of adult share of the household at time t (i.e. $c_t = \sum_{i=1}^{I} p_t^i c_t^i$), d_t is the aggregate consumption of elderly share of the household at time t (i.e. $d_t = \sum_{i=1}^{I} p_{t-1}^i d_t^i$), s_t is the aggregate savings at time t (i.e. $s_t = \sum_{i=1}^{I} p_t^i s_t^i$), h_t is the aggregate level of human capital in the economy at time t (i.e. $h_t = \sum_{i=1}^{I} p_t^i h_t^i$), n_t is the total number of children in the economy at time t (i.e. $n_t = \sum_{i=1}^{I} p_t^i n_t^i$), and e_t is the total education provision in the economy at time t (i.e $e_t = \sum_{i=1}^{I} p_t^i e_t^i$).

Due to the nature of the model, however, where one period of analysis correspond to one generation or 30 year, it is not feasible to formally construct the problem of the government who is searching for the best social outcome by minimising its loss function. This approach is not feasible because the composition and characteristics of the households are always changing in duration of analysis. Therefore, instead, the choices for the subsidies are formed exogenously, and then, given the behaviour of the households and producer, the tax rate(s) which balances the budget is(are) chosen endogenously.

2.4 Market equilibrium

Since in the present version of analysis, the stochastic process is introduced into the human capital formation function, it is not feasible to obtain a closed form analytical solution for the model economy. As a result, we rely on numerical methods to analyse the effects of government policies. Before moving forward, however, we follow Heer and Maussner (2009) and assume that stationary equilibrium for this model economy takes place when conditions below are satisfied.

1. Individual and aggregate behaviour are consistent. The aggregate variables are equal to the sum of individual variables.

2. The real wage rate and real interest rate solve the firm's optimisation problem by satisfying two conditions below, where real interest rate net of depreciation is equal to the marginal product of the physical capital stock per adult household and real wage rate is equal to the marginal product of effective labour force per adult household.

$$r_t = \alpha A {K_t}^{\alpha-1} L_t^{1-\alpha} - \delta \tag{10}$$

$$w_t = (1-\alpha) A K_t^{\alpha} L_t^{-\alpha} \tag{11}$$

3. Given the factor prices for physical capital stock and the effective labour force and the government policy in form of (exogenously chosen) subsidy rate(s) for education and/or fertility, the individual choices of adult households for consumption, savings, education and number of children given their endowment of human capital solve the optimisation problem of the households for each corresponding ability group.

4. Given the factor prices, subsidy rates and individual choices of households, the optimal tax rates for consumption, labour income and/or capital income are chosen to keep the government budget balanced.

5. Present population of adult households for each ability group depends on the past period fertility decisions and evolves according to process below.

$$p_t^i = p_{t-1}^i n_{t-1}^i \tag{12}$$

3 Discretisation of ε

To perform the numerical exercise for the proposed model economy, we have to discretise the random component $\exp^{\varepsilon}$ within the human capital formation function (1).

Let us assume that z is a standard normal random variable which has five equally distant nodes (i.e. $J = 5$) with the maximum value for z (i.e. z_j for $j = 5$) being equal to two (i.e. $z_5 = \sqrt{(J-1)}$). Given that the variance of z is equal to one (i.e. $var(z) = (z_5)^2/(J-1)$), the five nodes of z are -2, -1, 0, 1 and 2. In line with Devroye (1986), to approximate the normal distribution for z and to calculate the probability of each node occurring, we use the Binomial distribution and calculate Binomial probability according to (13) below.

$$b_j =_{J-1} C_{j-1} P^{j-1} P^{J-j} \tag{13}$$

where b_j is the Binomial probability of j's node; j is the index for a particular z's node where $j \in [1, 5]$; J is the total number of nodes for standard normal random variable z; ${}_{J-1}C_{j-1}$ is the number of combinations of $J-1$ nodes, taken $j-1$ at a time which is calculated using (14); where P is the probability of success on an individual trial which we assume is equal to 0.5.

$${}_{J-1}C_{j-1} = \frac{(J-1)!}{(j-1)!\big((j-1)(J-1)\big)!} \tag{14}$$

Therefore, the number of combinations and Binomial probability for each of the nodes of standard normal random variable z is given by the table 1 below.

jth node	z_j	$j-1$	$J-1$	${}_{J-1}C_{j-1}$	b_j
1	-2	0	4	1	$\frac{1}{16}$
2	-1	1	4	4	$\frac{4}{16}$
3	0	2	4	6	$\frac{6}{16}$
4	1	3	4	4	$\frac{4}{16}$
5	2	4	4	1	$\frac{1}{16}$

Table 1: The number of combinations and Binomial probability for five z nodes

Given the probability of occurrence for the each of these nodes, the mean of z is equal to zero, which indicates that with calculated probabilities for each node, z approximates a standard normal random variable.

Let us consider a new random variable ε which is given as $\varepsilon = \mu + \sigma z$. Since we concluded that z given the probability of each node approximates the standard normal random variable, then ε approximates the standard normal random variable as well with mean of μ and variance of σ^2 (i.e. $\varepsilon \sim N(\mu, \sigma^2)$). Now, let us suppose that x is another random variable which is defined as $x = \exp^{\varepsilon}$. As a result x approximates a log-normal random variable with parameters μ and σ. By normalizing the expected value of x to one (i.e. $E(x) = E(\exp^{\mu + \frac{\sigma^2}{2}}) = 1$), the expression is simplified to $\mu = -\frac{\sigma^2}{2}$.

Finally, we approximate the standard deviation of the idiosyncratic shock to the human capital with the standard deviation in lifetime labour income, which according to Bosworth et al (2000) and Millimet et al (2003) is equal to 20%. Given the value of $\sigma = 0.2$, the mean of ε is equal to -0.02 and five nodes for ε are -0.42, -0.22, -0.02, 0.18 and 0.38. Given these five nodes and given the Binomial probability distribution for each node indicated in table 1, we discretise the random process within the human capital formation function.

4 Calibration

For the purpose of our policy exercises, we follow original parametrisation of *de la* Croix and Doepke (2003) closely. We however, adjust the values for the parameters γ – the altruism factor of adult and elderly households toward children, A – productivity level and B – efficiency of human capital accumulation[5], in order to normalise the average decision adult households for children, the real wage rate and the average level of human capital to be equal to one at the initial steady-state without government presence. Furthermore, as we have mentioned in the previous section, we calibrate the standard deviation of the shocks in the human capital to approximate the standard deviation in the lifetime labour income, which according to Bosworth et al (2000) Millimet et al (2003) is equal to 20%. Overall full list of the structural parameters utilised within the model, and their corresponding values and the interpretation is given by table 2 below.

[5]In the original calibration of *de la* Croix and Doepke (2003), A and B parameters both set to one. For γ, the value of the paramiter has been set to 0.271, which results in zero growth of population in the balanced growth path.

parameter	value	interpretation
A	2.9504	productivity level
B	7.3478	efficiency of human capital accumulation
α	1/3	share of the capital income
β	0.99^{120}	discount factor
γ	0.17957	altruist factor
δ	1	depreciation rate of physical capital stock
η	0.5	relative significance of education for human capital
θ	0.0119	instrument parameter for non-zero human capital of children when education provision is zero
π	0.2	relative significance of human capital of parents for human capital of children
κ	0.1	relative significance of quality of education for human capital
ρ	$1.02^{30} - 1$	long-run growth rate of the human capital
σ	0.2	st. deviation of the idiosyncratic shocks to the human capital
ϕ	0.075	time-cost parameter to raise children

Table 2: Calibration for the structural parameters of the model economy

According to *de la* Croix and Doepke (2003), these structural parameters have been calibrated in order for the model economy in the balance growth path to follow the properties of the U.S. economy and population under the assumption that one period of analysis is equal to thirty years. To match the empirical evidence, *de la* Croix and Doepke (2003) set the capital income share parameter α to be equal to 1/3. For the discount factor β, the authors indicate that this parameter influences the ratio between human capital and physical capital stock in the balanced growth path, but since this ratio depends on the choice of units, the calibration of β has been found to be inconvenient. Therefore, *de la* Croix and Doepke (2003) have followed the real-business-cycle literature where discount factor for one quarter of analysis is often set to be equal to 0.99, and given that there are 120 quarters in 30 years (i.e. the time frame between one point of the observation and the next), the discount factor β is set to 0.99^{120}. Next, given that it takes thirty years between each period of analysis, *de la* Croix and Doepke (2003) considered the full depreciation of the physical capital stock, and therefore, the δ coefficient is equal to one. For parameter η which identifies the relative significance of education for development of human capital, the authors calibrate this parameter using the maximum fertility differential that they observe in the data set they implement. According to *de la* Croix

and Doepke (2003), $\frac{1}{(1-\eta)}$ defines the maximum fertility differential for the model economy, which for the case of Brazil has been reported to be equal to 2.74, which sets η to be equal to 0.635. For the purpose of our computational exercise, however, we normalise the maximum fertility differential to 2, and therefore, the η coefficient in our analysis is equal to 0.5. For θ we rely on the original parametrisation performed by *de la* Croix and Doepke (2003), however. According to the authors, in the ballanced growth path for the U.S economy, 7.3 percent of the GDP is dedicated to the total education expenditure, which corresponds to the value of θ of 0.0119. For π which measures relative significance of the human capital of parents for human capital of children, *de la* Croix and Doepke (2003) calculate the upper limit for the parameter in order to have individual stable dynamics, which according to the authors is equal to 0.246. With review of empirical evidence for the influence of parental education on child school performance, and taking into consideration the long-run nature of the analysis, *de la* Croix and Doepke (2003) set π to be equal to 0.2. In the case for the variable which depicts the relative significance of quality of education for the human capital formation κ, the authors followed the findings of Card and Kruger (1996) and Kruger and Lindahl (2001) and set this parameter to be equal to 0.1 (Croix and Doepke, 2003, p. 1099). Finally, *de la* Croix and Doepke (2003) set the value for the long-run growth rate of human capital ρ in line with the growth rate of the output in the balanced growth path, which for the case of United States averages at two pecent per year; and the time-cost parameter to raise children ϕ is chosen by the authors on the basis of the evidence presented by Haveman and Wolfe (1995) and Knowles (1999) who find that opportunity cost of a child is equal to 15 percent of parental time endowment, and with assumption placed by *de la* Croix and Doepke (2003) that children would spend 15 year out of 30 living with parents with "overall 50 percent of the time cost per year", the value for ϕ is set to 0.075 (Croix and Doepke, 2003, p. 1099).

5 Policy experiments

In order to perform the policy experiments and analyse the influence of government support for education and fertility, we adapt the computational algorithm presented by Heer and Maussner (2009). First, we have to establish the stable distribution of the adult households given the presence of uncertainty in the human capital accumulation process of children and without any government intervention. For a duration of the following discussion, we refer to this point as 'initial or original steady-state'. After a discussion

for the initial steady-state is completed, we introduce the government policy in our stochastic environment. As a result, the model economy reaches the second steady-state which we analyse alongside with the path that model economy takes to reach this second steady-state[6]. In the upcoming discussion, we look at the case of subsidy for education and fertility separately. For each version of analysis, however, the government finances its budget with three tax options – tax on consumption, tax on labour income and tax on capital income. Therefore, we focus on potential differences that result in the model economy from implementation of different tax options.

5.1 Initial steady-state

As a beginning point for our analysis, we discretise the model population of adult households into fifteen ability groups (i.e. $i \in [1, 15]$ where $i^{max} = 15$). We normalise the aggregate level of population to one, and, therefore, the share of adult households from each ability group is equal to $\frac{1}{15}$ at this initial stage. Given this uniform distribution of adult households, we introduce the process of human capital accumulation with idiosyncratic shocks in it. With the exogenously chosen (marginal) level of human capital for each ability group, and the process of extrapolation and interpolation which is shown below, the stable distribution of adult households forms the original steady-state without government presence. This original steady-state is depicted by table 3 below with the distribution for population presented by figure 1.

[6]We define the second steady-state as a case for the model economy which experiences the uncertainty in the human capital formation process with government policy present and which has the stable distribution of adult households

	Y	K	L	r	w			
	1.2757	0.1118	0.8504	2.8049	1.0000			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	1.0000	1.0000	0.0512	1.0000	0.6762	0.2024	0.4252	-0.7502
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0002	0.0000	1.6189	0.1365	0.0409	0.0858	-2.9271
2	0.2466	0.0015	0.0000	1.6189	0.1667	0.0499	0.1049	-2.6314
3	0.3012	0.0054	0.0000	1.6189	0.2037	0.0610	0.1281	-2.3356
4	0.3679	0.0141	0.0038	1.4234	0.2487	0.0745	0.1564	-2.0629
5	0.4493	0.0326	0.0099	1.2513	0.3038	0.0910	0.1911	-1.7902
6	0.5488	0.0692	0.0174	1.1386	0.3711	0.1111	0.2334	-1.5114
7	0.6703	0.1289	0.0265	1.0605	0.4532	0.1357	0.2850	-1.2284
8	0.8187	0.1948	0.0376	1.0040	0.5536	0.1657	0.3481	-0.9424
9	1.0000	0.2200	0.0512	0.9621	0.6762	0.2024	0.4252	-0.6543
10	1.2214	0.1772	0.0678	0.9303	0.8259	0.2472	0.5194	-0.3645
11	1.4918	0.1010	0.0881	0.9058	1.0087	0.3020	0.6343	-0.0735
12	1.8221	0.0412	0.1129	0.8867	1.2320	0.3688	0.7748	0.2184
13	2.2255	0.0117	0.1431	0.8716	1.5048	0.4505	0.9463	0.5111
14	2.7182	0.0020	0.1801	0.8596	1.8380	0.5502	1.1558	0.8044
15	3.3201	0.0001	0.2252	0.8501	2.2449	0.6721	1.4117	1.0982

Table 3: Original steady-state with presence of idiosyncratic shocks in the human capital and without presence of the government policies

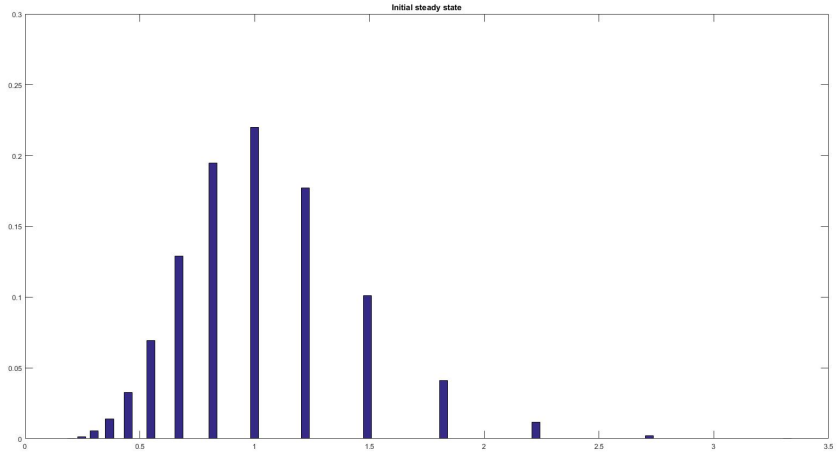

Figure 1: Distribution for the population of adult households at the initial steady-state.
Note: the (marginal) levels of the human capital are depicted on the horizontal axis; population share for each ability group is depicted on the vertical axis

According to our results, the first three ability groups optimally decide to have maximum number of children and provide their children with zero level of education. This optimal choice is formed due to low population weights and low levels of human capital for these three groups relative to the rest of population, and, therefore, to average level of human capital at this original steady-state. On the other side, top ability groups decide on having less children but provide their children with higher level of education. This decision is caused by large relative human capital, which, according to the analytical conclusions for the deterministic version of analysis[7] increases choices of adults for education provision for their children, which, in turn, causes a lower choice for fertility. Since the level of utility for adult households is increasing with their present and future consumption, as well as with the number and human capital of children, adult households with larger (marginal) level of human capital enjoy higher levels of utility at this original steady-state.

Extrapolation and Interpolation We adopt the process of (linear) extrapolation and interpolation presented by Judd (1998a and 1998b) to form

[7]Please refer to the A1 appendix section

<table>
<tr><td colspan="2">if $0 \leq h^i_{t,j} < h^1$</td></tr>
<tr><td>if $(h^2 - h^1) > (h^1 - h^i_{t,j})$
$p^1_t = \frac{(h^2 - h^1)}{(h^2 - h^i_{t,j})} p^i_{t-1} b^i_j + p^1_t$
$p^2_t = \frac{(h^1 - h^i_{t,j})}{(h^2 - h^i_{t,j})} p^i_{t-1} b^i_j + p^2_t$</td><td>if $(h^1 - h^i_{t,j}) > (h^2 - h^1)$
$p^1_t = \frac{(h^1 - h^i_{t,j}))}{(h^2 - h^i_{t,j})} p^i_{t-1} b^i_j + p^1_t$
$p^2_t = \frac{(h^2 - h^1}{(h^2 - h^i_{t,j})} p^i_{t-1} b^i_j + p^2_t$</td></tr>
<tr><td colspan="2">if $h^L \leq h^i_{t,j} \leq h^H$</td></tr>
<tr><td colspan="2">$p^L_t = \frac{(h^H - h^i_{t,j})}{(h^H - h^L)} p^i_{t-1} b^i_j + p^L_t$
$p^H_t = \frac{(h^i_{t,j} - h^L)}{(h^H - h^L)} p^i_{t-1} b^i_j + p^H_t$</td></tr>
<tr><td colspan="2">if $h^i_{t,j} > h^{15}$</td></tr>
<tr><td>if $(h^{15} - h^{14}) > (h^i_{t,j} - h^{15})$
$p^{14}_t = \frac{(h^i_{t,j} - h^{15})}{(h^i_{t,j} - h^{14})} p^i_{t-1} b^i_j + p^{14}_t$
$p^{15}_t = \frac{(h^{15} - h^{14})}{(h^i_{t,j} - h^{14})} p^i_{t-1} b^i_j + p^{15}_t$</td><td>if $(h^i_{t,j} - h^{15}) > (h^{15} - h^{14})$
$p^{14}_t = \frac{(h^{15} - h^{14})}{(h^i_{t,j} - h^{14})} p^i_{t-1} b^i_j + p^{14}_t$
$p^{15}_t = \frac{(h^i_{t,j} - h^{15})}{(h^i_{t,j} - h^{14})} p^i_{t-1} b^i_j + p^{15}_t$</td></tr>
</table>

Table 4: Extrapolation and interpolation procedure

the population of adult households after their realisation for the level of human capital as it is summarised by table 4. where $h^i_{t,j}$ is the realised level of human capital for adult from ability group i at a time t who received the shock j during human capital accumulation process at the childhood; p^i_{t-1} is the past period population size of adult households from ability group i; b^i_j is the (binomial) probability of a shock from a node j to influence the human capital for adult household from ability group i. We consider that the realised level of human capital for adult household who receives shock from a node j nests between the two predetermined (marginal) levels of the human capital – h^L and h^H. Therefore, the closer the realised value of human capital to a particular node, the larger population size that population group would have; where p^L_t is the resulted population size for the group with the (marginal) level of human capital corresponding to L (i.e. $i = L$), and p^H_t is the resulted population size for the group with the (marginal) level of human capital corresponding to H (i.e. $i = H$).

5.2 Subsidy for education

We discuss the results for the case of government support for education in this section. We consider that the government provides the subsidy rate of ten percent to every adult household in the model population established in the

previous section. We present the results for the second steady-state that takes place in environment with idiosyncratic shocks to the human capital and with the government support for education, and we compare it to the initial steady-state. Then, we discuss the transition path that our model economy takes from the original steady-state to the steady-state with government presence from the perspective of the mean (average) household. We also include the discussion for the changes that we observe at the individual level.

5.2.1 Second (final) steady-state

As we have indicated previously, the government can finance its budget with tax on consumption, labour income and capital income. Therefore, we consider three potential steady-states that can exist in the proposed environment with subsidy for education in it. The summary for these steady-states is given by the tables 5, 6 and 7 below, whereas the distribution of the adult households for each if three cases is depicted by figures 2, 3 and 4.

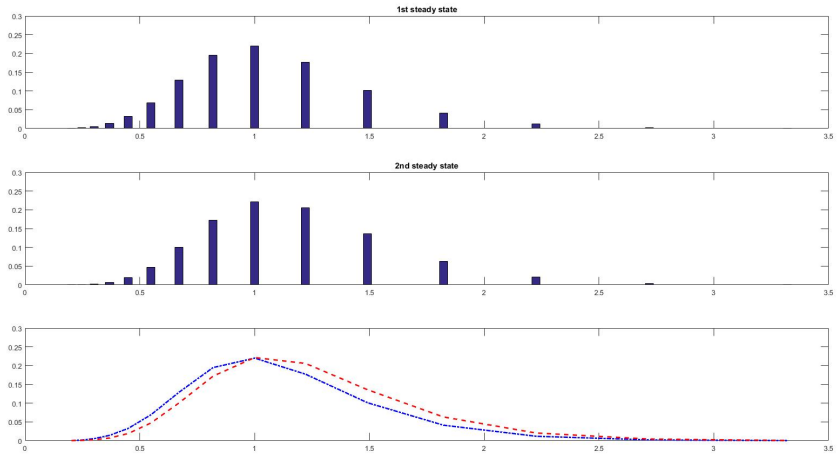

Figure 2: Comparison between distribution for the population of adult households at the initial steady-state with the distribution at the second (final) steady-state where government supports education and taxes consumption. Note: the (marginal) levels of the human capital are depicted on the horizontal axis; population share for each ability group is depicted on the vertical axis; blue dash-dotted line shows the distribution of adult households at the original steady-state; red dashed line illustrates the distribution of adult households at the second (final) steady-state

	Y	K	L	r	w	τ^c		
	1.3783	0.1240	0.9067	2.7048	1.0134	0.0052		
	(8.04%)	(10.96%)	(6.61%)	(-3.57%)	(1.34%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	1.0950	0.3603	0.0595	0.9739	0.7465	0.2246	0.4571	-0.6122
	(9.50%)	(-63.97%)	(16.25%)	(-2.61%)	(10.39%)	(10.96%)	(7.49%)	(18.39%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0001	0.0000	1.6189	0.1376	0.0414	0.0843	-2.9245
		(-62.81%)	—	(0%)	(0.82%)	(1.34%)	(-1.83%)	(0.09%)
2	0.2466	0.0006	0.0000	1.6189	0.1681	0.0506	0.1029	-2.6287
		(-60.97%)	—	(0%)	(0.82%)	(1.34%)	(-1.83%)	(0.10%)
3	0.3012	0.0023	0.0000	1.6189	0.2053	0.0618	0.1257	-2.3329
		(-57.23%)	—	(0%)	(0.82%)	(1.34%)	(-1.83%)	(0.11%)
4	0.3679	0.0070	0.0042	1.4078	0.2508	0.0755	0.1536	-2.0622
		(-50.50%)	(10.76%)	(-1.09%)	(0.82%)	(1.34%)	(-1.83%)	(0.03%)
5	0.4493	0.0191	0.0104	1.2415	0.3063	0.0922	0.1876	-1.7890
		(-41.35%)	(5.03%)	(-0.79%)	(0.82%)	(1.34%)	(-1.83%)	(0.07%)
6	0.5488	0.0473	0.0180	1.1319	0.3741	0.1126	0.2291	-1.5098
		(-31.68%)	(3.50%)	(-0.59%)	(0.82%)	(1.34%)	(-1.83%)	(0.11%)
7	0.6703	0.1004	0.0272	1.0557	0.4570	0.1375	0.2798	-1.2265
		(-22.09%)	(2.81%)	(-0.45%)	(0.82%)	(1.34%)	(-1.83%)	(0.15%)
8	0.8187	0.1719	0.0385	1.0005	0.5581	0.1680	0.3418	-0.9404
		(-11.76%)	(2.41%)	(-0.35%)	(0.82%)	(1.34%)	(-1.83%)	(0.22%)
9	1.0000	0.2215	0.0523	0.9595	0.6817	0.2051	0.4174	-0.6521
		(0.68%)	(2.17%)	(-0.27%)	(0.82%)	(1.34%)	(-1.83%)	(0.33%)
10	1.2214	0.2059	0.0692	0.9283	0.8326	0.2506	0.5098	-0.3623
		(16.18%)	(2.00%)	(-0.22%)	(0.82%)	(1.34%)	(-1.83%)	(0.62%)
11	1.4918	0.1357	0.0897	0.9042	1.0170	0.3060	0.6227	-0.0712
		(34.33%)	(1.88%)	(-0.17%)	(0.82%)	(1.34%)	(-1.83%)	(3.19%)
12	1.8221	0.0632	0.1149	0.8854	1.2422	0.3738	0.7606	0.2208
		(53.15%)	(1.79%)	(-0.14%)	(0.82%)	(1.34%)	(-1.83%)	(1.10%)
13	2.2255	0.0204	0.1456	0.8706	1.5172	0.4566	0.9290	0.5136
		(74.10%)	(1.72%)	(-0.11%)	(0.82%)	(1.34%)	(-1.83%)	(0.48%)
14	2.7182	0.0042	0.1831	0.8589	1.8531	0.5576	1.1347	0.8069
		(113.29%)	(1.67%)	(-0.09%)	(0.82%)	(1.34%)	(-1.83%)	(0.31%)
15	3.3201	0.0005	0.2289	0.8495	2.2633	0.6811	1.3859	1.1007
		(212.99%)	(1.63%)	(-0.07%)	(0.82%)	(1.34%)	(-1.83%)	(0.23%)

Table 5: Second (final) steady-state with presence of idiosyncratic shocks in the human capital and with the government support for education that is financed with the tax on consumption.
Note: for each of the individual, average and aggregate variable, the values inside of the parentheses indicate the percentage deviations from the initial steady-state given the government policy

	Y	K	L	r	w	τ^l		
	1.3716	0.1219	0.9080	2.7520	1.0070	0.0060		
	(7.52%)	(9.04%)	(6.77%)	(-1.89%)	(0.70%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	1.0893	0.4814	0.0590	0.9812	0.7373	0.2207	0.4572	-0.6244
	(8.93%)	(-51.86%)	(15.27%)	(-1.88%)	(9.04%)	(9.04%)	(7.52%)	(16.77%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0001	0.0000	1.6287	0.1366	0.0409	0.0847	-2.9290
		(-60.45%)	—	(0.60%)	(0.10%)	(0.10%)	(-1.29%)	(-0.06%)
2	0.2466	0.0006	0.0000	1.6287	0.1669	0.0500	0.1035	-2.6332
		(-58.60%)	—	(0.60%)	(0.10%)	(0.10%)	(-1.29%)	(-0.07%)
3	0.3012	0.0024	0.0000	1.6287	0.2039	0.0610	0.1264	-2.3374
		(-54.90%)	—	(0.60%)	(0.10%)	(0.10%)	(-1.29%)	(-0.08%)
4	0.3679	0.0073	0.0042	1.4172	0.2490	0.0745	0.1544	-2.0666
		(-48.29%)	(10.10%)	(-0.43%)	(0.10%)	(0.10%)	(-1.29%)	(-0.18%)
5	0.4493	0.0198	0.0104	1.2495	0.3041	0.0910	0.1886	-1.7934
		(-39.37%)	(4.73%)	(-0.14%)	(0.10%)	(0.10%)	(-1.29%)	(-0.18%)
6	0.5488	0.0484	0.0179	1.1392	0.3714	0.1112	0.2303	-1.5142
		(-30.02%)	(3.29%)	(0.05%)	(0.10%)	(0.10%)	(-1.29%)	(-0.19%)
7	0.6703	0.1021	0.0272	1.0624	0.4537	0.1358	0.2813	-1.2310
		(-20.81%)	(2.64%)	(0.18%)	(0.10%)	(0.10%)	(-1.29%)	(-0.21%)
8	0.8187	0.1735	0.0385	1.0068	0.5541	0.1659	0.3436	-0.9449
		(-10.95%)	(2.27%)	(0.27%)	(0.10%)	(0.10%)	(-1.29%)	(-0.26%)
9	1.0000	0.2218	0.0522	0.9654	0.6768	0.2026	0.4197	-0.6566
		(0.80%)	(2.03%)	(0.35%)	(0.10%)	(0.10%)	(-1.29%)	(-0.35%)
10	1.2214	0.2044	0.0691	0.9340	0.8267	0.2475	0.5126	-0.3668
		(15.34%)	(1.88%)	(0.40%)	(0.10%)	(0.10%)	(-1.29%)	(-0.61%)
11	1.4918	0.1336	0.0896	0.9098	1.0097	0.3023	0.6261	-0.0757
		(32.28%)	(1.76%)	(0.44%)	(0.10%)	(0.10%)	(-1.29%)	(-2.91%)
12	1.8221	0.0618	0.1148	0.8909	1.2332	0.3692	0.7648	0.2163
		(49.72%)	(1.68%)	(0.47%)	(0.10%)	(0.10%)	(-1.29%)	(-0.95%)
13	2.2255	0.0198	0.1454	0.8759	1.5063	0.4509	0.9341	0.5091
		(69.96%)	(1.62%)	(0.50%)	(0.10%)	(0.10%)	(-1.29%)	(-0.40%)
14	2.7182	0.0041	0.1829	0.8641	1.8397	0.5508	1.1409	0.8024
		(104.77%)	(1.57%)	(0.52%)	(0.10%)	(0.10%)	(-1.29%)	(-0.25%)
15	3.3201	0.0004	0.2287	0.8546	2.2471	0.6727	1.3935	1.0962
		(195.46%)	(1.53%)	(0.54%)	(0.10%)	(0.10%)	(-1.29%)	(-0.18%)

Table 6: Second (final) steady-state with presence of idiosyncratic shocks in the human capital and with the government support for education that is financed with the tax on labour income.
Note: for each of the individual, average and aggregate variable, the values inside of the parentheses indicate the percentage deviations from the initial steady-state given the government policy

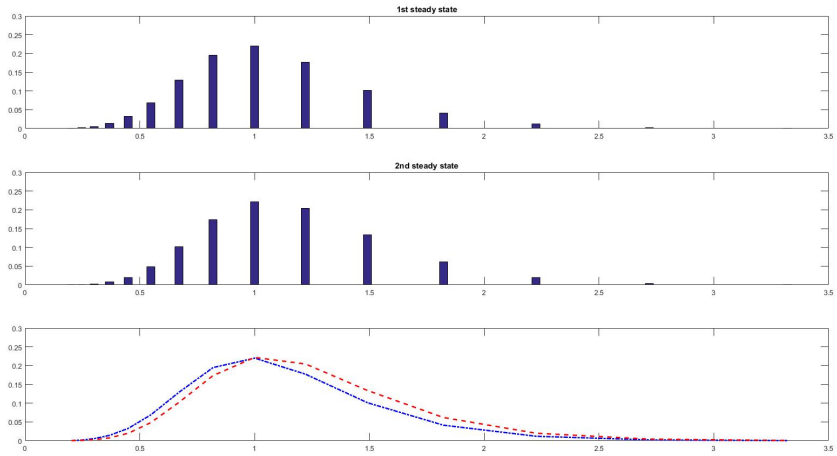

Figure 3: Comparison between distribution for the population of adult households at the initial steady-state with the distribution at the second (final) steady-state where government supports education and taxes labour income. Note: the (marginal) levels of the human capital are depicted on the horizontal axis; population share for each ability group is depicted on the vertical axis; blue dash-dotted line shows the distribution of adult households at the original steady-state; red dashed line illustrates the distribution of adult households at the second (final) steady-state

	Y	K	L	r	w	τ^k		
	1.3783	0.1240	0.9067	2.7048	1.0134	0.0102		
	(8.04%)	(10.96%)	(6.61%)	(-3.57%)	(1.34%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	1.0950	0.3603	0.0595	0.9739	0.7503	0.2246	0.4560	-0.6078
	(9.50%)	(-63.97%)	(16.25%)	(-2.61%)	(10.96%)	(10.96%)	(7.24%)	(18.98%)
i	h_i	pop.share	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0001	0.0000	1.6189	0.1383	0.0414	0.0841	-2.9200
		(-62.81%)	—	(0%)	(1.34%)	(1.34%)	(-2.06%)	(0.24%)
2	0.2466	0.0006	0.0000	1.6189	0.1690	0.0506	0.1027	-2.6243
		(-60.97%)	—	(0%)	(1.34%)	(1.34%)	(-2.06%)	(0.27%)
3	0.3012	0.0023	0.0000	1.6189	0.2064	0.0618	0.1254	-2.3285
		(-57.23%)	—	(0%)	(1.34%)	(1.34%)	(-2.06%)	(0.30%)
4	0.3679	0.0070	0.0042	1.4078	0.2521	0.0755	0.1532	-2.0578
		(-50.50%)	(10.76%)	(-1.09%)	(1.34%)	(1.34%)	(-2.06%)	(0.25%)
5	0.4493	0.0191	0.0104	1.2415	0.3079	0.0922	0.1871	-1.7846
		(-41.35%)	(5.03%)	(-0.79%)	(1.34%)	(1.34%)	(-2.06%)	(0.32%)
6	0.5488	0.0473	0.0180	1.1319	0.3761	0.1126	0.2286	-1.5054
		(-31.68%)	(3.50%)	(-0.59%)	(1.34%)	(1.34%)	(-2.06%)	(0.40%)
7	0.6703	0.1004	0.0272	1.0557	0.4593	0.1375	0.2792	-1.2221
		(-22.09%)	(2.81%)	(-0.45%)	(1.34%)	(1.34%)	(-2.06%)	(0.51%)
8	0.8187	0.1719	0.0385	1.0005	0.5610	0.1680	0.3410	-0.9359
		(-11.76%)	(2.41%)	(-0.35%)	(1.34%)	(1.34%)	(-2.06%)	(0.69%)
9	1.0000	0.2215	0.0523	0.9595	0.6852	0.2051	0.4165	-0.6477
		(0.68%)	(2.17%)	(-0.27%)	(1.34%)	(1.34%)	(-2.06%)	(1.01%)
10	1.2214	0.2059	0.0692	0.9283	0.8369	0.2506	0.5087	-0.3578
		(16.18%)	(2.00%)	(-0.22%)	(1.34%)	(1.34%)	(-2.06%)	(1.84%)
11	1.4918	0.1357	0.0897	0.9042	1.0222	0.3060	0.6213	-0.0668
		(34.33%)	(1.88%)	(-0.17%)	(1.34%)	(1.34%)	(-2.06%)	(9.23%)
12	1.8221	0.0632	0.1149	0.8854	1.2486	0.3738	0.7588	0.2253
		(53.15%)	(1.79%)	(-0.14%)	(1.34%)	(1.34%)	(-2.06%)	(3.14%)
13	2.2255	0.0204	0.1456	0.8706	1.5250	0.4566	0.9268	0.5180
		(74.10%)	(1.72%)	(-0.11%)	(1.34%)	(1.34%)	(-2.06%)	(1.35%)
14	2.7182	0.0042	0.1831	0.8589	1.8626	0.5576	1.1320	0.8114
		(113.29%)	(1.67%)	(-0.09%)	(1.34%)	(1.34%)	(-2.06%)	(0.86%)
15	3.3201	0.0005	0.2289	0.8495	2.2750	0.6811	1.3827	1.1052
		(212.99%)	(1.63%)	(-0.07%)	(1.34%)	(1.34%)	(-2.06%)	(0.63%)

Table 7: Second (final) steady-state with presence of idiosyncratic shocks in the human capital and with the government support for education that is financed with the tax on capital income.
Note: for each of the individual, average and aggregate variable, the values inside of the parentheses indicate the percentage deviations from the initial steady-state given the government policy

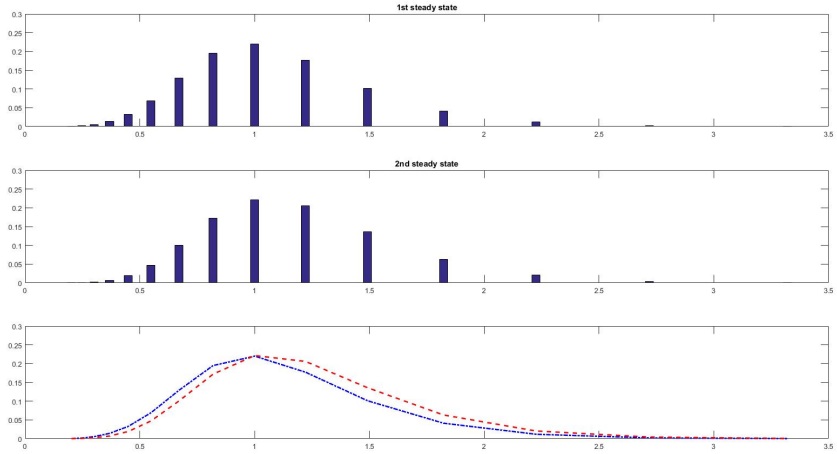

Figure 4: Comparison between distribution for the population of adult households at the initial steady-state with the distribution at the second (final) steady-state where government supports education and taxes capital income. Note: the (marginal) levels of the human capital are depicted on the horizontal axis; population share for each ability group is depicted on the vertical axis; blue dash-dotted line shows the distribution of adult households at the original steady-state; red dashed line illustrates the distribution of adult households at the second (final) steady-state

According to our results, at the second steady-state where the subsidy for education of ten percent is provided, the government has to levy the tax rate on consumable goods of 0.52% to keep its budget balanced. In the case if government finances its policy with the tax on labour income, however, this tax rate is set at 0.60%. When the capital income tax is utilised instead, the tax rate on the capital income at the second steady-state is chosen at a level of 1.02%. We observe this rise in the tax rates when we shift from tax on consumption to tax on labour income and to tax on capital income because, based on our assumptions, the tax on consumable goods covers the population of adult and elderly households simultaneously, whereas the tax on labour income is levied on the agents who are in the effective labour force (i.e. only adult households), and tax on capital income covers the share of population which receives the return on financial investment (i.e. elderly households).

With the continuous presence of the government support for education (together with the existence of idiosyncratic shocks to the human capital), we observe that at the second steady-state the level of average education

is 16.25% larger than at original steady-state when government utilises the taxes on consumption or capital income, and with the tax on labour income in place instead, the average level of education at the second steady-state is 15.27% larger than at original equilibrium. According to our previous result from the deterministic version of analysis which are provided in A1 section of appendix, only tax on labour income enters directly into the decisions of the adult households for education provision, which, according to our previous conclusions, reduces incentives of adult households for choice of education for their children.

Due to increase in the average level of education, the average level of human capital at the second steady-state increases by 9.50% when government levies tax on consumption or capital income. With the labour income tax, the average level of human capital at the second steady-state is 8.93% larger than at original equilibrium. Oppositely, however, the average level of fertility is found to fall by 2.61% when government utilises tax on consumption or capital income, whereas the average fertility at the second steady-state with support for education decreases by 1.88% when the labour income tax is in use. This conclusion for decrease in average level of fertility when the average level of education is increasing is consistent with analytical results for the deterministic environment, which indicated the presence of parental 'quality-quantity' trade-off for children. Due to decrease in the average fertility at the second steady-state under all tax options, the population of adult households becomes 63.97% smaller than at original steady-state when government uses tax on consumption or capital income, and it becomes 51.86% lower when government levies tax on labour income instead. Population size, however, continues to fall as time progresses.

With the lower of average fertility, lower population size of adult households, higher level of average education provision and higher average level of human capital, we observe an increase in the size of effective labour force per adult household. Its level increases by 6.61% when tax on consumption or capital income finance government budget, and by 6.77% when the labour income tax is in use instead. Based on the conclusions from the previous analysis for deterministic case, decrease in the average level of fertility would reduce the average time that parents need to take care for children, which would increase the average time that they can contribute towards the labour market. Decrease in average level of fertility at the second steady-state would also decrease the average number of teachers required, which, in turn, would create an inflow to the effective labour force. Oppositely, increase in the average education would require higher average number of teacher than at original steady-state to provide the desired level of education. Lastly, the increase in the average level of human capital combined with decrease in

population size of the adult households would result in smaller but more productive effective labour force.

As a result of increase in the effective labour force per adult household at the second steady-state when government subsidies the education, combined with increase in the physical capital stock per adult household, the real wage rate at this second steady-state is 1.34% larger when tax on consumption or capital income is in use, and it becomes 0.70% larger than at original steady-state when labour income tax is in place instead. This combined with increase in the average level of human capital and presence of taxes produce an increase in the average level of consumption and average level of savings of adult households at the second steady-state. With the tax rate on consumable goods, the average level of consumption increases by 10.39% and the average level of savings rises by 10.96%. With the tax rate on labour income these average figures both increase by 9.04%. Finally, the average consumption of adult households and their average savings both become 10.96% larger than at original equilibrium if government chooses capital income tax to finance its budget.

Due to increase in the average savings of adult households and decrease in the population of adult households, the physical capital stock per adult household at the second steady-state increases by 10.96% when government uses tax on consumption or capital income, and with the labour income tax in place it rises by 9.04%. With the increase in effective labour force per adult household being smaller than in the physical capital stock per adult household, the marginal product of the physical capital stock per adult household is lower at the second steady-state. Therefore, the real interest rate declines, and it becomes 3.57% lower than at original steady-state when government uses tax on consumption or capital income, and it falls by 1.89% with labour income tax in place. Consequently, with presence of taxes, higher average level of human capital, higher real wage rate and lower interest rate at the second steady-state, the average consumption of the elderly households increases by 7.49% with presence of tax on consumption, 7.52% with use of labour income tax, and 7.24% when capital income tax is in place.

Finally, according to our results at the second steady-state with the government subsidy for education of ten percent, the average level of utility (i.e. welfare of the economy) increases by 18.39%, 16.77% and 18.98% when tax on consumption, labour income or capital income is in place, respectively. The increase in the level of welfare in the model economy is the net effect of increase in the average level of consumption of adult and elderly households, increase in the average human capital of children, and decrease in the average level of fertility.

Now, when we have a comparison between original steady-state and the second steady-states for the case of the average (mean) household, we outline the differences that appear at the individual levels. It is still true, however, that when the government budget is financed by the labour income tax, the level of education received by all ability groups is lower than under all other tax options. Consequently, the individual levels of fertility are higher for all ability groups when labour income tax is in use. Additionally, we still observe that at the individual levels, the consumption of the elderly households at the second steady-state is lower with government budget being financed by capital income tax than with other tax options.

According to our results, at the second steady-state with idiosyncratic shocks to the human capital and with the government support for education, for our model population, the lowest three ability groups of adult households (i.e. $i \in [1, 3]$) still provide their children with zero level of education (and decide on maximum possible number of children). According to the past analysis for the deterministic environment, the value for the relative human capital is a key variable for decision on desired level of education in the utilised model economy. Therefore, with increase in the average level of human capital (due to the change in composition of population that results from the government policy) and with fixed (marginal) levels for human capital, the relative human capital for individual ability group would not increase as we observed for deterministic analysis, but it would decline. As a result we would either observe a moderate growth in decision for education (as with is the cases for $i \in [4, 15]$), or absence of any change (as with the cases for $i \in [1, 3]$).

Next, for the cases with tax levied either on consumption or on capital income, we observe anticipated decline in fertility at individual levels for the second steady-state with government support for education. For the case of the labour income tax, however, the individual fertility choices in general are higher at the second steady-state (which occurs together with increase in individual levels for education provision). This unexpected result for the case of the labour income tax could be explained as following. First, labour income tax distorts individual choices of the adult households for education provision, which with the presents of parental 'quality-quantity' trade-off for children would result in the marginally larger individual fertility levels than under tax on consumption or capital income. And second, the dampened response of the relative human capital (due to a smaller increase in the average human capital level) would dampen a rise in individual decisions for education, which would minimise the negative response in fertility decisions.

Moving forward, the individual choices for consumption of adult households and their savings are higher at the second steady-state with govern-

ment support for education. For the individual ability groups, these positive changes occur only due to increase in the real wage rate. Oppositely, the individual consumption for the elderly households across all ability groups is now lower. This occurs because the decrease in the real interest rate is only offset by moderate increase in the real wage rate[8].

Lastly, the distorting nature of the labour income tax has found to produce decreases in the individual utility levels of the adult households at the second steady-state. With other two tax options, individual utility levels are found to increase moderately.

Finally, from the data and the graphs for composition of population at the original steady-state and the second steady-state under three tax options, we observe a marginal narrowing of the distribution of adult households as the economy reaches the second steady-state. Furthermore, with increase in the average level of human capital due to the government policy, the distribution of adult households shifts to the right. These two changes could suggest that population of the model economy at the second steady-state with government support for education enjoys a higher level of human capital that is more equally[9] distributed. As a concluding note, it should be mentioned that even with fixed (marginal) levels of human capital for each ability group, the individual households still experience increase in their level of human capital which results in migration from lower ability groups to a higher ones.

5.2.2 Transition to the second (final) steady-state

We continue our discussion for the role of the government support for education in the heterogeneous stochastic environment with examination of the transition path that the model economy takes from original steady-state to a second steady-state. We consider the evolution of the model economy from the perspective of average (mean) household with an overview at the individual ability groups. The transition paths of the model economy for average household with the government placing tax on consumption, labour income and capital income are depicted by figure 5 where period one indicates the original steady-state, period forty represents the second steady-state, and one period of analysis is equivalent to the lifespan of one generation which is considered to be thirty years. Therefore, we proceed with our discussion starting from the second period of our simulation.

At the second period of analysis, the government enters into the model

[8]And this moderate increase in the real wage rate is not supported by increase in the individual (marginal) levels of human capital (i.e. what we have observed previously for the deterministic scenario)

[9]I.e.: the level of inequality in distribution of human capital potentially declines

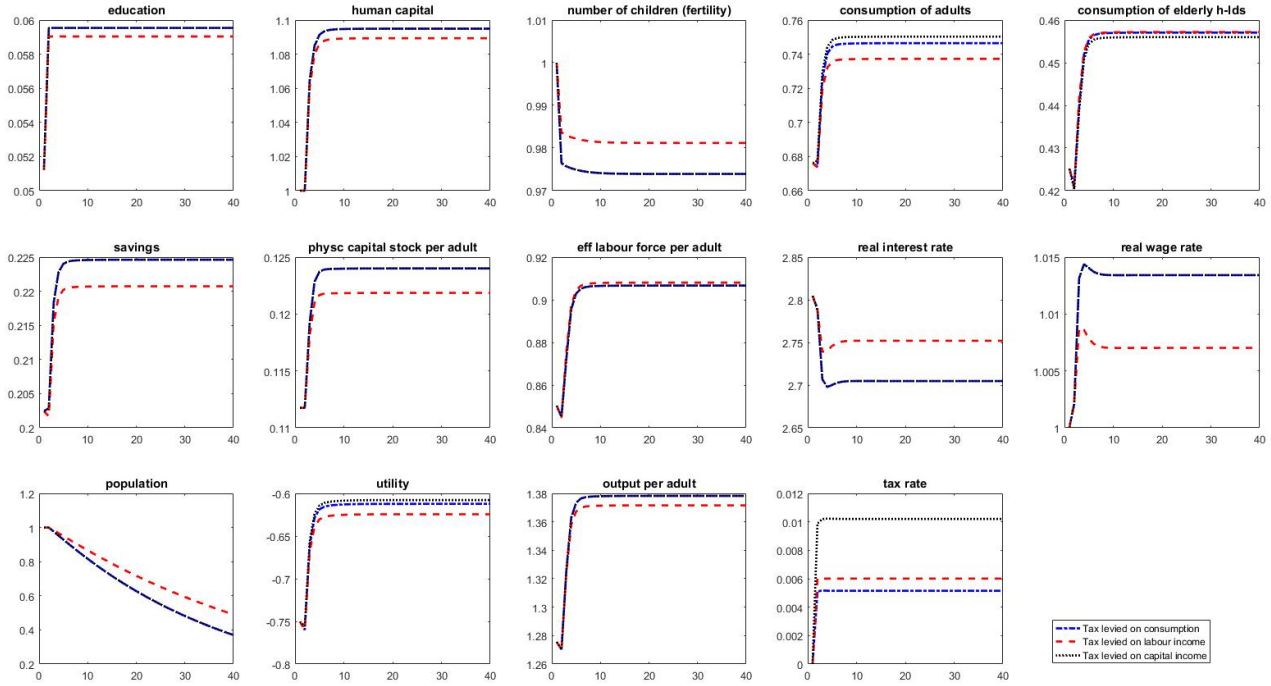

Figure 5: Transition path of the model economy with idiosyncratic shocks to the human capital from the perspective of the average household. Government provides subsidy for education of ten percent.
Note: transition path with tax on consumption is depicted by blue dash-dotted line; transition path with tax on labour income is presented by red dashed line; transition path with capital income tax is shown by black dotted line

economy. In order to provide the subsidy rate for education of ten percent to every heterogeneous adult household and balance its budget, the government levies the tax rate on consumption of 0.51%. In the case when government finances its budget with the tax on labour income, this tax rate is set at 0.60%, whereas, with the capital income tax in use, the tax rate is 0.99%.

With the government support for education, the average level of education received by the children increases by 16.25% from the initial steady-state when government uses tax on consumption or capital income. With labour income tax in place, the average level of education received becomes 15.27% higher compared to the initial equilibrium. At the individual level, the government support for education increases parental choices for education provision for all ability groups with exception for the first three lowest ones (i.e. $i \in [1, 3]$). For these three groups, adult households still optimally decide on zero education provision due to low levels of relative human capital, which remain identical to the initial steady-state values. An increase in these individual parental choices is proportionally declining to the marginal levels of human capital for individual ability groups, however. For instance, our results indicate, that for the fourth ability group (i.e. $i = 4$), the education attainment increases by 80.88% and 76.02% when government finances its budget with either consumption or capital income tax, and with labour income tax, respectively. This value increases by 16.28% and 15.30% for the ninth ability group, and by 12.58% and 11.82% for the fourteenth ability group, with tax on consumption or capital income and with tax on labour income in use, respectively.

These results can be rationalised based on the (past) analytical conclusions from the deterministic version of model economy. In the deterministic version, the individual choice for education has been defined by (15)

$$e_t^i = \frac{\eta\phi(1-\tau_t^l)x_t^i - \theta(1-sub_t^e) - \eta\bar{e}_t sub_t^n}{(1-\eta)(1-sub_t^e)} \tag{15}$$

with the first order partial derivative of the individual choice for education with respect to the subsidy rate for education defined by (16)

$$\frac{\partial e_t^i}{\partial sub_t^e} \approx \frac{\eta\phi(1-\tau_t^l)x_t^i - \eta\bar{e}_t sub_t^n}{(1-\eta)(1-sub_t^e)^2} \tag{16}$$

and second order partial derivative with respect to the relative human capital defined by (17)

$$\frac{\partial^2 e_t^i}{\partial sub_t^e \partial x_t^i} \approx \frac{\eta\phi(1-\tau_t^l)}{(1-\eta)(1-sub_t^e)^2} \tag{17}$$

which is less than one and with the current calibration for the model parameters is equal to 0.0926 for the case of tax on consumption or capital income being used to finance the government budget, and is equal to the 0.0920 when the labour income tax in place instead. This result suggest that with introduction of government support for education and with shift from one ability group to another (i.e. from lower marginal value of relative human capital to higher marginal value of human capital), the rate of increase in the optimal education provision would diminish.

Based on the previous results for the deterministic version of analysis, we would expect to observe an increase in both average and individual levels of human capital for the next generation of adult households, which would begin a change in the composition of the model population. We, however, could not anticipate the complete response of the model economy for the next period of analysis yet due to the presence of the idiosyncratic shocks in the human capital formation process.

Considering another factor that results in the change of a future composition of the model population and population size overall, we observe that the average level of fertility declines by 2.36% and 1.64% at the first period of the government presence when tax on consumption or capital income, and labour income are in use, respectively. This decline in fertility choices of the adult households is explained by the parental 'trade-off' for children which is an underlying characteristics of the utilised model economy. Furthermore, as we indicated previously, a smaller decline in (the average) fertility under the labour income tax can be explained by the distorting nature of this tax on the parental decisions for education provision. As we observed above, the level of education increases by smaller rate (both at individual and average levels) with the labour income tax financing government budget, which, according to the conclusions from the deterministic version of analysis, would result in a smaller decline in fertility choices compared to the other tax options.

At the individual level, the households from the lower ability groups experience much larger decline in fertility compared to the households from the higher ability groups. For instance, with the presence of tax on consumption or capital income, the adult households from the fourth, eights and top (fifteenth) ability groups reduce their fertility choices by 7.05%, 2.35% and 0.50%, respectively. For the case of the labour income tax, however, due to its distorting nature, for the top two ability groups (i.e. $i = 14, 15$), we even observe an increase in individual fertility choices at the second period of analysis. Overall, the adult households from the fourth and eights ability groups experience decline in fertility by 6.13% and 1.63%, respectively, whereas, the households from the top ability group experience an increase of 0.13% at the first period of government presence when the labour income tax is in use.

In order to address a decline in the individual negative responses of adult households for fertility when we shift from lower ability groups to higher ability ones, we utilise conclusions from the discussion for individual choices for education. According to our results above, when the government enters the economy and provides subsidy for education, the lower ability groups with lower levels of relative human capital would increase their choices for education by larger amount than the higher ability households with higher levels of relative human capital. This combined with the underlying properties for the behaviour of the households which indicate a negative relationship between choices for education and fertility results in larger decline in fertility for groups who experience larger increase in education (i.e. lower ability groups). For the households from higher ability groups, the increase in education has been found to be moderate, which translates in moderate decline in fertility as in the case of tax on consumption or capital income, or, even, a moderate increase as for the scenario with the labour income tax. According to our results, however, the only exception for this dynamics is presented by the three lowest ability groups (i.e. $i \in [1, 3]$) due to the absence of change in the optimal decisions for education at the second period of analysis, which results from the low levels of relative human capital for these two household groups.

With this observed dynamics for education attainment and fertility decisions, combined with absence of change in the composition of adult households (since it has been formed at the original steady-state before government has entered), our results indicate a decrease in the size of the effective labour force per adult household of 0.63% and 0.68% at the second period of analysis when government uses tax on consumption or capital income and tax on the labour income, respectively. This decline in the size of effective labour force per adult household is the net effect that results from the following changes. First, an increase in the choice for education provision combined with identical level of human capital of adult households to the initial steady-state requires an increase of number of teachers in the economy which reduces a number of adult households available for participation in the effective labour force. And, second, a wide decrease in the fertility choices creases a decrease in the number of teachers required and an increase in the free time that adult household can contribute to participation in effective labour force instead of taking care for their children.

Opposite to the observed decrease in the size of effective labour force per adult household, the physical capital stock per adult household remains unchanged at the first period of the government presence. This is the case, because according to the underling assumptions of the utilised model economy, the level of the physical capital stock per adult household is defined

by the aggregate investment of the past generation of the adult households (considering there is a full depreciation of the physical capital stock) and by the present population size of adult households, that in the case of the second period of discussion both defined by the generation of the initial steady-state.

Absence of change in the physical capital stock per adult household and observed decrease in the effective labour force per adult household, decreases the value of the marginal product of the former and increases it for the latter. Since both of these factors of production are paid their marginal products, for the second period of analysis we observe a decrease in the real interest rate of 0.57% when government uses tax on consumption or capital income, and it falls by 0.61% with labour income tax in place. For the real wage rate, however, our results indicate 0.21% increase when tax on consumption or capital income is utilised, and it rises by 0.23% when tax on labour income is considered. We observe a marginally larger increase in real wage rate and marginally smaller decrease in the real interest rate for the case of the labour income tax compared to other tax options because, according to formulation of the model economy, only labour income tax distorts the decisions of adult households for education provision. As we indicated earlier, the use of the labour income tax results in smaller increase of the education attainment, which results in a smaller decrease in fertility choices, which produces larger decrease in the effective labour force per adult household, which leads to a larger increase in the marginal product of the effective labour force per adult household and greater decrease in the marginal product of the physical capital stock per adult household at the second period of analysis compared to other two taxes.

With increase in the real wage rate and introduction of three potential tax options, our results indicate the decrease in the consumption of the adult households at the second period of analysis when government uses tax on consumption and labour income. With tax levied on consumption, the consumption of the adult households for all ability groups decreases by 0.30%, while with the labour income tax it falls by 0.37%. With the capital income tax in place, however, which does not affect the consumption decisions of the adult households, the consumption of the adult households from every ability group increases by 0.21%. Next, savings of the adult households, which are discouraged by the presence of the labour income tax only and increase with the rise in the real wage rate, grow by 0.21% for adult households from every ability group when tax on consumption or capital income is in use. With the labour income tax in place, however, savings of the adult households fall by 0.37%. Lastly, according to the conclusions from the deterministic version of the analysis, the consumption of the elderly households is increasing with past real wage rate and present real interest rate, and decreasing with the

past tax rate on labour income, and present tax rates on consumption and capital income. Therefore, according to our results, consumption of the elderly households for every ability group falls by 0.93%, 1.05% and 1.15% with the tax on consumption, tax on labour income, and tax on capital income in place, respectively.

As a result of these observed changes (in consumption of the adult and elderly households, and in the fertility choices) for the first period of the government presence, the level of welfare is decreasing under all tax options. For instance, with the tax on consumption, the level of welfare decreases by 1.32%, and with the tax on labour income and capital income this value is falling by 1.29% and 0.72%, respectively. Furthermore, at the individual level, we observe that households from the lower ability groups suffer larger decrease in the utility level compared to a higher ability groups, due to larger negative responses of the lower ability groups in their fertility decisions. Finally, due to reduction in the effective labour force per adult household and absence of change in value of the physical capital stock per adult household, we observe decrease in the level of the real output per adult household produced at the second period of analysis. Our results indicate 0.42% decrease in this value when the government uses tax on consumption or capital income, and it falls by 0.45% when the labour income tax is utilised instead.

Moving to the next period of analysis, we observe a change in the composition of population for the adult households due to past period government support for education. Our results indicate an increase in the population share for a higher adult households from tenth to fifteenth ability groups (i.e. $i \in [10, 15]$); whereas the share within the total population for the remaining ability groups is declining at the third period of analysis. There are few reasons that explain this change. First, due to the subsidy for education and resulted larger response in the education attainment for lower ability groups (compared to a higher ability ones), the lower ability groups end up with the realised level of human capital that is larger than the marginal level of human capital that characterises a particular ability group. As a result, past period young households from the lower ability group now have enough human capital to become a part of higher ability groups. And second, due to a larger negative reaction of the lower ability households in their fertility choices that we observed for the second period of analysis that results from a relatively larger increase in education attainment for these groups, there are relatively smaller number of adult households from the lower ability groups compared to a higher ability ones that exist at the third period of analysis. Therefore, the population of the adult households at the second period of the government support for education is characterised by the average (mean)

level of human capital that is 6.45% and 6.07% larger than at the original steady-state when government uses tax on consumption or capital income, and labour income, respectively. However, due to decrease in the past fertility choices, the total population size of the adult households is decreasing at the third period of analysis, and it becomes 2.35% lower than at original steady-state when government utilises tax on consumption or capital income. With labour income tax in use, the size of the adult households diminishes by 1.64%. (A smaller decrease in the population size with the labour income tax in use is explained by the distorting nature of this tax. As it has been indicated earlier, the use of the labour income tax by the government to finance the support for education produces response in education provision that is muted compared to the other tax options. With the parental trade-off in place, relatively smaller increase in education provision results in relatively smaller decline in fertility, that produces relatively smaller decrease in the population size of the future adult households).

In order to keep providing the support for education at ten percent rate and keep the government budget balanced, the government increases the tax rate on consumption up to 0.52% and tax rate on capital income up to 1.02%. The tax rate on the labour income is kept unchanged, however.

With a continuous government support for education and change in the structure of the population of adult households, our results indicate that the average level of education received at the third period of analysis is 16.2494% larger than at original steady-state when tax on consumption or capital income is in use, which, however, is 0.0018% lower than at the previous period of analysis. In the case of the labour income tax, the average level of education at the second period of government presence is 15.2732% above the original steady-state, which indicates a 0.0012% decrease compared to the previous generation.

This change in the average level of education attainment could be reasoned as following. First, since the marginal levels of the human capital for each ability group are kept unchanged, the increase in the average level of human capital due to past period government support for education decreases the values of the relative human capital for each ability group. As we discussed it earlier, the relative human capital is a key decision parameter for the adult households in the utilised model economy for education provision decisions. Therefore, according to our results, there is a decrease in the value of the relative human capital across all ability groups at the third period of analysis which diminish the optimal decisions of the current adult households for education provision for their children. Our results suggest, however, that the optimal choice for education at the second period of the government intervention is still larger than at original steady-state. For

instance, with the tax on consumption or capital income the education provision of the fourth, ninth and fourteenth ability groups are 31.87%, 6.41% and 4.96% higher than at original steady-state, respectively. With the tax on labour income in use the level of education attainment by these groups is 29.99%, 6.03% and 4.66% larger than at first equilibrium. Furthermore, according to our results, the lower ability groups experience larger decrease in optimal choice for education compared to a higher ability groups. The second reason for a very moderate decrease in the average level of education even when there are sizeable declines within this decision for each ability group is a change in composition of population for the adult households. As we have indicated above, with the government support for education the individual households from the lower ability groups (especially ones who faced positive idiosyncratic shocks during human capital formation process) become a part of the population groups that are defined by relatively larger level of the marginal human capital, who are able to provide their children with higher level of education. Since the share of the adult households from the higher ability groups is increasing at the second period of government support for education, the average level of education is kept relatively stable.

With decrease in the optimal choices for education within each ability group, our results indicate the increase in fertility levels. For instance, with the tax on consumption or capital income, the childbearing decisions of the adult households for the fourth, eights and fourteenth ability groups are increasing by 4.27%, 1.38% and 0.36%, and become 3.08%, 1.00% and 0.26% below the original steady-state, respectively. With the labour income tax, the fertility choices for these three ability groups are increasing by 4.05%, 1.31% and 0.34% from the previous period of analysis, and they become 2.33% and 0.34% below the original equilibrium for the fourth and eights ability groups. For the adult households from the fourteenth ability group, the level of fertility becomes 0.36% above the original steady-state, however. Despite these positive changes for the fertility levels for each ability group within the population of adult households (with exception for the bottom ability group which provide their children with zero education), the change in the overall composition of the model population results in the lower levels of the average fertility for the second period of the government presence. Therefore, according to our results, the average level of fertility at the third period of analysis is falling by 0.07% and becomes 2.43% below of original steady-state when tax on consumption or capital income is in place. With the labour income tax, the average level of fertility reduces by 0.07% and becomes 1.71% below the original equilibrium.

Given an increase in the average endowment of the human capital of adult households, the effective labour force becomes more productive com-

pared to the previous generation. Furthermore, an increase in the average level of the human capital at the third period of analysis results in the rise of ability of teachers, which decreases the number of teachers required that results in greater number of adult households participating in the effective labour force. Additionally, with decrease in the average fertility levels in the economy, the average time that adult households need to spend for taking care of the children is falling, which increases the time that adult households can participate in the effective labour force on average. Finally, decrease in the average fertility level reduces number of teachers required even further. Consequently, our results indicate that at the second period of the government presence the value of the effective labour force per adult household is increasing by 3.23% from the previous generation and becomes 2.58% above the original steady-state when tax on consumption or capital income finance the government budget. With the labour income tax, the effective labour force per adult household increases by 3.65% and is 2.94% above the initial steady-state. This larger increase in the value of the effective labour force per adult household in the case of the labour income tax is primarily driven by a smaller decrease in the population of the adult households and larger decrease in the average birth rate, all of which result in a larger number of adult households who can participate in the labour market.

Next, the physical capital stock per adult household, which is determined by the aggregate savings of the adult households from the past generation and by the present size of the population of adult households, becomes 6.68% larger than at original steady-state when the government implements tax on consumption or capital income. With the labour income tax, which has reduced incentives of the adult households for savings, our results indicate that the value for the physical capital stock per adult household at the third period of analysis is increases by 5.68% from original equilibrium. With the labour income tax the model economy experience increase in the per adult household value of this second factor input because the rate of fall in the present population size of the adult households outweighs the rate of decrease in saving decisions of the previous generation.

With each of these two factors of production being paid their corresponding marginal products, relatively larger increase in the physical capital stock per adult household compared to increase in the effective labour force per adult household diminishes the real interest rate and increases the real wage rate by 2.94% and 1.10% that become 3.50% below and 1.31% above the original steady-state, respectively, when tax on consumption or capital income is in use. With tax on labour income, the real interest rate falls by 1.74% and it is 2.35% below original equilibrium, whereas the real wage rate rises by 0.65% and becomes 0.88% above the initial steady-state.

Given the changes in the factor prices and revised tax rates, we observe a uniform increase in the consumption and savings of the adult households across all ability groups. Our results indicate that with the tax on consumable goods, the consumption of adult households increases by 1.095% and becomes 0.79% larger than at original steady-state, whereas savings of adult households inside of every group increase by 1.10% and are 1.31% larger than at first steady-state. With the labour income tax, both of these indicators for every ability group increase by 0.65% and become 0.27% larger than at initial equilibrium, whereas with capital income tax, these values increase by 1.10% and are 1.31% larger than at the original equilibrium. The consumption of the elderly households within of each ability group continues its decrease, however, due to the present period decline in the real interest rate. Therefore, the consumption of the elderly households in each ability group decreases by 1.97%, 1.06% and 1.98%, which becomes 2.88%, 2.10% and 3.10% below of original steady-state with tax on consumption, labour income and capital income in use, respectively. However, with the change in the structure of the model population, where households from the lower ability groups become the part of higher ability groups who enjoy higher consumption during adulthood and at the old age together with higher decisions for savings, we observe that the average consumption of the adult households increase by 7.62%, 6.76% and 7.62%, and becomes 7.30%, 6.36% and 7.85% larger than at the original steady-state, when tax on consumption, labour income or capital income is in place. The average savings of the adult households at the third period of analysis increase by 7.62%, 6.76% and 7.62%, and are 7.85%, 6.36% and 7.85% higher than at initial equilibrium; whereas, the average consumption of the elderly households at the second period of the government presence rises by 4.36%, 4.95% and 4.35%, which is 3.39%, 3.85% and 3.15% larger than at original steady-state, when tax on consumption, labour income or capital income is utilised, respectively.

With these changes in the composition of the population, in the decisions within the ability groups and in the production sector at the third period of analysis, we observe increase in the level of welfare by 12.98%, 12.06% and 13.06%, which becomes 11.84%, 10.92% and 12.43% above the original steady-state when government uses tax on consumption, labour income and capital income, respectively. Finally, with increase in both effective labour force per adult household and physical capital stock per adult household at the second period of the government presence with support for education attainment, the value of the real output per adult household rises by 4.36% and becomes 3.92% above the initial equilibrium when government uses tax on consumption or capital income. With labour income tax in place instead, the real output per adult household increases by 4.32% and is 3.85% above

the initial steady-state.

The dynamics described for the third period of analysis continues for all the variables with exception for consumption of the elderly households within a given ability groups until the economy reaches the second steady-state. At the fourth period of analysis the past period increase in the real wage rate is enough to counteract a fall in the real interest rate rate and (the negative) effect of taxes. As a result, the consumption of the elderly households within each ability group increases, which together with the change in the composition of the model population contributes for increase in the average consumption of the elderly households, level of welfare and individual utility levels for each ability group even further.

5.3 Subsidy for children

Moving our discussion forward, we consider the influence of the government support for fertility decisions of the heterogeneous households in the proposed stochastic environment next. We consider that the government provides every adult household in the model population with ten percent subsidy rate for childbearing decisions. Similarly to our previous discussion, the government has three tax options[10] that it can utilise in order to finance the subsidy program and keep its budget balanced. In the upcoming discussion we implement the same order of analysis as before: first, we examine the second steady-state that the model economy reaches with subsidy program for fertility, and next, we analyse the transition path that the model economy takes from the original steady-state to the second steady-state. In our discussion, we focus on the changes that take place both for mean household and individual household ability groups. The summary for the steady-states with government support for fertility that is financed with tax on consumption, labour income and capital income is presented by the tables 8, 9 and 10, whereas figures 6, 7 and 8 illustrate the initial and resulted distributions of the model population, respectively.

5.3.1 Second (final) steady-state

According to our results, in order to balance its budget and provide the subsidy rate for children of ten percent, the government levies the tax rate on consumption of 0.47% at the second steady-state. Alternately, with the labour income tax in place, the tax rate is set at 0.58%, and with the capital income tax in use, the tax rate balances at 0.84%.

[10] I.e.: tax on consumption, labour income and capital income

	Y	K	L	r	w	τ^c		
	1.2421	0.1000	0.8639	3.1417	0.9585	0.0047		
	(-2.63%)	(-10.55%)	(1.58%)	(12.01%)	(-4.15%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	0.9333	192.87	0.0467	1.1498	0.6020	0.1811	0.4121	-0.9095
	(-6.68%)		(-8.80%)	(14.98%)	(-10.97%)	(-10.55%)	(-3.09%)	(-21.23%)
i	h_i	p_i	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0010	0.0000	2.2733	0.1302	0.0392	0.0891	-2.9020
		(396.02%)	—	(40.42%)	(-4.60%)	(-4.15%)	(3.84%)	(0.86%)
2	0.2466	0.0077	0.0000	2.1181	0.1591	0.0478	0.1089	-2.6189
		(411.58%)	—	(30.84%)	(-4.60%)	(-4.15%)	(3.84%)	(0.47%)
3	0.3012	0.0236	0.0000	2.0060	0.1943	0.0584	0.1330	-2.3329
		(337.56%)	—	(23.91%)	(-4.60%)	(-4.15%)	(3.84%)	(0.11%)
4	0.3679	0.0426	0.0011	1.8417	0.2373	0.0714	0.1624	-2.0525
		(202.68%)	(-71.14%)	(29.39%)	(-4.60%)	(-4.15%)	(3.84%)	(0.51%)
5	0.4493	0.0601	0.0076	1.4959	0.2898	0.0872	0.1984	-1.7940
		(84.41%)	(-22.83%)	(19.55%)	(-4.60%)	(-4.15%)	(3.84%)	(-0.21%)
6	0.5488	0.0859	0.0156	1.2966	0.3540	0.1065	0.2423	-1.5239
		(24.13%)	(-9.94%)	(13.87%)	(-4.60%)	(-4.15%)	(3.84%)	(-0.83%)
7	0.6703	0.1281	0.0254	1.1691	0.4324	0.1301	0.2960	-1.2467
		(-0.61%)	(-4.06%)	(10.24%)	(-4.60%)	(-4.15%)	(3.84%)	(-1.49%)
8	0.8187	0.1728	0.0373	1.0819	0.5281	0.1589	0.3615	-0.9649
		(-11.31%)	(-0.74%)	(7.76%)	(-4.60%)	(-4.15%)	(3.84%)	(-2.38%)
9	1.0000	0.1860	0.0519	1.0197	0.6450	0.1940	0.4415	-0.6797
		(-15.46%)	(1.36%)	(5.99%)	(-4.60%)	(-4.15%)	(3.84%)	(-3.88%)
10	1.2214	0.1498	0.0697	0.9739	0.7878	0.2370	0.5393	-0.3922
		(-15.50%)	(2.78%)	(4.68%)	(-4.60%)	(-4.15%)	(3.84%)	(-7.58%)
11	1.4918	0.0887	0.0914	0.9393	0.9623	0.2894	0.6587	-0.1029
		(-12.25%)	(3.79%)	(3.70%)	(-4.60%)	(-4.15%)	(3.84%)	(-39.89%)
12	1.8221	0.0388	0.1180	0.9127	1.1753	0.3535	0.8045	0.1878
		(-5.84%)	(4.53%)	(2.94%)	(-4.60%)	(-4.15%)	(3.84%)	(-14.03%)
13	2.2255	0.0123	0.1504	0.8921	1.4355	0.4318	0.9827	0.4794
		(4.65%)	(5.08%)	(2.35%)	(-4.60%)	(-4.15%)	(3.84%)	(-6.20%)
14	2.7182	0.0025	0.1900	0.8759	1.7533	0.5274	1.2002	0.7719
		(25.96%)	(5.51%)	(1.89%)	(-4.60%)	(-4.15%)	(3.84%)	(-4.04%)
15	3.3201	0.0003	0.2383	0.8630	2.1415	0.6442	1.4659	1.0651
		(85.14%)	(5.84%)	(1.53%)	(-4.60%)	(-4.15%)	(3.84%)	(-3.02%)

Table 8: Second (final) steady-state with presence of idiosyncratic shocks in the human capital and with the government support for fertility that is financed with the tax on consumption.
Note: for each of the individual, average and aggregate variable, the values inside of the parentheses indicate the percentage deviations from the initial steady-state given the government policy

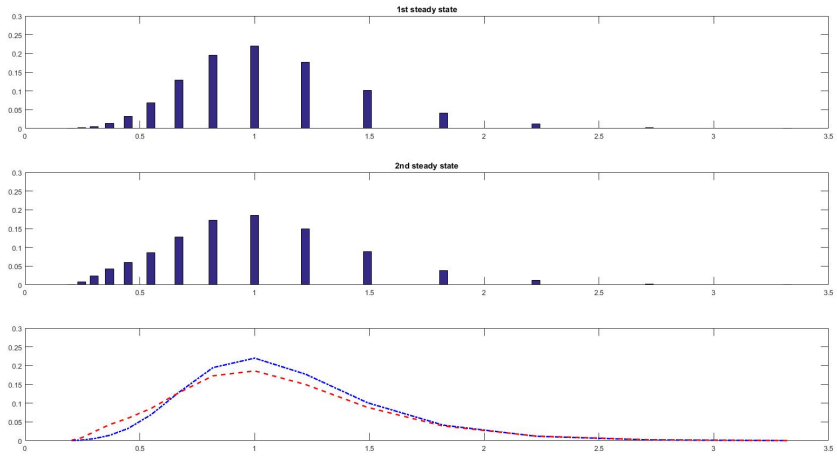

Figure 6: Comparison between distribution for the population of adult households at the initial steady-state with the distribution at the second steady-state where government supports fertility decisions and taxes consumption. Note: the (marginal) levels of the human capital are depicted on the horizontal axis; population share for each ability group is depicted on the vertical axis; blue dash-dotted line shows the distribution of adult households at the original steady-state; red dashed line illustrates the distribution of adult households at the second steady-state

	Y	K	L	r	w	τ^l		
	1.2362	0.0983	0.8652	3.1928	0.9526	0.0058		
	(-3.09%)	(-12.06%)	(1.73%)	(13.83%)	(-4.74%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	0.9286	255.78	0.0463	1.1583	0.5946	0.1780	0.4121	-0.9216
	(-7.14%)		(-9.57%)	(15.82%)	(-12.06%)	(-12.06%)	(-3.09%)	(-22.84%)
i	h_i	p_i	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0010	0.0000	2.2796	0.1293	0.0387	0.0896	-2.9073
		(408.22%)	—	(40.81%)	(-5.29%)	(-5.29%)	(4.36%)	(0.68%)
2	0.2466	0.0078	0.0000	2.1256	0.1579	0.0473	0.1094	-2.6241
		(422.10%)	—	(31.30%)	(-5.29%)	(-5.29%)	(4.36%)	(0.28%)
3	0.3012	0.0240	0.0000	2.0141	0.1929	0.0577	0.1337	-2.3380
		(345.65%)	—	(24.41%)	(-5.29%)	(-5.29%)	(4.36%)	(-0.10%)
4	0.3679	0.0434	0.0011	1.8489	0.2356	0.0705	0.1633	-2.0575
		(207.89%)	(-70.78%)	(29.90%)	(-5.29%)	(-5.29%)	(4.36%)	(0.26%)
5	0.4493	0.0610	0.0076	1.5028	0.2877	0.0861	0.1994	-1.7990
		(87.30%)	(-22.74%)	(20.09%)	(-5.29%)	(-5.29%)	(4.36%)	(-0.49%)
6	0.5488	0.0870	0.0156	1.3030	0.3514	0.1052	0.2435	-1.5288
		(25.75%)	(-9.93%)	(14.43%)	(-5.29%)	(-5.29%)	(4.36%)	(-1.15%)
7	0.6703	0.1292	0.0254	1.1751	0.4292	0.1285	0.2975	-1.2516
		(0.24%)	(-4.08%)	(10.81%)	(-5.29%)	(-5.29%)	(4.36%)	(-1.89%)
8	0.8187	0.1732	0.0373	1.0877	0.5243	0.1570	0.3633	-0.9697
		(-11.10%)	(-0.78%)	(8.34%)	(-5.29%)	(-5.29%)	(4.36%)	(-2.89%)
9	1.0000	0.1852	0.0519	1.0253	0.6404	0.1917	0.4438	-0.6845
		(-15.82%)	(1.30%)	(6.57%)	(-5.29%)	(-5.29%)	(4.36%)	(-4.61%)
10	1.2214	0.1482	0.0696	0.9793	0.7821	0.2342	0.5420	-0.3969
		(-16.36%)	(2.71%)	(5.26%)	(-5.29%)	(-5.29%)	(4.36%)	(-8.89%)
11	1.4918	0.0873	0.0914	0.9446	0.9553	0.2860	0.6620	-0.1076
		(-13.60%)	(3.71%)	(4.28%)	(-5.29%)	(-5.29%)	(4.36%)	(-46.35%)
12	1.8221	0.0380	0.1179	0.9179	1.1668	0.3493	0.8086	0.1830
		(-7.80%)	(4.45%)	(3.52%)	(-5.29%)	(-5.29%)	(4.36%)	(-16.21%)
13	2.2255	0.0119	0.1503	0.8972	1.4251	0.4267	0.9876	0.4747
		(1.80%)	(5.00%)	(2.94%)	(-5.29%)	(-5.29%)	(4.36%)	(-7.12%)
14	2.7182	0.0024	0.1898	0.8809	1.7406	0.5211	1.2062	0.7672
		(21.40%)	(5.42%)	(2.48%)	(-5.29%)	(-5.29%)	(4.36%)	(-4.63%)
15	3.3201	0.0003	0.2382	0.8680	2.1260	0.6365	1.4733	1.0603
		(75.77%)	(5.75%)	(2.11%)	(-5.29%)	(-5.29%)	(4.36%)	(-3.45%)

Table 9: Second (final) steady-state with presence of idiosyncratic shocks in the human capital and with the government support for fertility that is financed with the tax on labour income.
Note: for each of the individual, average and aggregate variable, the values inside of the parentheses indicate the percentage deviations from the initial steady-state given the government policy

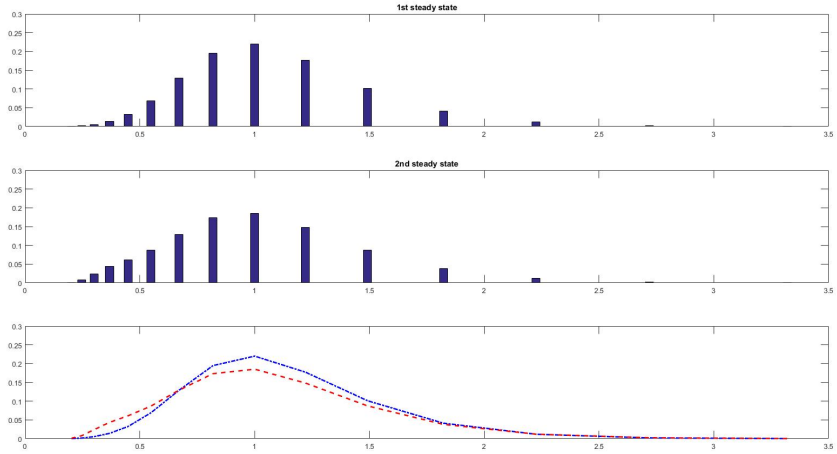

Figure 7: Comparison between distribution for the population of adult households at the initial steady-state with the distribution at the second steady-state where government supports fertility decisions and taxes labour income. Note: the (marginal) levels of the human capital are depicted on the horizontal axis; population share for each ability group is depicted on the vertical axis; blue dash-dotted line shows the distribution of adult households at the original steady-state; red dashed line illustrates the distribution of adult households at the second steady-state

	Y	K	L	r	w	τ^k		
	1.2421	0.1000	0.8639	3.1417	0.9585	0.0084		
	(-2.63%)	(-10.55%)	(1.58%)	(12.01%)	(-4.15%)			
	$\bar{h}$	tot.pop.	$\bar{e}$	$\bar{n}$	$\bar{c}$	$\bar{s}$	$\bar{d}$	$\bar{u}$
	0.9333	192.87	0.0467	1.1498	0.6048	0.1811	0.4114	-0.9052
	(-6.68%)		(-8.80%)	(14.98%)	(-10.55%)	(-10.55%)	(-3.26%)	(-20.66%)
i	h_i	p_i	e_i	n_i	c_i	s_i	d_i	u_i
1	0.2019	0.0010	0.0000	2.2733	0.1308	0.0392	0.0890	-2.8978
		(396.02%)	—	(40.42%)	(-4.15%)	(-4.15%)	(3.66%)	(1.00%)
2	0.2466	0.0077	0.0000	2.1181	0.1598	0.0478	0.1087	-2.6147
		(411.58%)	—	(30.84%)	(-4.15%)	(-4.15%)	(3.66%)	(0.63%)
3	0.3012	0.0236	0.0000	2.0060	0.1952	0.0584	0.1328	-2.3287
		(337.56%)	—	(23.91%)	(-4.15%)	(-4.15%)	(3.66%)	(0.29%)
4	0.3679	0.0426	0.0011	1.8417	0.2384	0.0714	0.1622	-2.0483
		(202.68%)	(-71.14%)	(29.39%)	(-4.15%)	(-4.15%)	(3.66%)	(0.71%)
5	0.4493	0.0601	0.0076	1.4959	0.2912	0.0872	0.1981	-1.7898
		(84.41%)	(-22.83%)	(19.55%)	(-4.15%)	(-4.15%)	(3.66%)	(0.02%)
6	0.5488	0.0859	0.0156	1.2966	0.3557	0.1065	0.2419	-1.5197
		(24.13%)	(-9.94%)	(13.87%)	(-4.15%)	(-4.15%)	(3.66%)	(-0.55%)
7	0.6703	0.1281	0.0254	1.1691	0.4344	0.1301	0.2955	-1.2425
		(-0.61%)	(-4.06%)	(10.24%)	(-4.15%)	(-4.15%)	(3.66%)	(-1.15%)
8	0.8187	0.1728	0.0373	1.0819	0.5306	0.1589	0.3609	-0.9606
		(-11.31%)	(-0.74%)	(7.76%)	(-4.15%)	(-4.15%)	(3.66%)	(-1.93%)
9	1.0000	0.1860	0.0519	1.0197	0.6481	0.1940	0.4408	-0.6755
		(-15.46%)	(1.36%)	(5.99%)	(-4.15%)	(-4.15%)	(3.66%)	(-3.24%)
10	1.2214	0.1498	0.0697	0.9739	0.7916	0.2370	0.5384	-0.3880
		(-15.50%)	(2.78%)	(4.68%)	(-4.15%)	(-4.15%)	(3.66%)	(-6.42%)
11	1.4918	0.0887	0.0914	0.9393	0.9668	0.2894	0.6576	-0.0987
		(-12.25%)	(3.79%)	(3.70%)	(-4.15%)	(-4.15%)	(3.66%)	(-34.15%)
12	1.8221	0.0388	0.1180	0.9127	1.1809	0.3535	0.8032	0.1920
		(-5.84%)	(4.53%)	(2.94%)	(-4.15%)	(-4.15%)	(3.66%)	(-12.10%)
13	2.2255	0.0123	0.1504	0.8921	1.4423	0.4318	0.9810	0.4837
		(4.65%)	(5.08%)	(2.35%)	(-4.15%)	(-4.15%)	(3.66%)	(-5.37%)
14	2.7182	0.0025	0.1900	0.8759	1.7616	0.5274	1.1982	0.7761
		(25.96%)	(5.51%)	(1.89%)	(-4.15%)	(-4.15%)	(3.66%)	(-3.51%)
15	3.3201	0.0003	0.2383	0.8630	2.1517	0.6442	1.4634	1.0693
		(85.14%)	(5.84%)	(1.53%)	(-4.15%)	(-4.15%)	(3.66%)	(-2.63%)

Table 10: Second (final) steady-state with presence of idiosyncratic shocks in the human capital and with the government support for fertility that is financed with the tax on capital income.
Note: for each of the individual, average and aggregate variable, the values inside of the parentheses indicate the percentage deviations from the initial steady-state given the government policy

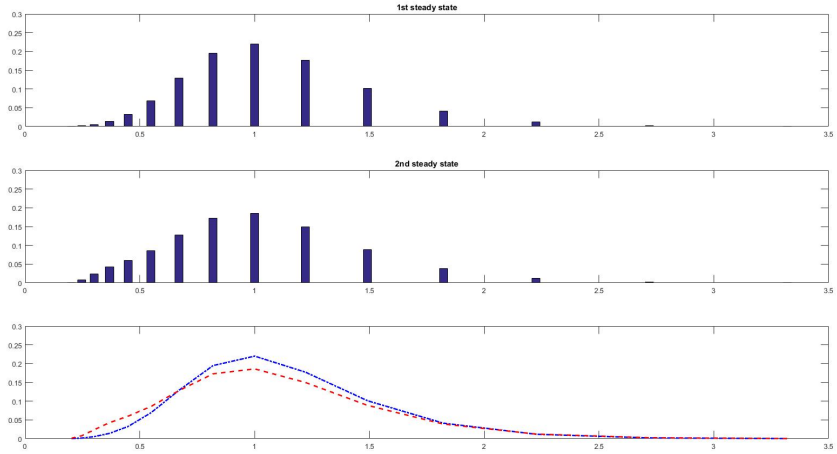

Figure 8: Comparison between distribution for the population of adult households at the initial steady-state with the distribution at the second steady-state where government supports fertility decisions and taxes capital income. Note: the (marginal) levels of the human capital are depicted on the horizontal axis; population share for each ability group is depicted on the vertical axis; blue dash-dotted line shows the distribution of adult households at the original steady-state; red dashed line illustrates the distribution of adult households at the second steady-state

With continuous government support for fertility and with the presence of idiosyncratic shocks to the human capital, we observe that the average childbearing decisions of adult households increase by 14.98% when government implements tax on consumption or capital income, and with the labour income tax, this value is 15.82% larger than at original equilibrium. Due to presence of the parental 'quality-quantity' trade-off and change in the structure of the population, however, the increase in number of children born reduces the optimal choice of the adult households for education provision. Therefore, with tax on consumption and capital income, the average education attainment at the second steady-state falls by 8.80%, and with the labour income tax in place the level of average education received decreases by 9.57%. This larger rate of response for the case of the labour income tax can be explained by the distorting nature of this tax option on the optimal choice for education provision, which according to the previous analytical conclusions results is creating further disincentive for the choice of education that leads to a larger increase in the average level of fertility. Therefore, our results indicate that at the second steady-state with the government support

for fertility, the size of the model population is 192.87 times larger than at original equilibrium when government uses tax on consumption or capital income. With the labour income tax, the size of population is 255.78 times larger than at original equilibrium. (Please note, the population would continue to grow even after the economy reaches the second steady-state). This increase in the size of the model population, however, comes at a cost of decrease in the average level of human capital of adult households, which is 6.68% lower than at original steady-state when government uses tax on consumption or capital income, and it falls by 7.14% when tax on labour income is levied instead.

Due to these changes, the level of the effective labour force per adult household for the second steady-state is 1.58% larger than at original equilibrium when government finances its budget with tax on consumption or capital income. With the labour income tax is in place instead, this value rises by 1.73% from the original steady-state. Therefore, at the second steady-state, the effective labour force becomes less skilled, however it is (significantly) larger than at the initial equilibrium due to growth in the size of the model population. Furthermore, this change in the effective labour force is the net effect of decrease in education provision, which reduces number of teachers required and, therefore, increases participation in the effective labour force; and increase in the fertility level, which decreases the time that adult households can spend in the labour market due to associated increase in time required for taking care for the children, and increases number of teachers required to provided the education for increased population of children.

Moving forward, our results indicate 10.55% lower level of the physical capital stock per adult household for the steady-state with government support for fertility when the tax on consumption and capital income is in place. With tax on labour income, the physical capital stock per adult household falls by 12.06% instead. For our policy experiments with the full depreciation of the physical capital stock, the level for this second factor input is fully determined by the average level of savings of the adult households (dynamics for which is discussed below).

With increase in the effective labour force per adult household and decrease in the physical capital stock per adult household, the marginal productivity for the former factor input is decreasing, whereas the marginal productivity of a latter factor input is rising. Since the factor price for each of these factors of production is equivalent to their marginal products, the real wage rate at the second steady-state is 4.15% below the original equilibrium when government uses tax on consumption or capital income, and it is 4.74% lower than original steady-state when labour income tax is in use. The real interest rate at the second steady-state is 12.01% above original

equilibrium when tax on consumption or capital income is levied, and with the labour income tax the real interest rate rises 13.83% above original value.

With each tax option in place combined with lower level of the average human capital, lower real wage rate and higher real interest rate, our results suggest that the average level of consumption at the second steady-state becomes 11.97%, 12.06% and 10.55% lower than original steady-state when tax on consumption, labour income or capital income is in place, respectively. The average savings of the adult households become 10.55% lower than at original steady-state if government finances subsidy for children program with tax on consumption or capital income, and with the labour income tax in use, the average savings of adult households at the second steady-state are 12.06% below the original steady-state. Finally, the average consumption of the elderly households at the second steady-state is 3.0937%, 3.0901% and 3.2584% below original equilibrium when government places tax on consumption, labour income or capital income, respectively.

As a result of these changes, the level of welfare for the second steady-state is 21.23% lower than at original steady-state with absence of policy intervention when government taxes consumption. For the case which considers utilisation of the labour income tax, the level of welfare at the second steady-state reduces by 22.84% from original equilibrium, and with capital income tax it falls by 20.66%. Finally, due to considerable decrease in the physical capital stock per adult household and moderate increase in the effective labour force per adult household, the real output per adult household becomes 2.63% below original equilibrium when government uses tax on consumption or capital income, and with the labour income tax the level of real output per adult household diminishes by 3.09% when the model economy reaches second steady-state with the government support for fertility.

At the individual level, due to decrease in the average level of human capital, which results from the change in the composition of the model population as a consequence of the government support for fertility, combined with the stationary levels of the marginal values of human capital that define each ability group, the value for the relative human capital becomes larger for all ability groups at the second steady-state. Similarly to our earlier discussion, the increase in the relative human capital results in a dampening of the responses of adult households to change in the environment of the economy, which produces not fully anticipated results where the government support for childbearing decisions leads to simultaneous increase in fertility and education provision of the higher ability households who belong to ability groups from nine to fifteen (i.e. $i \in [9, 15]$). For the households from ability groups from four to eight (i.e. $i \in [4, 8]$), the conclusions are more in

line with the predictions of the deterministic version of analysis, where subsidy for fertility leads to increase in childbearing decisions of the households, which due to parental 'quality-quantity' trade-off results in pronounced decrease in optimal choice for education. Finally, our results indicate that the lowest three ability groups (i.e. $i \in [1, 3]$) do not change their education choice and still provide zero education, however their fertility decisions increase considerably. It must be stressed, however, that these results are true for the ability groups of the households but not necessary for the individual households. Individual adult households (with exception for those who are the part of ability groups that provide their children with zero education) who receives subsidy for fertility increases their optimal choice for children and reduces the education provision for their children. As a result, decrease in the education provision combined with idiosyncratic shocks to the human capital results in the change of composition for the model population of the adult households that becomes more scattered across lower ability groups for the second steady-state.

For the consumption and savings of the adult households at the individual ability level, we observe that the outcome for the second steady-state closely follow dynamics of the real wage considered earlier, given each of the tax options in place. On the other hand, due to increase in the real interest rate, the consumption of the elderly households at the individual ability level is above the original equilibrium.

Given these changes, however, the level of utility at the second steady-state decreases below original equilibrium for households in ability groups from five to fifteen (i.e. $i \in [5, 15]$) in the case of tax being levied on consumption or capital income. with the labour income tax, we additionally observe that the level of utility for a third ability group decreases below original steady-state as well. Absence or minority of change in education attainment and fertility decisions combined with pronounce increase in the consumption of the elderly households, results in increase in the level of fertility for the lowest four ability groups (i.e. $i \in [1, 4]$) when economy reaches the second steady-state, however, and the fertility subsidy program is financed with tax on consumption or capital income. with the labour income tax, the individual utility levels increase only for the first, second and fourth ability groups, however. This difference in the individual utility responses, and generally lower levels of individual utilities with the labour income tax in place, is attributed to the distorting nature that this tax has on the education and consumption choices. Finally, based on the visual examination of the distribution of the human capital across the model population at the second steady-state with government support for fertility (which becomes relatively wider), it may be suggested that economy experiences increase in the level

of inequality in the distribution of human capital across the members of the population.

5.3.2 Transition to the second (final) steady-state

Continuing with our discussion for the government support of fertility in the environment with uncertainty in the formation of human capital, we analyse the transition path that economy takes from the original steady-state to the second steady-states presented in the previous section. Figure 9 below depict the evolution of the model economy with tax on consumption, labour income and capital income in place, from the perspective of the average household.

According to our results, in the first period of government support for the childbearing decisions, the government levies the tax rate on consumption of 0.47% to keep its budget balanced. In the case when government uses the labour income tax, the rate is set at 0.56% to provide the subsidy rate for fertility of ten percent. Finally, if capital income tax is implemented instead, the tax rate is equal to 0.92% at the first period of government presence.

In the second period of analysis with the government support for fertility, the average level of childbearing decisions has been found to increase by 10.29% with the tax on consumption or capital income financing government budget. When government levies tax on labour income instead, the average level of fertility is increasing 11.68% above original steady-state. This increase in the average level of fertility takes place for few reasons. First, in the first period of the government presence, with absence of change in the composition of the model population, provision of subsidy for fertility reduces the cost of having children across all ability groups. And, second, from the results for the deterministic version of analysis (presented by (18) below), subsidy for children provides disincentives for the education attainment, which with the presence of the 'quality-quantity' trade-off of parents for their children promotes fertility further, which further reduces optimal choice for education. According to (19), however, this effect is non-linear and households with lower human capital compared to the rest of the population would experience larger decrease in the choice for education, which according to the results from our simulations translates to a larger increase in fertility for them.

From (15)

$$\frac{\partial e_t^i}{\partial sub_t^n} \approx -\frac{\eta \bar{e}_t}{(1-\eta)(1-sub_t^e)} \tag{18}$$

and

$$\frac{\partial^2 e_t^i}{\partial sub_t^n \partial x_t^i} \approx -\frac{\eta \frac{\partial \bar{e}_t}{\partial x_t^i}}{(1-\eta)(1-sub_t^e)} \tag{19}$$

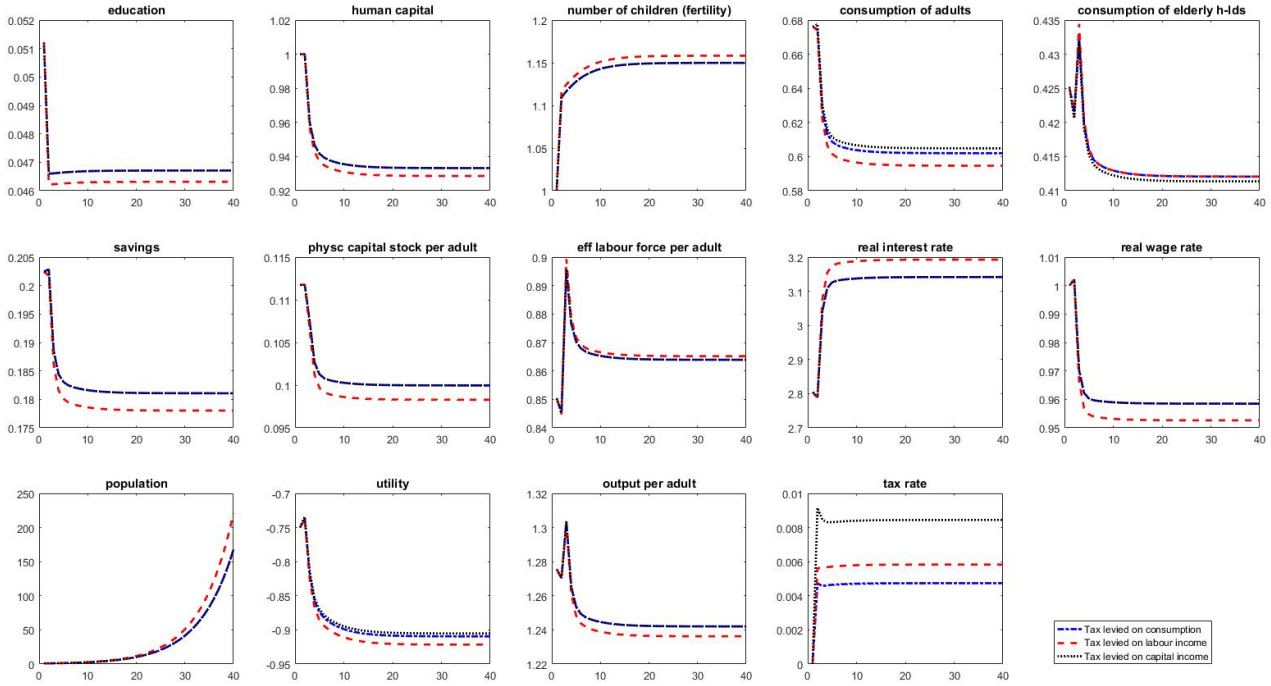

Figure 9: Transition path of the model economy with idiosyncratic shocks to the human capital from the perspective of the average household. Government provides subsidy of ten percent for childbearing decisions.
Note: transition path with tax on consumption is depicted by blue dash-dotted line; transition path with tax on labour income is presented by red dashed line; transition path with capital income tax is shown by black dotted line

where $\frac{\partial \bar{e}_t}{\partial x_t^i} > 0$ and absolute value of (21) is larger for ability groups with lower (relative) human capital and smaller for ability groups with larger (relative) human capital. Therefore, according to our results, the education provision of fourth, fifth, ninth and fourteenth ability groups falls by 100%, 47.08%, 9.10% and 2.59% when government uses tax on consumption or capital income, respectively. As a result, the fertility indicators for these ability groups increase by 36.85%, 27.19%, 7.97% and 2.49%, which raises average level of fertility. The average level of education, however, falls by 9.01% in the first period of government support when tax on consumption or capital income is in place. With use of the labour income tax, the education decisions of fourth, fifth, ninth and fourteenth ability groups decline by 100%, 48.60%, 9.85% and 3.20%, which increases fertility choices of these groups by 37.55%, 28.32%, 8.69% and 3.10%, respectively. Consequently, the average level of fertility increases but the average education attainment falls by 9.75%. As before, according to the results from the deterministic version of analysis, labour income tax produces direct negative incentives for the parental choice for education, which results in a larger increase in fertility when childbearing decisions are subsidised. Additionally, similar to the case of subsidy for education, the households from the lower three ability groups still optimally choose zero level of education for their children. However, the fertility decisions for these three groups now change and they found to increase substantially.

With increase in fertility and decrease in education provision, we observe 0.62% decrease in the effective labour force per adult household when tax on consumption or capital income is in place. With labour income tax, the value of effective labour force per adult household drops by 0.67% at the first period of government presence. Since the effective labour force is the product of the population of adult households, human capital and time that adult households dedicate to the labour force net of share of the households who are required for education provision and are not utilised by the production sector as factor input, the increase in the fertility has two channels through which it affects the effective labour force. First, due to increase in fertility adult households have to spend more time for taking care for children which reduces the time they can spend for the labour market participation. And second, with increase in the number of children born the number of teacher should follow in order to provide the optimally chosen level of education. Decrease in optimal decision for education, however, impacts the size of the effective labour force as well by decreasing the number of teachers required.

Given the full depreciation of the physical capital stock, the level of this second factor input is completely determined by the aggregate savings of the

past generation. Therefore, for the second period of analysis, the level of physical capital stock per adult household remains unchanged.

With both of these factor inputs being paid their corresponding marginal products, decrease in the size of effective labour force per adult household and absence of change in the physical capital stock per adult household raises the marginal productivity of the effective labour force and decreases marginal productivity of the physical capital. Therefore, the real wage is found to be increasing by 0.21%, whereas, the real interest rate decreases by 0.56% at the first period of government presence when tax on consumption or capital income is implemented. With tax on labour income the real wage rate rises by 0.22% and real interest rate reduces by 0.61%. Consequently, with the tax on consumption in place, the consumption of adult households from every ability group is decreasing by 0.26% whereas the consumption of elderly households is decreasing by 0.88%. Savings of the adult households increase by 0.21% however. With the labour income tax in use, consumption and savings of adult households are found to diminish by 0.34%. The consumption of the elderly households decreases by 1.01% across all ability groups. Finally, with capital income tax, the consumption and savings of adult households increase by 0.21% at the first period of government presence, but the consumption for the elderly households diminishes by 1.08%.

As a result of these changes that take place at the second period of analysis, however, the level of welfare in the model economy increases by 1.60%, 1.61% and 2.15% when government levies tax on consumption, labour income or capital income to finance the program of support for fertility, respectively. At the individual level, however, due to non-linear responses of individual fertility decisions and decisions for education provision to fertility subsidy, the level of utility for lower ability households is found to increase more than for higher ability groups. Moreover, for the top three ability groups[11] the level of utility is declining at the first period of government presence when tax on consumption or labour income finance the government budget. According to our results, the utility of the households from the fourth and ninth ability groups is increasing by 2.4743% and 1.2962% when government taxes consumption, respectively. With the labour income tax, the utility level for these two ability groups is increasing by 2.4648% and 1.3065%, and with the tax on capital income these values rise by 2.6732% and 1.9233%. For the fourteenth ability group, however, the utility falls by 0.1098% and 0.1159% with tax levied on consumption and labour income, respectively, but with the tax on capital income the level of utility for this group increases 0.4003% above original steady-state. Finally, with a reduction in size of effective

[11] I.e. $i \in [13, 15]$

labour force and absence of change in the physical capital stock, our results suggest that at the first period of the government support for fertility in the environment with uncertainty in formation of human capital the amount of real output per adult household declines by 0.41% when government uses tax on consumption or capital income, and it falls by 0.45% when the labour income tax is in use.

Due to the government support for fertility and reaction of the previous generation of adult households to this policy in form of increase in childbearing decisions and decrease in the education provision, we observe a change in the structure of the model population of the adult households for the third period of analysis. The population share of adult households of the lower ability groups increases, whereas the population share of the higher ability ones declines. As a result, the average level of human capital for adult households declines by 3.99% at the second period of government presence when government uses tax on consumption or capital income to finance its budget. With the labour income tax the average level of human capital for adult households diminishes by 4.31%. The population size of the adult households increases, however, by 10.92% when tax on consumption or capital income is in use, and with the labour income tax the population size of adult households increases by 11.68%. With these changes, when the government uses tax on consumption to provide the subsidy rate for fertility of ten percent and to keep the balanced budget, the tax rate is set at 0.46% at the third period of analysis. For the case with tax on capital income income, the tax rate is re-adjusted to 0.85%, whereas, for scenario with the labour income tax, the tax rate is equal to 0.56%.

Given these changes in the composition of adult households and in the tax rates, however, with the continuous government support for childbearing decisions, our results indicate that the average level of fertility increases by 0.66% and becomes 11.66% above original equilibrium when government uses tax on consumption or capital income. With the labour income tax, the average level of fertility rises 0.68% and is 12.44% above the initial steady-state. For the individual ability groups, however, we observe the results that are not fully anticipated. For instance, for the fifth, ninth and fourteenth ability groups the childbearing decisions reduce by 3.71%, 1.12% and 0.35%, and become 22.47%, 6.77% and 2.13% above original equilibrium when government implements tax on consumption or capital income, respectively. With the labour income tax, the fertility decisions for these three ability groups fall by 4.01%, 1.20% and 0.38%, and become 23.17%, 7.38% and 2.71% above original steady-state. Furthermore, with decline in decisions of adult households for having children (relative to the previous period of analysis) and with the

presence of parental 'quality-quantity' trade-off for children, the individual decisions for education within ability groups begin to increase at the second period of government presence. For example, with the tax levied on consumption or capital income, the optimal choice for education of fifth, ninth and fourteenth ability groups increase by 26.73%, 6.70% and 4.83%, and become 32.93% and 3.01% below original steady-state for fifth and ninth, whereas for fourteenth the individual education provision increases 2.12% above original steady-state. With the labour income tax, the choice for education of these three groups increases by 29.60%, 7.26% and 5.23% from the previous generation, and becomes 33.39% and 3.30% below original steady-state for the fifth and ninth ability groups, respectively. For the fourteenth ability group, the education provision is 1.86% above original steady-state at the second period of government presence. Finally, at the average level, the education attainment increases by 0.02% and becomes 8.99% below original equilibrium when government uses tax on consumption or capital income. With the labour income tax, the average education provision rises by 0.01%, and is 9.74% lower than initial steady-state.

This increase in education attainment (for all groups with exception for the lowest three ones) combined with decrease in fertility decisions among individual ability groups while receiving subsidies for fertility can be explained by the fixed nature of the marginal levels of the human capital that is required for the process of extrapolation and interpolation. With fixed marginal levels of human capital corresponding to each ability group, the government support for fertility reduces average level of human capital which in turn increases the relative human capital. As we have indicated previously, the change in relative human capital produces equal-directional change in education attainment which in turn results in the change in fertility decisions in the opposite direction of relative human capital and of education choices. The increase in education attainments and decrease in fertility decisions at the level of individual ability groups does not, however, result in the same dynamics for the each individual adult households. According to our results, the past period decrease in education provision across all ability groups results in lower endowment of human capital for adult households (especially for ones who have faced negative shocks during human capital accumulation in a childhood), which forces these individuals to move to lower ability groups which are characterised by having a lower provision of education to their children but by having larger number of children. The net of these changes for the ability groups and for the composition of adult households results in the reported changes for the average values of education and fertility for the third period of analysis.

With increase in the population of adult households, our result suggest

an increase in the effective labour force per adult household by 5.99% when government uses tax on consumption or capital income, and with the labour income tax in place the value of the effective labour force per adult household increases by 6.46%. Therefore, at the second period of the government presence the effective labour force rises 5.33% above original steady-state with tax levied on consumption or capital income, and it grows to be 5.75% above original equilibrium with the government taxing the labour income. Next, with the past period increase in the aggregate amount of savings with the tax levied either on consumption or capital income, the physical capital stock value increases when either of these two tax options is in place. With the labour income tax, however, we observed a decrease in savings of the past generation of adult households, which, therefore, reduced the value of the physical capital stock at the third period of analysis. However, with increase in the population size at the second period of government presence, we observe a decrease in the per adult household value of the physical capital stock under all three tax options. Our results indicate that with the tax imposed on consumption or capital income, the value of the physical capital stock per adult household falls 3.79% below original steady-state, whereas with the labour income tax in place, this indicator reduces 4.63% below initial equilibrium. With decrease in the physical capital stock per adult household accompanied with increase in effective labour force per adult household, the real wage rate falls by 3.17% and becomes 2.97% below the first steady-state when government uses tax on consumption or capital income, and with the labour income tax the real wage rate reduces by 3.60% and is 3.39% below the original equilibrium. The real interest rate, however, rises by 9.06% with tax on consumption or capital income present and it becomes 8.44% larger than the first equilibrium, whereas it increases by 10.34% and is 9.67% above the original steady-state.

As a result of decrease in the real wage rate and adjustment in the tax rates, the consumptions of adult households among different ability groups is decreasing by 3.16% and becomes 3.42% below originals steady-state when government taxes consumption. With the tax imposed on the labour income, the consumption of adult households among different ability groups reduces by 3.61% and is 3.93% below first equilibrium. And, finally, with the tax on capital income this value falls by 3.17% and becomes 2.97% lower than original steady-state. The savings of adult households among different ability groups reduce by the same amount as consumption when government implements tax on labour income or on capital income. With the tax on consumption, however, the savings of adult households between different ability groups fall by 3.17% and is 2.97% lower than the first steady-state. Lastly, with the past period increase in the real wage rate and present period increase

in the real interest rate, the consumption of the elderly households between different ability groups increases by 6.90%, 7.85% and 6.93% that becomes 5.94%, 6.74% and 5.75% above the initial steady-state with tax on consumption, labour income and capital income in place, respectively. At the average level, however, the amount by which the consumption and savings of adult households plummets is larger and the rate at which the consumption of the elderly households increases is smaller. This is the case because the composition of adult households has changed where households from higher ability groups become a part of a lower ability groups with lower consumption and savings. Therefore the average level of consumption of adult households falls by 7.03%, 7.56% and 7.04%, that becomes 7.27%, 8.07% and 6.85% when the tax on consumption, labour income and capital income finance the government budget, respectively. For the tax option levied on labour income and capital income, the change in average level of saving is identical to the change in the average level of consumption, whereas the average savings of the adult households with the tax levied on consumption reduce by the same amount as in the case of taxing a capital income. Finally, the average level of consumption for the elderly households is found to increase by 2.64%, 3.20% and 2.66%, which is 1.73%, 2.17% and 1.54% above the initial steady-state when government levies tax on consumption, labour income and capital income, respectively.

As a concluding point of observation for the third period of analysis with the government support for fertility in the environment with uncertainty in the human capital formation process, the level of welfare decreases by 10.85%, 11.79% and 10.92% that becomes 9.07%, 9.98% and 8.54% lower than the original steady-state when tax on consumption, labour income and capital income is in place, respectively. Lastly, our results indicate the increase in the level of output produced per adult household by 2.62% that is 2.20% larger than at first steady-state when government implements tax on consumption or capital income. With the labour income tax, the per adult household value of the real output increases by 2.63% and is 2.17% larger than before government presence.

With further change in the composition of population due to variation in fertility decisions of past generations and in the optimal education provision which further alters the endowment of the human capital, our results indicate the decreases in the average level of human capital by 1.39% which becomes 5.32% lower than initial steady-state when government uses tax on consumption or capital income. With the labour income tax the average level of human capital of adult households at the third period of government presence reduces by 1.49% and is 5.73% lower than original equilibrium. The

population size of adult households, increase, however, further by 11.66% when government uses tax on consumption or capital income, whereas with the labour income tax it rises by 12.44%, which therefore is 23.86% and 25.58% larger than at original steady-state, respectively. Finally, in order to balance its budget and continue to provide the subsidy for fertility of ten percent, the government sets the tax rate on consumption to 0.46%, on labour income to 0.57% or on capital income to 0.83% at the fourth period of analysis.

Given a continued decrease in the average level of human capital and fixed values for the marginal levels of human capital for each ability group, the values for the relative human capital continue to rise. With increase in the relative human capital for the ability groups, the decisions for education attainment within these groups continue to increase but the fertility choices continue to fall. Our results indicate that the education attainment by fifth, ninth and fourteenth ability groups increase by 7.42%, 2.21% and 1.62% when government uses tax on consumption or capital income to finance its budget and provide subsidy rate for fertility of ten percent. The education attainment of these three ability groups becomes 27.96% and 0.87% below original steady-state for fifth and ninth group, whereas for fourteenth – it is 3.78% above original equilibrium. With the labour income tax, the choices for education of these three groups increase by 8.00%, 2.38% and 1.74%, respectively, which is 28.06% and 1.00% below original steady-state for the fifth and ninth ones, and it is 3.64% larger than original equilibrium for the fourteenth ability group. (Due to the large fertility and low relative level of human capital, however, the lowest three ability groups still provide their children with zero education). The fertility choices at the third period of the government presence for the fifth, ninth and fourteenth groups decrease by 1.22%, 0.37% and 0.12%, respectively, when government implements tax on consumption or capital income, and with the labour income tax, these values decrease by 1.30%, 0.39% and 0.12% that become 20.98%, 6.37% and 2.01% above original steady-state when government levies tax on consumption or capital income, when with the labour income tax the fertility decisions for these three groups are 21.57%, 6.95% and 2.58% larger than at original equilibrium. (The change in childbearing decisions shown for the three representative groups above is consistent with change across all ability groups including the lowest three ones). At the individual level, however, due to continued change in the composition of the model population, the childbearing decisions continue to rise with the government support for fertility, which is depicted by the dynamics in the average level of fertility. According to our results, the average level of fertility increases by 0.5453% and becomes 12.27% above original steady-state when tax on consumption or labour income is in

place, and with the labour income tax this value increases by 0.5493% and is 13.05% larger than at the steady-state without government presence. The average level of education still continues to be below the initial steady-state; however our results indicate moderate increase in this indicator. Based on the outcomes of the simulations, the average level of education increases by 0.0381% and becomes 8.96% below the first steady-state when government uses tax on consumption or capital income, and it increases by 0.0341% and is 9.71% lower than initial equilibrium when government implements labour income tax.

As a result of these alterations, the value of the effective labour force per adult household decreases by 2.13% and is 3.09% above original steady-state when tax on consumption or capital income is in place, and it decreases by 2.30% and becomes 3.32% larger than initial equilibrium when tax on labour income is utilised. The per adult household value of the physical capital stock decreases as well at the fourth period of simulation, and it falls by 4.52% and is 8.14% lower than at original steady-state when government budget is financed with tax on consumption or capital income, while with the labour income tax it reduces 5.04% and becomes 9.44% below initial steady-state. (The decrease in the effective labour force per adult household is primarily driven by decrease in the endowment of human capital which reduces the productivity of the labour, increase in the fertility levels which reduces the time available for parents to participate in the labour force and increases the number of teachers required to provide the optimally chosen level of education, and increase in the size of population which spreads effective labour force more thinly; whereas, the decrease in the value of the physical capital stock per adult household is explained by decrease in the decisions to save of the past generation of adult households, and, again, increase in the present population size). With decrease in the physical capital stock being more pronounced than in the effective labour force, the marginal productivity of the former is found to be increasing while for the latter it is declining. Therefore, with the tax levied on consumption or capital income, the real interest rate increases 2.21% from the previous generation and it becomes 10.84% larger than at original steady-state. The real wage rate, however, declines by 0.82% and is 3.77% lower than initial steady-state. With the labour income tax, the real interest rate increases by 2.54% and real wage rate falls by 0.95%, and they become 12.46% above and 4.30% below the initial steady-state, respectively.

Given these changes, the consumption of adult households from every ability group diminishes by 0.83%, 0.95% and 0.82%, that becomes 4.21%, 4.84% and 3.77% lower than at original steady-state when government finances subsidy for fertility with the tax on consumption, labour income and

capital income, respectively. The reduction in savings within every ability group is equivalent to the reduction in consumption when government uses labour income tax or tax on capital income, whereas with the tax on consumption savings of adult households decrease by 0.82% and are 3.77% below original equilibrium. The consumption of the elderly households for different ability groups is found to be falling too, which is the net result of considerable decrease in the previous period real wage rate, increase in the present real interest rate and adjustment in the tax rates. Therefore, this indicator decreases by 1.57%, 1.76% and 1.56% from the previous period which becomes 4.30%, 4.89% and 4.12% above initial steady-state with tax levied on consumption, labour income and capital income, respectively. Due to change in composition of the model population, however, our results indicate more pronounced decrease in the average consumption of adult and elderly households and average level of savings. This larger decrease at the average level compared to reduction experienced by each ability groups indicates that at the individual level (i.e. for each individual household) the decrease in these three variables is larger than it is for the ability groups. Based on our simulation, the average consumption and savings of adult households decreases by 2.4266% and 2.1984%, and become 10.30% and 8.89% below original equilibrium when government levies tax on labour income and capital income, respectively. With the tax on consumable goods, the average consumption of adult households reduces by 2.2024% while the average savings fall by 2.1984%, and they become 9.31% and 8.89% below initial steady-state, respectively. Lastly, the average consumption of the elderly households decreases by 2.9347%, 3.2217% and 2.9241% and which is 1.2545%, 1.1249% and 1.4230% below the first steady-state when the government budget is financed with tax on consumption, labour income and capital income, respectively.

Lastly, our results indicate that at the third period of the government presence with the government support for fertility and with uncertainty that takes place in the human capital accumulation process, the level of welfare decreases by 4.85%, 5.22% and 4.87% and it is 14.37%, 15.73% and 13.82% below original steady-state when tax on consumption, labour income and capital income finance the government budget, respectively. Furthermore, when government uses tax on consumption or capital income the level of output per adult household is found to fall by 2.93% which becomes 0.80% lower than initial steady-state. With the labour tax, the output per adult household decreases by 3.22% and is 1.12% lower than original equilibrium.

Overall, the dynamics expresses for the fourth period of the analysis continues until the model economy reaches the second steady-state which has

been described in the previous section.

5.4 Inequality

In our discussion, where comparison between the original steady-state and potential second steady-states under different government policy options has been drawn, we have proposed that the level of inequality in the distribution of the human capital could be increasing with the subsidy for fertility and reducing with subsidy for education program based on the visual examination of the distribution functions. To address these propositions, we calculate the measures of inequality for the distributions of the human capital across model population which are reported in table 11 below.

steady state	Gini coefficient	Coefficient of variation	Mean absolute deviation	Kuznets ratios $\leqslant 40\%$	Kuznets ratios $\geqslant 60\%$	The range
original	0.2018	0.3724	0.2818	0.1230	0.3332	3.1190
$sub^e + \tau^c$	0.1968↓	0.3637↓	0.2864↑	0.0764↓	0.4299↑	2.8474↓
$sub^e + \tau^l$	0.1971↓	0.3640↓	0.2866↑	0.0786↓	0.4241↑	2.8622↓
$sub^e + \tau^k$	0.1968↓	0.3637↓	0.2864↑	0.0764↓	0.4299↑	2.8474↓
$sub^n + \tau^c$	0.2389↑	0.4376↑	0.3442↑	0.2209↑	0.2924↓	3.3402↑
$sub^n + \tau^l$	0.2391↑	0.4383↑	0.3449↑	0.2242↑	0.2881↓	3.3580↑
$sub^n + \tau^k$	0.2389↑	0.4376↑	0.3442↑	0.2209↑	0.2924↓	3.3402↑

Table 11: Indicators for inequality in distribution of the human capital for the model population of the original steady-state without government presence and of the second steady-state with various policy options.
Note: ↑ indicates that the level of inequality in distribution of the human capital across the population increases at the second steady-state; ↓ indicates that the level of inequality in distribution of the human capital across the population reduces at the second steady-state

Adopting these inequality measures presented in Ray (1998) to the distribution of human capital, the range is calculated as the difference between the human capital of the top ability group and the bottom one divided by the average level of human capital. The Kuznets ratios show the share of the human capital that belongs to the bottom and to the top 40% of the population. The mean absolute deviation is calculated as the total sum of absolute values for the distances between the average level of human capital and the marginal human capital levels for the all ability groups. The co-

efficient of variation indicates the standard deviation in the distribution of human capital across population, and, finally, the Gini coefficient is obtained by calculating the total absolute differences in the level of human capital between all pairs of ability groups that our model population consists of.

Based on the results presented in the table 11, we, however, cannot form unambiguous conclusion for the effect that discussed subsidy programs would have on the level of inequality in the distribution of the human capital, and therefore, we are not able to either accept or reject hypotheses that we have formed previously about the change in inequality level as economy reaches different steady-states based upon visual examination of distribution functions. According to table 11, results for Gini coefficient, coefficient of variation and the range, however, are in line with our earlier predictions, and they suggest that the economy at its final steady-state would have smallest level of inequality when there is absence of policy instruments that provides disincentive for education attainment, while with the subsidy for fertility and/or with the presence of labour income tax, the level of inequality in distribution of human capital is larger than under alternative cases considered. Furthermore, according to these three inequality measures, the level of inequality in the distribution of human capital at the original steady state appears to be smaller than in the case of subsidy for fertility independently of the tax option in place, but it is larger than with the subsidy for education. These results, however, are challenged by the results for the mean absolute deviation and Kuznets ratios. Based on the mean absolute deviation, the original steady-state has the lowest level of inequality in the distribution of the human capital, while with the government presence, the level of inequality increases, which, however, is larger if the government policy in question creates disincentive for education attainment. Finally, based on the Kuznets ratios that we utilise, there is absence of a unified response for inequality in the distribution of the human capital to the considered government policies. According to the Kuznets ratio that corresponds to the bottom 40% of the population, the conclusions for the change in the inequality remain consistent with Gini coefficient, coefficient of variation and the range. For the Kuznets ratio that illustrates the change in the top 40% of the population, predictions are complete opposite this ratio increases with subsidy for education and falls with subsidy for fertility. This dynamics correlates with change in the average level of human capital directly, which suggest that with subsidy for education being financed through the tax with non-distorting properties for individual choices for education, population of the top ability household increases most. With subsidy for fertility, however, increasing part of the population becomes of lower ability groups, and population share of the top 40% of ability groups diminishes as a result.

6 Conclusion

In this paper we have studied the long-run impacts of the government support for education and fertility in the environment with uncertainty in the human capital formation. We utilised the model economy of *de la* Croix and Doepke (2003) to formalise the behaviour of households and producer. We extended this model economy with the government sector, which provides households with subsidies for education and fertility, and collects taxes on consumption, labour income and capital income. Furthermore, we considered that households receive idiosyncratic shocks when the human capital accumulates. For our discussion, we focused on comparison between initial steady-state with the presence of this uncertainty and without government, with the steady-states that resulted from the government policies. Additionally, we performed the analysis for the transition paths that the model economy has taken under different scenarios that we have considered, and our discussion included the analysis for the variations that take both at individual, average and aggregate levels.

7 Appendix

7.1 A1: Analytical solution for the problem of the households for the deterministic environment

$$\max_{c_t^i, s_t^i, e_t^i, n_t^i} u_t^i = \ln c_t^i + \beta \ln d_{t+1}^i + \gamma \ln n_t^i h_{t+1}^i$$

$$(1+\tau_t^c)c_t^i + s_t^i + e_t^i n_t^i w_t \bar{h}_t = (1-\tau_t^l) w_t h_t^i (1-\phi n_t^i) + n_t^i w_t \bar{h}_t (e_t^i sub_t^e + \bar{e}_t sub_t^n)$$

$$(1+\tau_{t+1}^c) d_{t+1}^i = \frac{1}{(1+\rho)} \big(1 + r_{t+1}(1-\tau_{t+1}^k)\big) s_t^i$$

$$h_{t+1}^i = \frac{1}{(1+\rho)} B(\theta + e_t^i)^\eta (h_t^i)^\pi \bar{h}_t^\kappa$$

First Order Conditions:

$$\frac{\partial \mathcal{L}_t^i}{\partial c_t^i} = \frac{1}{c_t^i} - \lambda_t^i (1+\tau_t^c) = 0$$

$$\frac{\partial \mathcal{L}_t^i}{\partial s_t^i} = \beta \frac{1}{s_t^i} - \lambda_t^i = 0$$

$$\frac{\partial \mathcal{L}_t^i}{\partial e_t^i} = \gamma\eta \frac{1}{(\theta + e_t^i)} + \lambda_t^i n_t^i w_t \bar{h}_t sub_t^e - \lambda_t^i n_t^i w_t \bar{h}_t = 0$$

$$\frac{\partial \mathcal{L}_t^i}{\partial n_t^i} = \gamma \frac{1}{n_t^i} - \lambda_t^i (1-\tau_t^l) w_t h_t^i \phi + \lambda_t^i w_t \bar{h}_t (e_t sub_t^e + \bar{e}_t sub_t^n) - \lambda_t^i e_t^i w_t \bar{h}_t = 0$$

where λ is Lagrange multiplier
Optimal decisions of the households:

$$e_t^i = \frac{\eta\phi(1-\tau_t^l)x_t^i - \theta(1-sub_t^e) - \eta \bar{e}_t sub_t^n}{(1-sub_t^e)(1-\eta)}$$

$$c_t^i = \frac{(1-\tau_t^l)}{(1+\tau_t^c)} \frac{1}{(1+\beta+\gamma)} w_t h_t^i$$

$$s_t^i = (1-\tau_t^l) \frac{\beta}{(1+\beta+\gamma)} w_t h_t^i$$

$$d_{t+1}^i = \frac{1}{(1+\rho)} \frac{(1-\tau_t^l)}{(1+\tau_{t+1}^c)} \frac{\beta}{(1+\beta+\gamma)} \big(1 + r_{t+1}(1-\tau_{t+1}^k)\big) w_t h_t^i$$

$$n_t^i = \frac{\gamma}{(1+\beta+\gamma)} x_t^i \Big[(1-\tau_t^l)\phi x_t^i + e_t^i (1-sub_t^e) - \bar{e}_t sub_t^n \Big]^{-1}$$

where x_t^i is the relative human capital and $x_t^i = h_t^i / \bar{h}_t$

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