# Supplementary Materials –

## Part 1: Technical description of the time-varying UNU-UK-EU mapping protocols for POM and WG

## Notations

Let us introduce the following notations:

* – Net imports (tonnes) for CN code in year
* – production (tonnes) PCC code in year
* – POM (tonnes) for PCC code in year
* – POM (tonnes) for UNU key in year
* – POM for UK category in year
* – POM for EU category in year
* – binary variable equal to 1 if CN code maps onto PCC code in year , and equal to 0 otherwise (WOT CN-PCC mapping)
* – binary variable equal to 1 if CN code maps onto UNU key in year , and equal to 0 otherwise (WOT CN-PCC-UNU mapping)
* – binary variable equal to 1 if CN code maps onto UK code in year , and equal to 0 otherwise (CN-UK mapping provided by REPIC and WEEE Europe)
* – binary variable equal to 1 if UNU key maps onto EU category in year , and equal to 0 otherwise (WOT UNU-EU mapping)
* – fraction of POM for PCC code mapping onto UK category in year
* – fraction of POM for UNU key mapping onto UK category in year
* – fraction of POM for UK category mapping onto EU category in year
* – fraction of POM for UK category mapping onto UNU key in year
* – fraction of POM for EU category mapping onto UK category in year
* – fraction of WG for UNU key mapping onto UK category in year
* – fraction of WG for UK category mapping onto EU category in year

The weight fractions define the intermediate PCC-UK protocol, while the weight fractions and define the required UNU-UK and UK-EU protocols for POM.

The weight fractions and define the inverse UK-UNU and EU-UK protocols for POM.

The weight fractions and define the UNU-UK and UK-EU protocols for WG.

We note that , and are not protocols since each CN code always maps onto single PCC, UNU and UK categories, respectively. Instead, they are binary variables (not fractions/percentages) defining regular mappings of CN codes onto PCC, UNU and UK categories, respectively (labels only, not weights). Likewise, is also a binary variable, defining the UNU-EU mapping.

Net imports are known from Eurostat data for trade (CN level), while production is known from PRODCOM data for production (PCC level). Combining the two, and using the CN-PCC mapping defined by the binary variables, we get POM on the PCC level:

Equation 1

The summation is performed over all CN codes. This step is possible because of the assumed uniqueness of mapping from CN to PCC, which is supported by Eurostat product code conversion tables.

All the subsequent steps apart from the UNU-EU mapping require protocols. The reason for this is that from the PCC level onwards, moving towards more aggregate categories like UNU and UK often results in splits of a single PCC code (or UNU key) into multiple aggregate categories. This is contrary to the expectation that the codes on a more detailed level should merge into a single aggregate category. The only exception is the UNU-EU mapping which links each UNU key with a single EU category.

In order to create a protocol, a more detailed set of product categories that underpins the mapping between the two categories linked by the protocol is required. For the PCC-UK protocol the underlying detailed categories are CN codes, while for the UNU-UK protocol PCC codes could be used as the link. The UK-EU protocol mapping relies on the UNU codes.

## PCC-UK protocol for POM

Before we can define the PCC-UK protocol, we need to know which PCC codes map on which UK categories. This mapping is defined by the following binary variable, which is based on the underlying CN codes:

For example, if a PCC code maps onto two UK categories and in year , we have

This PCC code consists of several CN codes with the CN-UK mapping defined by the binary variable .

The question is how best to approximate the POM for each of these CN codes in order to define the PCC-UK protocol for this particular PCC code. The section below provides a possible approximation.

## Approximation for CN-level POM

The total production for PCC code in year is . Thus, if the production corresponding to CN code that belongs to PCC code in year , which is unknown, is denoted as , the sum of these CN-level productions should add to :

*Equation 2*

Here we used the binary filter to ensure only those CN codes that belong to the given PCC code are included.

Now, the net imports for these CN codes are known and are denoted as . There are two options depending on the sign of the net imports:

* If , then it is possible that this particular CN code is not produced in the UK, i.e. and all the consumption is met through the imports.
* If , then it is possible that this particular CN code is not consumed in the UK, i.e. and all the production serves to meet he exports.

Both of these cases provide a theoretical minimum for the UK production of a given CN code. Thus, a summation of these lowest possible levels of production across all the CN codes within the given PCC code should not exceed :

*Equation 3*

If this condition does not hold, it means the CN-level data for trade is inconsistent with the PCC-level data for production, and either the trade or production figures have to be adjusted for the left hand side and right hand side of Equation 1 to match. This could either be done by reducing for the CN codes with negative net imports, or by increasing for the entire PCC code in question. The latter of the two is the easiest way of correcting the possible inconsistencies between the CN and PCC data, which we are going to adopt here.

Thus, if the condition given by Equation 3 does not hold, we restore the balance between imports, exports and productions by setting

*Equation 4*

If the condition given by Equation 3 holds and the inequality sign prevails, it means the lower-end estimate for the production on the CN level given by Equation 4 needs to be upgraded. There is no unique way of doing this based on the existing data. Therefore, we are going to use the simplest approximation that production levels for all CN codes are higher than their respective lower-end estimates in Equation 4 by the same amount , which is unique to the given PCC code where the relevant CN codes belong to:

*Equation 5*

Plugging this into gives:

Therefore, the quantity which defines the production on the CN level according to is equal to

*Equation 6*

The CN-level POM for the CN codes corresponding to PCC code is therefore given by

## Defining the weights for the PCC-UK protocol for POM

With the CN-level approximation for POM introduced above, we can define the required PCC-UK protocol that maps the POM for PCC code to UK category in year :

Here is either given by or, depending on the outcome of the test provided by:

* If = FALSE, then is given by
* If = TRUE, then is given by (and is defined in

The summations are performed over all CN codes and UK categories under consideration.

This methodology applies to all PCC codes that split into multiple UK categories, no matter how many UK categories are involved in the split.[[1]](#footnote-1) The total number of UK codes that the given PCC code maps onto in year is given by

To complete defining the protocol, those PCC codes that map onto a single UK category, i.e. those that have , need to be assigned the weight conversion factor , while for the PCC codes not mapping on any of the UK categories, , we set .

Finally, if for a given PCC code the POM in year is zero,

the corresponding PCC-UK conversion factors should also be set to zero for all , , even if some of the mapping binary variables are non-zero (i.e. PCC and UK “labels” are mapped by the weights are not).

With the PCC-UK protocol defined, the PCC-level POM can be converted into POM in UK categories as follows:

In this formula the summation is performed over all PCC codes.

All the parameters in these relationships correspond to year and are expected to be time-variable. For any year and any PCC code , however, the following normalising condition holds:

## UNU-UK protocol for POM

Now let us define the UNU-UK protocol. Using the underlying CN codes to provide the splits between the weight flows is not practical for the UNU-UK mapping since POM is not known at the CN level. We cannot possibly rely on the trade data alone (without production) to define the weight splits across as many as 50% of all the UNU keys that map onto multiple UK categories.

Instead, we propose to use the underlying PCC codes that belong both to a given UNU key and to the multiple UK categories which this key splits into. The advantage here is that POM data is available on the PCC level, and the mapping between PCC and UK codes is straightforward for over 95% of PCC codes, while the remainder of the PCC codes with the irregular UK mapping are covered by the newly defined approximate PCC-UK protocol.

Before we can define the UNU-UK protocol, we need to know which UNU keys map on which UK categories. This mapping is defined by the following binary variable:

We also need to know which PCC codes correspond to a given UNU key. For this purpose, the following binary variable is introduced:

The UNU level POM is therefore given by

The corresponding weight fraction defining the respective mapping of the POM for UNU key onto UK category in year is expressed as follows:

The summations are performed over all PCC codes and UK categories under consideration.

This methodology applies to all UNU keys that split into multiple UK categories, no matter how many UK categories are involved in the split.[[2]](#footnote-2) The total number of UK codes that the given UNU key maps onto in year is given by

To complete defining the protocol, those UNU keys that map onto a single UK category, i.e. those that have , need to be assigned the weight conversion factor , while for the UNU keys not mapping only any of the UK categories, , we set .

Finally, if for a given UNU key the POM in year is zero,

the corresponding PCC-UK conversion factors should also be set to zero for all , , even if some of the mapping binary variables are non-zero (i.e. PCC and UK “labels” are mapped by the weights are not).

With the UNU-UK protocol defined, the UNU-level POM can be converted into POM in UK categories as follows:

*Equation 7*

The summation is performed over all UNU keys.

All the parameters in these relationships correspond to year and are expected to be time-variable. For any year and any UNU key , however, the following normalising condition holds:

## UK-EU protocol for POM

The mapping between UNU keys and EU6 categories is unique in each year , and is given by the binary variable which is provided by the WOT model.

Here we use EU6+PV as a default set of the target EU categories, but exactly the same mapping procedure could be applied to other types of EU categories (EU6, EU10, EU10+PV) by altering the UNU-EU binary variables accordingly.

Before we can define the UK-EU protocol, we need to know which UK categories map on which EU categories. This mapping is defined by the following binary variable:

The corresponding weight fraction defining the respective mapping of the POM for UK category onto EU category in year is expressed as follows:

The summations are performed over all UNU keys and EU categories under consideration.

The total number of EU categories that the given UK category maps onto in year is given by

To complete defining the protocol, those UK categories that map onto a single EU category, i.e. those that have , need to be assigned the weight conversion factor , while for the UK categories not mapping only any of the EU categories, , we set .

Finally, if for a given UK category the POM in year is zero,

the corresponding UK-EU conversion factors should also be set to zero for all , , even if some of the mapping binary variables are non-zero (i.e. UK and EU “labels” are mapped by the weights are not).

With the UK-EU protocol defined, the UK-level POM, which is denoted as , can be converted into POM in EU categories as follows:

*Equation 8*

The summation is performed over all UK categories.

All the parameters in these relationships correspond to year and could be time-variable. For any year and any UK category , however, the following normalising condition holds:

Equation 8 can be used to convert the reported UK-level POM and WEEE collected data into the required pool of EU categories (either EU6, EU6+PV, EU10 or EU10+PV).

##

## Inverse UK-UNU protocol for POM

Following the logic of linking any pair of aggregate categories using their respective mapping on a more granular category, we can introduce the inverses of the UNU-UK and UK-EU mapping protocols.

The weight fraction defining the respective mapping of the POM for UK category onto UNU key in year is expressed as follows:

This is the inverse of the UNU-UK mapping protocol with the weights . Note that the underlying one-to-one mapping between PCC codes and UNU key effectively means that the relevant binary mapping labels also act as the mapping weights.

The inverse UK-UNU protocol allows one to convert the UK-level POM into UNU-level POM:

*Equation 9*

The summation is performed over all UK categories.

All the parameters in these relationships correspond to year and are expected to be time-variable. For any year and any UK category , however, the following normalising condition holds:

Combining Equation 9 with, we arrive at the following condition that makes the inverse UK-UNU mapping consistent with the original UNU-UK mapping:

## Inverse EU-UK protocol for POM

The corresponding weight fraction defining the respective mapping of the POM for EU category onto UK category in year is expressed as follows:

This is the inverse of the UK-EU mapping protocol with the weights . Note that the underlying one-to-one mapping between UNU key and EU categories effectively means that the relevant binary mapping labels also act as the mapping weights.

The inverse EU-UK protocol allows one to convert the EU-level POM into UK-level POM:

*Equation 10*

The summation is performed over all EU categories.

All the parameters in these relationships correspond to year and could be time-variable. For any year and any EU category , however, the following normalising condition holds:

Combining Equation 10 with Equation 8, we arrive at the following condition that makes the inverse EU-UK mapping consistent with the original UK-EU mapping:

## Using the UNU-UK protocol for POM to project UNU-level POM and WG from WOT model onto UK categories

To obtain an adequate estimate of WG in the UK categories, we need to apply the UNU-UK protocols for the past years when the sales took place, which are denoted here as , rather than in the current year when the WG is calculated:

*Equation 11*

Here is Weibull distribution for the UNU key corresponding to sales in year , evaluated in the current year , and the outer summation is performed over all UNU keys.

This expression is different from the more intuitive but incorrect methodology of applying the UNU-UK protocols for the current year when the WG is calculated:

The issue with the latter formula is that the POM-based protocols in the current year do not match with the actual protocols of the products that are being discarded as WEEE in this year, since these product originate from earlier years when the protocols were different. This mismatch could lead to errors in the estimated weight per each of the UK categories calculated using UNU figures.

We should, therefore, use the above to work out POM and Equation 11 to calculate WG in UK categories, based on the historic POM in the UNU keys.

## UNU-UK protocol for WG

UNU-level WG in year is expressed via the relevant POM and residence time distributions:

*Equation 12*

According to Equation 11, the fraction of this WG that feeds into the UK category is given by

Here are historic UNU-UK mapping protocols for POM.

Therefore, the UNU-UK protocol weights for WG mapping are given by

*Equation 13*

Here given by Equation 12.

It is easy to see that the WG protocol weights satisfy the condition

for all the UNU keys with non-zero POM in any of the historic years (i.e. when the first part of Equation 13 holds).

Using this protocol, one can convert UNU-level WG in year into UK category WG in the same year according to the following expression:

## UK-EU protocol for WG

Following from the definition of the UK-EU mapping protocol for POM, the weight fraction defining the respective mapping of the WG for UK category onto EU category in year is expressed as follows:

*Equation 14*

Here is WG for UNU key in year calculated from Equation 12, are the weights for the UNU-UK WG mapping protocol defined in Equation 13, and are the binary UNU-EU mapping variables introduced in the previous sections that also act as mapping weights. The summations are performed over all UNU keys and EU categories under consideration.

Using this protocol, one can convert UK-level WG in year into EU category WG in the same year according to the following expression:

*Equation 15*

This formula could be used to convert the WEEE figures that are officially reported in UK categories into EU categories.

Equation 14 can be verified by expressing the WG for EU category in year directly via UNU-level POM:

Plugging this into Equation 15, along with the UK-level WG from Equation 11 and the UK-EU WG mapping weights from Equation 14, we should arrive at the same result on both sides.

## Small mixed WEEE protocol

Small mixed WEEE (SMW) is defined as WEEE in the UK categories 2 to 10. The SMW protocol[[3]](#footnote-3) provides an estimated conversion of the total mass flow across these categories into each individual UK category (from 2 to 10), which is used to report the collected SMW without having to sort all the items into their respective categories.

The WOT results for WG allow us to estimate the SMW protocol both for the historic years and in the near-term future (based on the underlying UNU-level POM forecasts):

Here is the WG for UK category in year given by Equation 11. The protocol depends on assumptions regarding unit weights as well as residence time distributions made in the WOT model, which affect the estimates for .

## Part 2: Technical description of the prototype dynamic model for POM and WG of fridges in UK (number of units)

The main goal of this pilot study, which focuses on fridges (UNU key 108, part of UK category 12), was to make the residence time distribution for a given product respond dynamically to changing socio-economic and market conditions. This is not trivial because:

* There appears to be little information in the literature about consumers’ behaviour when it comes to replacing their EEE products, which affects the products household residence times
* In particular, correlations between the residence times and changing socio-economic and market conditions have not been studied extensively

The direction of modelling presented below provides just one of several possible ways of incorporating the required dynamic effects in a model. The assumptions on the socio-economic and market drivers made here need to be verified further before the results could be used to advice on WEEE policy.

Consider a single batch of units of a given product sold in year . Assume that at the time of the sales, the products’ subsequent residence time was set to follow Weibull distribution with parameters and specific to the year :

It means that in a subsequent year , the following percentage of all the units from the given batch is set to be discarded during a time period :

Thus, the dynamics of how the units get discarded in the future is pre-defined throughout their residence times set in the year of the sale. If is the **number of units sold** (POM) in year , the total units of WG in year , , is given by adding the POM weighted by the respective residence time distributions over all the preceding years (going at least as far back as 1980):

Here we noted that a single year corresponds to .

Both the WOT model and the EU Excel tool are based on this assumption, which produces smooth results for WG even if the input data for POM shows market fluctuations

In reality, the market fluctuations seen in the POM data, which are based on the underlying changes in the demand and supply driven by various socio-economic factors, are expected also to be reflected in WG. This is because in saturated markets, WG is associated with replacing an older unit of a given product with a newer one, and therefore accelerations (decelerations) in the sales should translate into accelerations (decelerations) in WG.

To make the residence distribution of a given product respond to the current market conditions in year , we propose to introduce the following boost/break factor:

*Equation 16*

Definitions: – index-linked GDP; – population; – inflation-adjusted price per unit; – elasticity of the replacement behaviour depending on the disposable income relative to the unit’s price.

The boost/break factor acts to accelerate or decelerate the replacement rate in year of the units from the given batch sold back in year , depending on how the factor itself changes between these two years:

Indeed, if , then and a larger fraction of units from the batch gets discarded as WEEE during the period in year that expected according to the static Weibull distribution.

We cannot use common price-demand elasticities to model changes in the replacement behaviour directly since at any given moment in time units of different ages, i.e. coming from different historic batches, are being replaced. One would expect that the replacement rates and their elasticity to changes in socio-economic and market conditions are going to be different for each batch. In other words, products sold in different years in the past will exhibit their own price-demand elasticities in the current year . Thus, a combined price-demand elasticity across all the units that are being replaced in year does not capture sufficient behavioural detail.

The cumulative percentage of the units from the given batch discarded between years and is therefore

Here we accounted for all the years in between, and noted that a single year corresponds to .

Because the cumulative share of the discarded products from the batch cannot exceed 100%, we define the adjusted probability distribution for the residence times as follows:

*Equation 17*

Using this definition, we introduce the new dynamic model for POM and WG, both of which are measure in the number of units per year:

*Equation 18*

Here is the total stock at time (units).

If the stock is specified, for example by linking the number of fridges to the number of households, then Equation 18 serves to reconstruct the historic POM, from the onset of the problem (1950) until the current year , that would have been required to sustain the given stock. The equation then allows us to work out the corresponding evolution of WG driven by replacements, unless the stock declines in some periods (in which case there is no replacement for the WG).

With all components of the model introduced, it remains to define the Weibull distribution parameters, and , at the time of the sale. While the actual distribution responsible for WG, defined in Equation 17, evolves relative to the Weibull function as the units for the given batch sold in year become older, we expect the Weibull parameters at the time of the sale to reflect on the market shifts in the years leading to this sale. One way to do this is to relate the Weibull parameters to the Mean and SD of the actual age distribution of the products that are being replaced in a given year , which are given by

*Equation 19*

It only makes sense to apply these formulae from the year when most of the initial stock going back to 1950 would have been replaced. The first year when the and of the units becoming WG are going to be dynamically linked with the Weibull parameters of the newly sold units will be obtained as part of the fitting algorithm described below.

Prior to , the shape and scale of the Weibull distributions defining the residence times are modelled as linear functions of time (Wang et al., 2013):

Here is the first year when the POM and WG are calculated, and the constants , , , are going to be obtain through the optimisation algorithm. For , and are adjusted according to the age distribution of the products discarded as waste in each year .

The Mean and SD for the Weibull distribution in year are given by

*Equation 20*

Assuming that the main statistical parameters characterising the age of the units being discarded match with the corresponding Weibull parameters for the newly sold units that replace the discarded ones, we have

Since and are known if Equation 18 for the historic sales has a solution, these relations allow us to work out and . Excluding from Equation 20, we get the following transcendental equation for :

*Equation 21*

The left hand side of this equation is plotted in Figure 1 as a function of . The plot indicates that there is a unique solution in the range , which is close to the linear function approximation, . However, relative discrepancies between the linear solution and the actual nonlinear left hand side of Equation 21 become significant when exceeds 1.5, and therefore a lookup table needs to used to find a more accurate solution. In addition to the value lookup, we are also going to use a first order correction based on the value of the slope inferred numerically from Equation 21 at the nearest lookup point:

The subscript “” stands for the lookup values corresponding to the pre-computed left hand side of Equation 21.

With defined, the second Weibull parameter is calculated as



*Figure 1. The ratio of Weibull SD over Weibull Mean plotted as a function of , where is the shape parameter. The is plotted for reference.*

We model the total stock of fridges (number of units) according to the following formula:

Here is the number of UK households known from historic records reported by the Office of National Statistics, is an estimated number of the households with two fridges, and the exponential function with the shape and scale approximates the historic uptake of the fridges by UK households beginning in year (i.e. percentage of the households owning a fridge in a given year ). In the absence of credible records for the households with two fridges, we approximate the function by the GDP per capita relative the price of the fridge:

In this formula, is calibration year and is the fraction of the households with two fridges in this year. Along with multiple other parameters of the model, will need to be determined by means of an optimal fitting algorithm introduced below. The GDP- and price-based term is similar to the one used in the boost/break factor for the residence times, but has a unit elasticity. A more complex formula for could be proposed if necessary.

Sample numerical solutions and plots

The test model uses the following input data:

* UK household data, 1996-2017, ONS
* UK household size in 1950, Holmans (2005)
* UK population data, 1950-2016, ONS
* UK index-linked GDP data, 1950-2017, ONS
* Historic EEE prices from <http://www.thepeoplehistory.com/50selectrical.html>
* Inflation multipliers from

<https://www.saving.org/inflation/inflation.php?amount=1&year=1952>

* POM 1995-2021 POM estimates, WOT1.2 (based on Eurostat and PRODCOM data for net imports and production)



*Figure 2. Index-linked UK GDP per capita between 1950 and 2017. ONS data.*

Using the input data and assigning reasonable values to the remaining parameters of the model, we solve the model forward in time from 1950 to produce POM () and WG (), both measure in the number of units sold and discarded annually. We compare this solution for POM against the corresponding POM from WOT1.2 (apparent consumption methodology), and calculate a mean square difference between the two for the period from 1995 to 2021. We then employ a combination of the “Evolutionary” and “GRG Nonlinear” algorithms in the Excel Solver to find the values of the parameters of the problem that minimise the mean square difference, which yields the optimal solution presented in Table 1.

All the key technical results illustrating the optimal solutions for UK fridges obtained from the prototype dynamic model are summarised in a series of Figures below, as well as Figure 3 (POM) and Figure 4 (WG) in the Results section of the paper.



*Figure 3. Stock of the fridges according to the historic number of households between 1950 and 2017, and the optimal solution for the uptake percentage as a function of time, including households with two units.*

*Table 1. An optional solution for the parameters of the model that have not been determined from the data. The solution corresponds to the lowest mean square deviation between the calculated POM and the POM from WOT1.2 over the period from 1995 until 2021 (both POMs measured by the number of units sold in the UK per year). Product type: fridges.*

|  |  |  |
| --- | --- | --- |
| **Parameter** | **Notation** | **Solution** |
| First dynamic year |  | 1985.3 |
| Initial Weibull shape (k) |  | 3.11 |
| Initial Weibull scale (lambda), years |  | 24.1 |
| Initial rate of change for the shape, 1/yr |  | 0.030 |
| Initial rate of change for the scale, yr/yr |  | -0.13 |
| Year when fridges appeared |  | 1944.7 |
| Stock build-up shape |  | 0.93 |
| Stock build-up timescale, years |  | 25.0 |
| % of households with two units in 2017 |  | 17.62% |
| Price baseline in 2000, £(2017) |  | 1158 |
| Price baseline in 2008, £(2017) |  | 808 |
| Price baseline in 2017, £(2017) |  | 563 |
| Price baseline in 2021, £(2017) |  | 614 |
| Replacement boost elasticity to GDP and prices |  | 0.82 |



*Figure 4. Inflation-adjusted price of a fridge reconstructed prior to 2000 based on historic records, and optimised from 2000 to provide the closes match with the WOT1.2 data for POM (number of units)*



*Figure 5. Evolution of the time-dependent Mean and SD of the products residence times, obtained as part of the optimal solution that provides the closes match with the WOT1.2 data for POM (number of units). The mean residence times apply to any given year while the product is being used. For example, if a fridge had been sold in 1970, its mean residence time would have been just under 20 years, but if it was still in use in 1990, the mean residence times would have dropped to 16 years, making the older fridge more likely to be replaced than according to the original distribution with the 20-year mean.*

1. Based on the current mapping, no PCC codes splits into 5 or more UK categories. [↑](#footnote-ref-1)
2. Based on the current mapping, no UNU keys splits into 6 or more UK categories. [↑](#footnote-ref-2)
3. <https://www.gov.uk/government/publications/weee-evidence-and-national-protocols-guidance/waste-electrical-and-electronic-equipment-weee-evidence-and-national-protocols-guidance> [↑](#footnote-ref-3)