A Modified Fractionally Co-integrated VAR for Predicting Equity and Index Returns

Xingzhi Yao

Department of Economics, Lancaster University

November 2015

Motivation

Modelling of multivariate time series

- Traditional approach

Motivation

Modelling of multivariate time series

- Traditional approach
  - VAR (*Sims,1980*): $I(0)$ variables

Motivation

Modelling of multivariate time series

- Traditional approach
  - VAR (*Sims,1980*): $I(0)$ variables
  - VECM (*Johansen,1991*): long- and short-run relationships among $I(1)$ variables

Motivation

Modelling of multivariate time series

- Traditional approach
  - VAR (*Sims,1980*): $I(0)$ variables
  - VECM (*Johansen,1991*): long- and short-run relationships among $I(1)$ variables
- New approach: many economic & financial variables are fractionally integrated/long memory

Motivation

Modelling of multivariate time series

- Traditional approach
  - VAR (*Sims,1980*): $I(0)$ variables
  - VECM (*Johansen,1991*): long- and short-run relationships among $I(1)$ variables
- New approach: many economic & financial variables are fractionally integrated/long memory
  - fractionally integrated VAR (FIVAR) (*Nielsen, 2004*): $I(d)$ variables

Motivation

Modelling of multivariate time series

- Traditional approach
  - VAR (*Sims,1980*): $I(0)$ variables
  - VECM (*Johansen,1991*): long- and short-run relationships among $I(1)$ variables
- New approach: many economic & financial variables are fractionally integrated/long memory
  - fractionally integrated VAR (FIVAR) (*Nielsen, 2004*): $I(d)$ variables
  - fractionally co-integrated VAR (FCVAR) (*Johansen, 2008*): long- and short-run relationships among $I(d)$ variables

Motivation

- Our approach: a modified FCVAR containing both $I(0)$ and $I(d)$ variables

$$\Delta^d X_t = \alpha\beta'\Delta^{d-b} L_b X_t + \sum_{i=1}^{k} \Gamma_i \Delta^d L_b^i X_t + \varepsilon_t$$

Motivation

- Our approach: a modified FCVAR containing both $I(0)$ and $I(d)$ variables

$$\Delta^d X_t = \alpha\beta'\Delta^{d-b} L_b X_t + \sum_{i=1}^{k} \Gamma_i \Delta^d L_b^i X_t + \varepsilon_t$$

- Traditional model system: $X_t \sim I(d)$

Motivation

- Our approach: a modified FCVAR containing both $I(0)$ and $I(d)$ variables

$$\Delta^d X_t = \alpha\beta'\Delta^{d-b} L_b X_t + \sum_{i=1}^{k} \Gamma_i \Delta^d L_b^i X_t + \varepsilon_t$$

- Traditional model system: $X_t \sim I(d)$
- Our model system: $X_t = (x_t, y_t, z_t)'$, where $x_t, y_t \sim I(d)$, $z_t \sim I(0)$

Motivation

- Our approach: a modified FCVAR containing both $I(0)$ and $I(d)$ variables

$$\Delta^d X_t = \alpha\beta'\Delta^{d-b}L_b X_t + \sum_{i=1}^{k}\Gamma_i\Delta^d L_b^i X_t + \varepsilon_t$$

- Traditional model system: $X_t \sim I(d)$
- Our model system: $X_t = (x_t, y_t, z_t)'$, where $x_t, y_t \sim I(d)$, $z_t \sim I(0)$
  - does the introduction of $I(0)$ variable affect the fractional co-integration between $I(d)$ variables?

Background

- Little consensus on how to test for fractional co-integration

Background

- Little consensus on how to test for fractional co-integration
  - Semiparametric: Regression-based estimator; Spectral matrix estimator

Background

- Little consensus on how to test for fractional co-integration
  - Semiparametric: Regression-based estimator; Spectral matrix estimator
  - Parametric: FCVAR model

Background

- Little consensus on how to test for fractional co-integration
    - Semiparametric: Regression-based estimator; Spectral matrix estimator
    - Parametric: FCVAR model
- Factors affecting the co-fractional relation between long memory variables

Background

- Little consensus on how to test for fractional co-integration
  - Semiparametric: Regression-based estimator; Spectral matrix estimator
  - Parametric: FCVAR model
- Factors affecting the co-fractional relation between long memory variables
  - regime switching

Background

- Little consensus on how to test for fractional co-integration
  - Semiparametric: Regression-based estimator; Spectral matrix estimator
  - Parametric: FCVAR model
- Factors affecting the co-fractional relation between long memory variables
  - regime switching
    - *Haldrup and Nielsen (2006)*: fractional co-integration in the full sample with no separation into regimes $\Longrightarrow$ misleading inference

Background

- Little consensus on how to test for fractional co-integration
    - Semiparametric: Regression-based estimator; Spectral matrix estimator
    - Parametric: FCVAR model
- Factors affecting the co-fractional relation between long memory variables
    - regime switching
        - *Haldrup and Nielsen (2006)*: fractional co-integration in the full sample with no separation into regimes $\Longrightarrow$ misleading inference
    - structural breaks

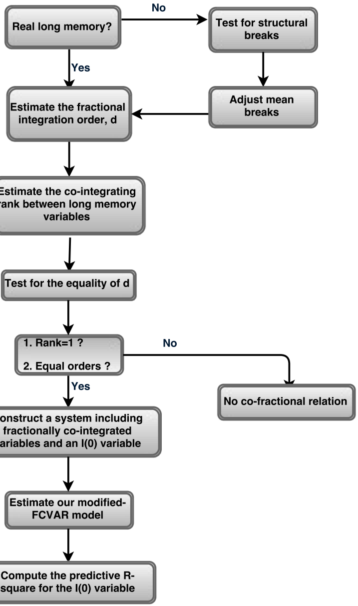
Background

- Little consensus on how to test for fractional co-integration
  - Semiparametric: Regression-based estimator; Spectral matrix estimator
  - Parametric: FCVAR model
- Factors affecting the co-fractional relation between long memory variables
  - regime switching
    - *Haldrup and Nielsen (2006)*: fractional co-integration in the full sample with no separation into regimes $\Longrightarrow$ misleading inference
  - structural breaks
    - *Christnesen and de Magistris (2010)*: common level shifts $\Longrightarrow$ fractional co-integration

Background

- Little consensus on how to test for fractional co-integration
  - Semiparametric: Regression-based estimator; Spectral matrix estimator
  - Parametric: FCVAR model
- Factors affecting the co-fractional relation between long memory variables
  - regime switching
    - *Haldrup and Nielsen (2006)*: fractional co-integration in the full sample with no separation into regimes $\Longrightarrow$ misleading inference
  - structural breaks
    - *Christnesen and de Magistris (2010)*: common level shifts $\Longrightarrow$ fractional co-integration
    - *Kellard et al. (2015)*: uncommon breaks $\Longrightarrow$ spurious fractional co-integration

Econometric Framework



Traditional FCVAR Model–implied and realisd variance

- Let $X_t \sim I\left(d\right)$

$$\Delta^d X_t = \underbrace{\alpha\beta'\Delta^{d-b}L_b X_t}_{\text{long-run equilibrium}} + \underbrace{\sum_{i=1}^{k}\Gamma_i\Delta^d L_b^i X_t}_{\text{short-run dynamics}} + \varepsilon_t$$

- $\Delta^d = (1-L)^d$, $L_d = 1-\Delta^d$, ε_t is $i.i.d.(0,\Omega)$
- α: the vector of loadings, β' : the cointegrating vector, Γ_i governs the short-run dynamics
- $\beta' X_t \sim I\left(d-b\right)$, $1 > d \geq b > 0$
  - weak co-fractional relation: $0 < b < 0.5$
  - strong co-fractional relation: $0.5 < b \leq d$

M-FCVAR Model–adding an I(0) variable

- (1) Does the introduction of $I(0)$ variable affect the fractional co-integration between $I(d)$ variables?

M-FCVAR Model–adding an I(0) variable

- (1) Does the introduction of $I(0)$ variable affect the fractional co-integration between $I(d)$ variables?
  - yes by using FCVAR, shocks associated with the $I(0)$ variable have a permanent effect on $I(d)$ variables

M-FCVAR Model–adding an I(0) variable

- (1) Does the introduction of $I(0)$ variable affect the fractional co-integration between $I(d)$ variables?
  - yes by using FCVAR, shocks associated with the $I(0)$ variable have a permanent effect on $I(d)$ variables
  - solution: a particular model design to ensure shocks are transitory

M-FCVAR Model–adding an I(0) variable

- (1) Does the introduction of $I(0)$ variable affect the fractional co-integration between $I(d)$ variables?
  - yes by using FCVAR, shocks associated with the $I(0)$ variable have a permanent effect on $I(d)$ variables
  - solution: a particular model design to ensure shocks are transitory
    - the first row of α matrix consists of zero elements only

M-FCVAR Model

- (2) How to make the two error correction terms both result in the $I(0)$?

M-FCVAR Model

- (2) How to make the two error correction terms both result in the $I(0)$?
- $$\Delta^b Y_t = \alpha\beta' L_b Y_t + \sum_{i=1}^{k} \Gamma_i \Delta^b L_b^i Y_t + \varepsilon_t$$

M-FCVAR Model

- (2) How to make the two error correction terms both result in the $I(0)$?
- $$\Delta^b Y_t = \alpha\beta' L_b Y_t + \sum_{i=1}^{k} \Gamma_i \Delta^b L_b^i Y_t + \varepsilon_t$$
  - $Y_t = (\Delta^{d-b} RV_t, \Delta^{d-b} VIX_t^2, r_t)'$

M-FCVAR Model

- (2) How to make the two error correction terms both result in the $I(0)$?
-
$$\Delta^b Y_t = \alpha\beta' L_b Y_t + \sum_{i=1}^{k} \Gamma_i \Delta^b L_b^i Y_t + \varepsilon_t$$
  - $Y_t = (\Delta^{d-b} RV_t,\, \Delta^{d-b} VIX_t^2,\, r_t)'$
  - $\beta' = \begin{pmatrix} 1 & \beta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, thus $\beta' Y_t \sim I(0)$ (note: $\alpha_{11} = 0$ and $\alpha_{12} = 0$)

Predictive R-square Implied by the M-FCVAR

- Write the M-FCVAR in a moving average representation and let $e3' \equiv (0, 0, 1)$, the impulse responses, Φ_j, is constructed by (α, β, k, b)

$$r_t = e3' \sum_{i=0}^{\infty} \Phi_i \varepsilon_{t-i}$$

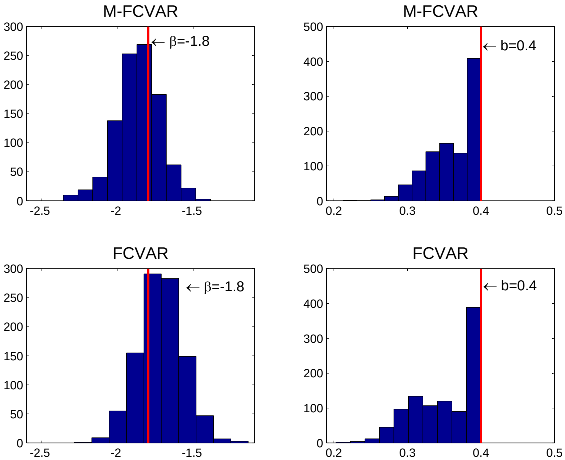
- Decompose return into the expected and unexpected part

$$r_t^h = \underbrace{e3' \sum_{j=0}^{h-1} \sum_{i=j+1}^{\infty} \Phi_i \varepsilon_{t+j-i}}_{\text{expected (A)}} + \underbrace{e3' \sum_{j=0}^{h-1} \sum_{i=0}^{j} \Phi_i \varepsilon_{t+j-i}}_{\text{unexpected (B)}}$$

- Return predictability over h horizons implied by the M-FCVAR model

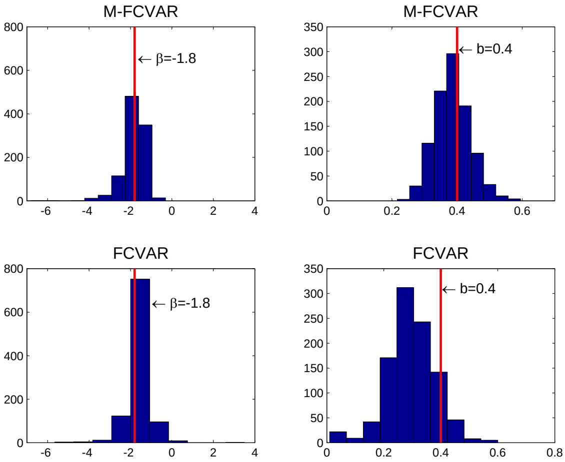
$$R_h^2 = \frac{var(A)}{var(A) + var(B)}$$

Simulation Outcome: d=b=0.4



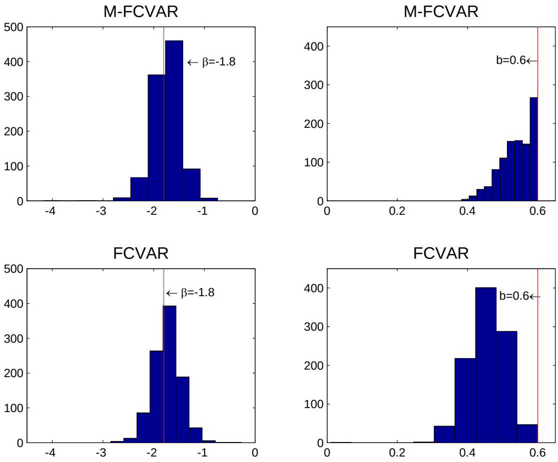
d=b=0.4	$\widehat{\beta}_1$			$\widehat{b}$		
	Mean	Std	Conf. int	Mean	Std	Conf. int
M-FCVAR	-1.865	0.143	(-2.183, -1.584)	0.364	0.034	(0.294, 0.400)
FCVAR	-1.716	0.147	(-2.004, -1.414)	0.352	0.044	(0.265, 0.400)

Simulation Outcome: d=0.6, b=0.4



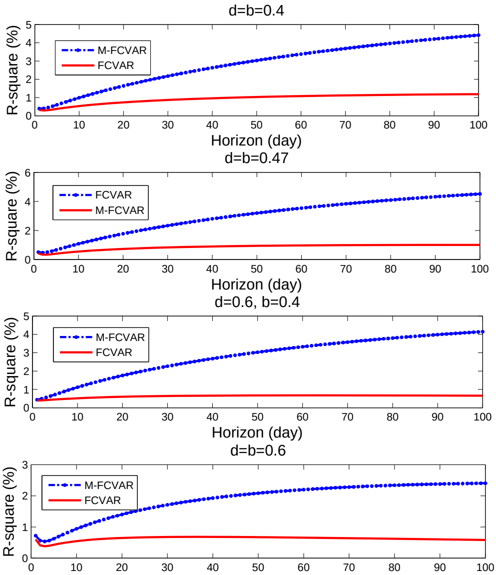
d=0.6, b=0.4	$\widehat{\beta}_1$			$\widehat{b}$		
	Mean	Std	Conf. int	Mean	Std	Conf. int
M-FCVAR	-1.848	0.559	(-3.242, -1.088)	0.386	0.054	(0.285, 0.496)
FCVAR	-1.579	0.571	(-2.760, -0.679)	0.295	0.087	(0.107, 0.456)

Simulation Outcome: d=b=0.6



d=b=0.6	$\widehat{\beta}_1$			$\widehat{b}$		
	Mean	Std	Conf. int	Mean	Std	Conf. int
M-FCVAR	-1.741	0.279	(-2.283, -1.217)	0.540	0.048	(0.431, 0.600)
FCVAR	-1.737	0.279	(-2.282, -1.156)	0.456	0.055	(0.351, 0.559)

Simulation Outcome: R-square



Comments

Thank you!