GROUP P1

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INTRODUCTION

As the list indicates, the backgrounds of the members of this group were extremely diverse. The Mathematics courses offered in the institutions represented varied greatly. However after a wide-ranging sharing of experience several 'perennial difficulties' were identified which were common to all. These are dealt with below; many of the details, produced by subgroups, are included in the Appendices.

1. VARIED BACKGROUNDS

Students entering Higher Education will have studied various A-level syllabuses which fall under the heading of Mathematics. It is well known that these syllabues vary greatly in content ranging from 'traditional' to 'modern'. It is anticipated that this situation will be improved when the common core produced by SCUE is included in all syllabuses - this should be achieved in the June 1986 papers. However it should be remembered that the common core is only about 40% of an A-level syllabus.

In any given course there will also be some students who have done a minimal amount of Mathematics at A-level (eg Pure Mathematics and Statistics) and those who have done a great deal more (eg those who have done Further Mathematics). In addition, the knowledge one may assume known will vary from year to year. The use of diagnostic tests at the beginning of a course to determine this has been found by several to be useful.

We isolated the following list of topics on which it was necessary to provide some material; they were required in later courses but were not usually sufficiently well known:

Complex numbers, hyperbolic functions, trigonometric functions,, relations and equations. Curve sketching, conics, differentiation, chain rule, Leibniz. Elementary differential equations. Integration techniques. Inverse functions. Sets, matrices, determinants, permutations and combinations. Maelaurin, Taylor, Induction.

It was felt that the best way to deal with this problem was to provide a package of units which could be worked through at the student's own pace (self-paced learning). This group recommends that some effort be made by this conference to get a suitable set of such units produced which would then be available to all.

2. THE INTRODUCTION OF ABSTRACT MATHEMATICS

It was agreed that, in this area, the major difficulty probably arose from the difference between A-level Mathematics and first year Pure Mathematics courses. The more conceptual approaches to Analysis and Abstract Algebra are quite a shock and bear little resemblance to the mainly algorithmic approach at A-level. Appendices 1 and 2 contain our thoughts on these topics.

3. 'DROP OUTS'

Too often, students fail to complete the course they embark on. Whilst the problem of 'drop outs' is not special to Mathematics we gave some thought to this problem and details of our discussion are reported in Appendix 3.

4. CHANGE OF ENVIRONMENT

The change of environment involved in entering Higher Education causes problems to many. These problems are again common to all students - opening a bank account, leaving home, making new friends etc. It was agreed that it was necessary to provide adequate pastoral care. The provision of a personal tutor in his subject area was believed to be very useful and the importance of early diagnosis of problems due to change in environment was noted.

The problems arising from independent working needed to be brought to a student's attention and an introduction to so-called 'study skills' should be given to all students. Some periodic contact with tutors and/or other staff was felt to be necessary during the first year.

5. LACK OF ENTHUSIASM

It has been observed that many of our students are lethargic and show little enthusiasm for the subject. Some heart-searching revealed that on many occasions the attitude of staff was at fault. An enthusiastic lecturer will usually communicate enthusiasm.*

Visits to schools were thought to be a useful means of communicating enthusiasm for the subject. Sixth form conferences many also be useful here.

A well organised Mathematical Society has been found by some to be useful, particularly if a programme with interesting speakers is organised.

Involvement in projects was believed by some to be a useful means of stirring up enthusiasm.

6. PRE-SIXTH FORM PROBLEMS

It was agreed that there may be problems going back to before sixth form. These are discussed in Appendix 4.

7. PROBLEMS WITH NEW AREAS

Many students found problems with new subject areas such as Statistics and Computing. A fear of computers had been detected by some. It was suggested that this may usually be overcome by a personal introduction. It was felt that the wide-spread use of calculators had now made statistics courses more oriented towards concepts.

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* An enthusiastic lecturer is a necessary but not sufficient condition for enthusiasm in students. Ed.

APPENDIX 1 INTRODUCING ANALYSIS

Analysis is rigorous (by nature) and students need to see the necessity for rigour and also to be dissatisfied with less. They need to appreciate the nature of 'proof'; to be familiar with the language of mathematics; and to have seen the unsuspected richness of the real line.

One way of 'softening the blow' is to present a short lecture course, at the beginning of the first year which

(i) introduces the tools of the trade - logic of sets, functions, proofs etc,

(ii) draws attention to the hidden properties of the real line,

(iii) indicates the desirability of axiomatising and pursues elementary consequences.

The following should not be regarded as a syllabus. Some of the material could equally well appear in an algebra course and also complement work in mathematical methods courses.

(a) <u>The language of mathematics</u>

The aim is to introduce some of the basic tools and promote an appreciation of the nature of a mathematical proof. Logical connections should be introduced in an entertaining but mathematically serious manner. Methods of proof should be discussed via concrete mathematical examples. A discussion of quantifiers should be included. The importance of counter-examples should be stressed. The algebra of sets should be established and familiarity and confidence in its use developed. The basic nature of functions should be covered.

(b) <u>Don't take the real numbers for granted</u>

Starting from the intuitive idea that the real numbers (R) <u>are</u> the points on an infinite line aim to

(i) develop the consequences of this notion, and

(ii) provide enough 'counter-intuitive' properties of the reals to get the students interested in doing it properly.

Define the rationals (Q) and discuss their density property. Point out that "there exist infinitely many rationals in [0,1] but not all numbers in [0,1] are rational". Discuss $\sqrt{2}$ which is a length but not rational. Show that irrationals are far from rare - lead on to countability. Establish that the reals contain the rationals and that $R \sim [0,1]$ etc. Recall surprise that the rationals, although infinite and dense have measure zero.

(c) <u>What is R?</u>

Present axioms for R. Show that school algebra can be deduced from the axioms of arithmetic. Use axioms of order to motivate solution of inequalities. Show how completeness axiom distinguishes Q from R.

Define the subsystems N, Z and Q and pursue topics such as induction, elementary number theory in Z etc.

APPENDIX 2 INTRODUCING ABSTRACT ALGEBRA

The main problems are caused by the abstractness of the material and the axiomatic method. Students in the first year are not used to 'proving' results. They often do not appreciate the <u>need</u> for definitions and mathematical rigour. It is important that they are led into abstraction gently and that time is given for the assimilation of new ideas. Rigour should not be introduced before a need for it has been perceived. (It should be realised that a desire for mathematical elegance may hinder the teaching process.)

Practical examples should be used to exemplify ideas and interest students. (Clearly students should tackle themselves a range of problems both concrete and theoretical.) An attempt should be made to relate abstract ideas to other parts of the course. An historical approach can often add interest and help the students appreciate how the need for the ideas arose.

It is essential that they see many simple examples of different types of proof arising in a situation with which they are basically familiar but in such a way as to avoid appearing to be mathematically pedantic. It is felt that these general aims can be achieved in a number of ways. Two suggestions, rather different in character, are offered. Neither is a complete course - Syllabus A allows some depth of treatment as it is mainly concerned with rings, while Syllabus B allows a shallower treatment. Each has been successfully used.

SYLLABUS A

1. The basic language

Sets, functions, relations, binary operations, mathematical induction.

2. Properties of Z

Divisibility, the division algorithm, gcd's and the Euclidean algorithm, prime numbers, fundamental theorem of arithmetic (congruences, Chinese Remainder Theorem).

3. Real Polynomials

Factorisation, division algorithm, hcf's, irreducible polynomials, Unique Factorisation Theorem (Chinese Remainder Theorem).

4. Fields

Point out the similarities of development of the previous two chapters and use this as a motivation for axiomatic systems. Useful systems arise out of abstracting that which is common to different situations. Field axioms approached as abstractions of real numbers, complex numbers.

5. Polynomials Revisited

Now speak of F[x], rather than R[x]. Special properties of C[x] (Fundamental Theorem of Algebra), irreducible polynomials in C[x], R[x].

6. Rings

History of non-Euclidean geometry, quaternions and their influence in proliferating axiomatic systems. Elementary properties of rings, ideals, isomorphism, factor rings (perhaps a discussion of Euclidean domains as generalisation of Chapters 2 and 3 if time allows: and applications in number theory).

SYLLABUS B

The basic language

Sets, functions, relations, binary operations. Methods of proof: induction, if and only if, contradiction, contrapositive.

Properties of Z, Z_n

Divisibility, the division algorithm, gcd's and the Euclidean algorithm, lcm's, prime numbers, fundamental theorem of arithmetic. Elementary properties of $+_n$, X_n.

Permutations

Cycles, transpositions, odd and even permutations, Cauchy's theorem.

One-set, one-operation structures

Groupoid, semi-group, monoid, group. Commutativity operation table. Intuitive idea of isomorphism. Groups of order $\leqslant 8$.
Subgroups; criterion for a subgroup. Use of group table for finite groups.
Cosets, Lagrange's theorem (plus statement of Sylow's theorem). Cyclic groups.

One-set, two-operation structures

Rings; ring with identity; commutative ring. Ring tables. Integral domain, division ring, field sub-structure; criteria for sub-ring, sub-field. Rings with no proper divisors of zero; characteristic.

Notes

1. Concrete examples brought in wherever possible, such as cardboard models for groups of symmetries; use of groups in campanology, square dancing, game of 15; Pauli spin matrices; applications to computer science etc.

2. Historical context given with respect to creation of group theory by Galois, Abel, Ruffini et al and with some mention of Quaternions.

APPENDIX 3 'DROP OUTS'

In most courses a number of students fail to complete the course, failure usually being in the first year. By the time of failure (departure), it is often too late to apply any remedy other than suggesting a job or alternative course (ie the student is lost to the course).

The reasons for a particular failure may be several and it may be difficult to ascertain these; one reason may be given thus obscuring others. The failure may be the result of compounding several effects/factors. The student may even leave without notice and further contact may be very difficult.

It is desirable to determine at an early stage which students are at risk in order to take remedial action.

When a student chooses a course, the opportunity to change/transfer to another course often involves a penalty, usually financial. It is important that the potential student is well aware of what the course involves. It is worthwhile, as well as interviewing all students, allowing them to talk to final year students about their experience on the course. Although seemingly far in the future, there is the possibility of discussing career prospects.

The problem of measuring motivation is still unsolved; academic achievement in school examinations is a poor indicator of future achievements at the tertiary level.

Many students of mathematical studies can run into problems when faced with essays and reports. Course structures are being altered in some instances to allow time to remedy the difficulties in this area; the number of failures as a result of a 'non-literate' student being unable to write essays/reports is being reduced.

It is becoming more common to issue to the students at the beginning of each sessions a guide which includes a calendar of assessments and their evaluation.

It is important that a system of counselling is clearly established. Many colleges operate a personal tutor system but the effectiveness depends on the enthusiasm of the tutors who must have professional counselling back-up services. If the students meets the tutor on the first day of his college career, he has a focus for discussion of anything, not only academic problems. If a sound relationship is established, the student has a source of advice including careers, testimonials and references; the tutor also has a contact to other (lecturing) staff. With larger student numbers the system can become cumbersome and may even stagnate with a consequent increase in the drop-out rate. The student has to cope with the idea of independent learning; the problems which can arise here should be detected early by personal tutors with the help of feedback from the lecturers.

For many students, going to college is an opportunity to get away from home; however, many are not ready to cope with the problems of the different environment and its responsibilities. This is more likely when there is strife between parents and offspring; parents may also put pressure on their children to go to college to gain a degree/professional qualification. The financial situation does cause one or two students to leave a course each year; the expected grant does not materialise (ie too small or nil) and the student has to leave; in other cases the parents cannot or will not provide their full financial contribution. There seems to be little that can be done in this case.

If a student falls ill or has a serious accident, this can be traumatic, especially in self-catering units. As this situation is fairly rare to lecturers/tutors, there is seldom any advice/help given other than the request for the essential medical certificate.

APPENDIX 4 PERENNIAL DIFFICULTIES DUE TO GAPS IN A-LEVEL MATHEMATICS

A surprisingly high proportion of students show gaps in very fundamental areas of Mathematics taught in the 4th and 5th forms. The most usual topics in which these gaps appear are the following:

1. Calculation with fractions, especially division and addition of fractions, eg 1/6 + 1/10 would be calculated as 10/60 + 6/60 rather than simply use the least common multiple of 30 of the two denominators. (Also cases have been noticed when the student was unable to get a correct answer by whatever means).

2. Calculation with powers, especially when the index is negative or fractional (or both). Confusion of handling the fractional powers such as $a^{\frac{1}{2}} = 1/a^2$ does occur too often not to be mentioned.

3. The relationship between $y = \log_c x$ and $x = c^y$ very often has not been impressed deep enough into the student's mind, and the student may not be able to transcribe a condition involving logarithms into the exponential form.

4. Cyclometric functions. Many of the formulae (needed eg for substitutions into integrals) are not remembered. In particular the formulae for sin or cos of a sum of two angles, a double angle, a half angle, sum of sines or cosines. Also the visual interpretation of the values of sin, cos, tan and cotan as certain lengths in the unit circle is often missing.

5. Trigonometry. Ability to use the cyclometric functions to solve practical examples involving triangles is sometimes low. The sine and cosine theorems may have to be recalled.

6. Solution of algebraic equations - quadratic in particular. The student does not remember the formula and relies entirely on the possibility of factorisation, and often spends a very long time in trying to obtain one (even if none is possible in the set of the integers). Equations of a higher degree leave some students completely helpless and the idea of trying and guess a root (and be then able to reduce the degree of the equation) is not a natural one to them. (On the other hand, some special tricks for very special types of equation seem superfluous).

7. Algebra of sets. The students are usually alright, but since this topic is not at A-level, a revision is advisable.

8. Matrices and linear equations. This is again an example of a topic done at O-level and not included in most A-level programmes at all.

The students forget the elementary calculations with matrices (in particular the multiplication and the corresponding compatibility of the dimensions of the matrices involved) and the calculation of determinants (of order 2 and 3) so these topics need revision. It is even more essential for the solution of systems of linear equations to make certain that <u>all</u> the students have mastered the method based on the Frobenius condition, namely row-operations on the augmented matrix.

9. Functions. Correct definition of function to be used not omitting to mention the definition domain; graphs of the absolute value $|x|$ function; the fact that $\sqrt{x}$ is $\geqslant 0$ (in real analysis), the fact that $x^2 = a$ $(a > 0)$ has solution $|x| = \sqrt{a}$, ie $x = \pm\sqrt{a}$ are some of the details the student should be reminded of.

10. Manipulative ability. This is of course not a single topic but a very important part of the student's knowledge which he/she is expected to bring with him to University. Factorisation, cancellation (not to forget to investigate the case when the cancelled factor is equal to zero), are just a few points to be checked to make sure that the manipulative techniques of the student are at an acceptable level.