



## RESEARCH ARTICLE

10.1029/2018JA025530

## Key Points:

- We have investigated *F* region irregularities produced by high power HF X-mode radio waves
- Plasma instabilities responsible for decametric irregularities in the *F* region are described via a two-step process
- Our theoretical characteristic time for development of decametric irregularities is of the order of 1 min

## Correspondence to:

F. Honary,  
f.honary@lancaster.ac.uk

## Citation:

Borisov, N., Honary, F., & Li, H. (2018). Excitation of plasma irregularities in the *F* region of the ionosphere by powerful HF radio waves of X-polarization. *Journal of Geophysical Research: Space Physics*, 123, 5246–5260. <https://doi.org/10.1029/2018JA025530>

Received 13 APR 2018

Accepted 26 MAY 2018

Accepted article online 5 JUN 2018

Published online 29 JUN 2018

©2018. The Authors.

This is an open access article under the terms of the Creative Commons Attribution-NonCommercial-NoDerivs License, which permits use and distribution in any medium, provided the original work is properly cited, the use is non-commercial and no modifications or adaptations are made.

## Excitation of Plasma Irregularities in the *F* Region of the Ionosphere by Powerful HF Radio Waves of X-Polarization

N. Borisov<sup>1</sup>, F. Honary<sup>2</sup> , and H. Li<sup>3</sup>

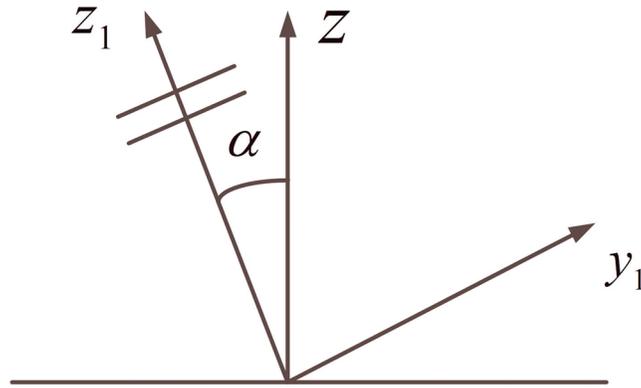
<sup>1</sup>IZMIRAN, Moscow, Russia, <sup>2</sup>Department of Physics, Lancaster University, Lancaster, UK, <sup>3</sup>School of Physics and Optoelectronic Engineering, Xidian University, Xi'an, China

**Abstract** We discuss theoretically the excitation of artificial plasma irregularities in the auroral ionosphere by high-frequency X-mode radio wave. This is done via a two-step process. As a first step we adopt the thermal self-focusing instability excited in the *F* region of the ionosphere under the action of a strong high-frequency (HF) radio wave. This instability causes the formation of perturbations of the electron temperature and plasma concentration across the magnetic field. In addition, the plasma becomes depleted in the regions of the electron temperature enhancements and vice versa, since the gradients of plasma concentration and the electron temperature have opposite signs. In such conditions the temperature gradient instability comes into play. As a second step we consider plasma and electron temperature inhomogeneities that appear due to this instability to be responsible for the generation of irregularities with transverse sizes smaller than the typical scales of the self-focusing instability. Alternative mechanisms such as excitation of the gradient-drift and the current-convective instabilities, which are often attributed to the generation of plasma irregularities in the *F* region and can contribute to the formation of artificial irregularities in the case of X-mode heating, are also discussed.

### 1. Introduction

The physical effects in the ionosphere caused by the action of high-frequency (HF) radio waves of O-polarization have been investigated both experimentally and theoretically for more than 40 years. These investigations have reported interesting and complex nonlinear plasma processes mainly in the *F* region and partly in the *E* region due to the interaction of powerful HF radio waves with ionospheric plasma. Among them are the following: the broadband anomalous absorption, excitation of plasma irregularities of different sizes (from centimeters up to kilometers), artificial optical emissions, the so-called magnetic zenith effect, and the stimulated electromagnetic (EM) emissions. The results of investigations are published in hundreds of original papers and summarized in monographs and reviews (Ginzburg, 1970; Gurevich, 1978; Kelley, 1989; Robinson, 1989; Stubbe & Hagfors, 1997; Stubbe, 1996). According to present understanding, most of the effects in the *F* region are initiated by very efficient transformation of the EM pump wave into the upper hybrid electrostatic oscillations. This process takes place between the reflection height of the pump wave and the so-called upper hybrid resonance (UHR) height (a few kilometers below the reflection height). According to the theory, this resonance exists only for the wave of O-mode polarization. Hence, the mentioned above physical processes should be absent when the HF radio wave of X-mode polarization is radiated into the ionosphere. That is why the physical processes in the ionosphere caused by HF pump wave of X-mode polarization were not actively investigated.

However, recent experiments with X-mode waves performed at Tromsø indicate that the heating of the ionosphere by very powerful extraordinary waves is accompanied by effects similar to those that are caused by O-mode heating (e.g., generation of plasma irregularities, significant additional heating of electrons, artificial optical emissions, and excitation of plasma and ion lines; Blagoveshchenskaya et al., 2014, 2011). So it is important to investigate possible mechanism for the generation of plasma perturbations in the ionosphere by X-mode HF pump wave. So far, only excitation of centimeter-size artificial irregularities in the ionosphere due to X-mode heating has been predicted by Vas'kov and Ryabova (1998). Also, Wang et al. (2016) reported that the threshold for the excitation of the oscillating two-stream instability and the parametric decay instability can be exceeded at the reflection height of a powerful X-mode wave under certain conditions. Hence, plasma and ion lines (as in the case of O-mode heating) can be detected near the reflection height.



**Figure 1.** Illustration of the coordination system  $z$ ,  $z_1$ , and  $y_1$ , with  $z_1$  direction being along the local magnetic field line and  $y_1$  is orthogonal to  $z_1$ . The inclination of magnetic field direction is shown by angle  $\alpha$ . The double line perpendicular to  $z_1$  axis represents the wave front of propagating electromagnetic wave.

It should be mentioned that the appearance of elongated decametric plasma irregularities in the  $F$  region due to X-mode heating has been clearly confirmed by experiments utilizing Cutlass radar (Blagoveshchenskaya et al., 2011). Nevertheless, no theoretical explanation for the excitation of plasma perturbations with transverse sizes from tens of meters up to hundreds of meters due X-mode heating has been reported. The aim of this paper is to investigate possible mechanisms that can produce such irregularities in the case of X-mode heating and provide explanation for the experimental observations.

## 2. Basic Equations

We consider an EM wave of X-polarization propagating into the ionosphere along the magnetic field line (which coincides with the  $z_1$  axis; Figure 1). It is convenient to use the system of coordinates  $x_1, y_1$ , and  $z_1$ , where the axis  $y_1$  is in the meridional plane and it is orthogonal to the  $z_1$  axis. Let us introduce an additional system of coordinates  $x, y$ , and  $z$  where  $z$  is vertical and  $x = x_1$ . Suppose that the angle  $\alpha$  between the  $z_1$  and the  $z$  axes is small ( $\alpha \ll 1$ ) that corresponds to the high-latitude conditions. Assume that the nondisturbed ionosphere is inhomogeneous only in the vertical direction.

We would like to discuss the heating of electrons in the  $E$  region of the ionosphere by the X-mode pump wave. For this purpose equations describing the propagation of the X-mode wave and hydrodynamic equations for the electron temperature and plasma concentration are used. An equation for propagation of X-mode along the magnetic field line in a weakly inhomogeneous auroral ionosphere is described by the second-order differential equation (Ginzburg, 1970).

$$\frac{d^2 E^{(+)}}{dz_1^2} + k_0^2 \left( 1 - \frac{v}{1 - \sqrt{u}} \right) E^{(+)} = 0. \quad (1)$$

Here  $E^{(+)} = E_x + iE_{y_1}$ ,  $E_{x'}$ , and  $E_{y_1}$  are the components of the electric field along the corresponding axes,  $k_0 = \frac{\omega_0}{c}$ ,  $\omega_0$  is the frequency of the pump wave,  $c$  is the speed of light,  $v = \frac{\omega_{pe}^2}{\omega_0^2}$ ,  $\omega_{pe}$  is the electron plasma frequency,  $\omega_{pe}^2 = \frac{4\pi e^2 N}{m}$ ,  $e, m$  are the charge of electron and its mass,  $N$  is the concentration of plasma,  $u = \frac{\omega_{He}^2}{\omega_0^2}$ ,  $\omega_{He} = \frac{eH_0}{mc}$  is the electron gyrofrequency,  $H_0$  is the Earth's magnetic field. In equation (1) we have neglected the interaction between the X- and O-modes, which is possible in a weakly inhomogeneous ionosphere rather far away from the reflecting height of the O-mode. As the angle  $\alpha$  between the axes  $z$  and  $z_1$  is small the scale of plasma inhomogeneity along the  $y_1$  axis is much larger than the corresponding scale along the  $z_1$  axis. Therefore, we have neglected the derivatives of the inhomogeneity along the  $y_1$  axis.

An approximate hydrodynamic equation for the electron temperature  $T_e$  (neglecting slow drift motion) describes the changes of energy in time taking into account the heating, the losses of energy, and the longitudinal thermodiffusion (Gurevich, 1978):

$$\frac{\partial T_e}{\partial t} - \frac{\partial}{\partial z_1} D_{Te}^{(II)} \frac{\partial T_e}{\partial z_1} + \delta_{em} v_{em} (T_e - T_m) + \delta_{ei} v_{ei} (T_e - T_i) = \frac{2}{3N} (\mathbf{E} \delta_{\omega} \mathbf{E}). \quad (2)$$

Here  $D_e^{(\parallel)}$  is the thermal conductivity of electrons along the magnetic field line,  $\nu_{em}, \nu_{ei}$  are the collision frequencies of electrons with neutral molecules and ions,  $\delta_{em}, \delta_{ei}$  are the fraction of energy transferred in collisions with neutrals and ions,  $\hat{\sigma}_{ew}$  is the tensor of the electron conductivity, and  $\mathbf{E}$  is the electric field of the pump wave. Since the electric field in the  $E$  region is transverse, only the transverse components of the electron conductivity need to be substituted in equation (2). The corresponding source of the electron heating by X-mode pump wave is as follows:

$$\frac{2}{3N} \mathbf{E} \hat{\sigma}_{ew} \mathbf{E} = \frac{e^2 E_0^{(+2)} \nu_e}{3m(\omega_0^2 - \omega_{He}^2)}. \quad (3)$$

Here  $\nu_e = \nu_{em} + \nu_{ei}$  is the total collision frequency of electrons,  $E_0^{(+)}$  is the amplitude of the pump wave, and  $\hat{\sigma}_{ew}$  is the tensor of the electron conductivity for the frequency  $\omega$ . It should be mentioned that in the case of X-mode wave in the denominator of the heating source in equation (3) there exists the difference  $\omega_0^2 - \omega_{He}^2$ , while for O-mode heating there should be the sum of the same terms. This means that the ohmic heating by the X-mode is stronger than the heating by O-mode with the same frequency especially if the frequency  $\omega_0$  is not too high compared to the gyrofrequency  $\omega_{He}$ . In deriving equation (3) it was assumed that the temperatures of ions and neutrals are equal and that the velocities of ions and neutrals are equal to 0 (due to rather frequent collisions between ions and neutrals).

The changes of plasma concentration in the ionosphere under the action of HF pump wave are described by

$$\frac{\partial N}{\partial t} - \frac{\partial}{\partial z_1} D_a^{(\parallel)} \frac{\partial N}{\partial z_1} - \nabla_{\perp} D_a^{(\perp)} \nabla_{\perp} N - \frac{\partial}{\partial z_1} D_{Ta}^{(\parallel)} \frac{\partial T_e}{\partial z_1} - \nabla_{\perp} D_{Ta}^{(\perp)} \nabla_{\perp} T_e = Q_N. \quad (4)$$

Here  $D_a^{(\parallel)}, D_a^{(\perp)}$  are the coefficients of the ambipolar diffusion along and across the magnetic field,  $D_{Ta}^{(\parallel)}, D_{Ta}^{(\perp)}$  are the coefficients of the ambipolar thermodiffusion, and  $Q_N$  is the source of electrons, which is connected with different processes (e.g., ionization and recombination).

We suppose that the total plasma concentration  $N$  in the quasi-stationary conditions consists of three parts:

$$N = N_0 + N_1 + N_2, \quad (5)$$

where  $N_0$  is the plasma concentration in nondisturbed ionosphere,  $N_1$  is a large-scale perturbation caused by the action of the pump wave, and  $N_2$  is a small perturbation associated with the self-focusing instability. Similarly, we present the electron temperature:

$$T_e = T_{e0} + T_{e1} + T_{e2}. \quad (6)$$

After integrating equation (2) along the magnetic field line, we arrive at the following equation:

$$\frac{d}{dt} \int_0^{\infty} T_{e1}(z_1) dz_1 + \int_0^{\infty} \delta \nu_e(z_1) T_{e1}(z_1) dz_1 = \int_0^{\infty} \frac{e^2 E_0^2 \nu_e(z_1)}{3m(\omega_0^2 - \omega_{He}^2)} dz_1. \quad (7)$$

Here  $z_1 = 0$  corresponds to the bottom of the  $E$  region,  $\delta \nu_e = (\delta_{em} \nu_{em} + \delta_{ei} \nu_{ei})$ ,  $\nu_e = \nu_{em} + \nu_{ei}$ . In deriving equation (7) we have taken into account that at the bottom of the ionosphere  $D_{Te}^{(\parallel)}$  is equal to 0, while at the upper end of the field line the perturbation is absent. Equation (7) shows that the total energy of electrons in a given flux tube grows with time, while the perturbation  $T_{e1}$  is small enough. The stationary state is achieved when the input of energy given by the term in the right-hand side in equation (7) is compensated by the energy transfer to ions and neutrals.

In the stationary conditions assuming that the temperatures of ions and neutrals are close to each other ( $T_i = T_m$ ) equation (2) reduces to

$$\delta \nu_e T_{e1} - \frac{\partial}{\partial z_1} D_{Te}^{(\parallel)} \frac{\partial T_{e1}}{\partial z_1} = \frac{e^2 E_0^2 \nu_e}{3m(\omega_0^2 - \omega_{He}^2)}. \quad (8)$$

In the  $E$  region and in the lower part of the  $F$  region (the heights  $z \leq 200$  km) for rather large-scale perturbations, taking into account that diffusion and thermodiffusion are weak enough, it follows from equations (8) and (4) that there is a simple link between perturbations of the electron temperature and plasma concentration (Gurevich, 1978):

$$\frac{N_1}{N_0} \approx \kappa \frac{T_{e1}}{T_{e0}}, \quad (9)$$

where  $\kappa$  is the coefficient of recombination.

For perturbations changing across the magnetic field line as  $\propto \exp(\pm ipk_0x)$  we may use equations (2) and (4) for perturbations of the electron temperature  $T_{e2}$  and plasma concentration  $N_2$ . At the same time electric field perturbations are described by more general equation than equation (1):

$$\frac{d^2 E_1^{(+)}}{dz_1^2} + k_0^2 \left( 1 - \frac{v_0}{1 - \sqrt{u}} - p^2 \right) E_1^{(+)} = k_0^2 \frac{\delta v}{1 - \sqrt{u}} E_0^{(+)}. \quad (10)$$

Here  $E_0^{(+)} = E_{0,x} + iE_{0,y_1}$  is a nondisturbed X-mode pump wave,  $E_1^{(+)} = E_{1,x} + iE_{1,y_1}$  is a small variation caused by plasma perturbation  $N_2$ ,  $v_0 = 4\pi e^2(N_0 + N_1)/m\omega_0^2$ ,  $\delta v = 4\pi e^2 N_2/m\omega_0^2$ . It is convenient to separate out the phase describing the variation in space and in time for the functions presented above:

$$\begin{aligned} E_0^{(+)} &= \tilde{E}_0^{(+)} \exp \left[ i(\omega_0 t - k_0 \int_0^{z_1} q(z_2) dz_2) \right] + c.c., \\ E_1^{(+)} &= \exp \left[ i \left( \omega_0 t - k_0 \int_0^{z_1} q(z_2) dz_2 \right) \right] [E_{1,1}^{(+)} \exp(ik_0 p x) + E_{1,2}^{(+)} \exp(-ik_0 p x)] + c.c., \\ N_2 &= N_{2,1}(y_1) \exp(ik_0 p x) + N_{2,2}(y_1) \exp(-ik_0 p x), \\ T_{e2} &= T_{e2,1}(y_1) \exp(ik_0 p x) + T_{e2,2}(y_1) \exp(-ik_0 p x) \end{aligned} \quad (11)$$

Here  $\tilde{E}_0^{(+)}$ ,  $E_{1,1}^{(+)}$ ,  $E_{1,2}^{(+)}$ ,  $N_{2,1}$ ,  $N_{2,2}$ ,  $T_{e2,1}$ , and  $T_{e2,2}$  are the slowly varying along the  $z_1$  axis amplitudes,  $q(z_1) = 1 - \frac{v_0(z_1)}{1 - \sqrt{u}}$ . We assume that the introduced perturbations  $N_{2,1}$ ,  $N_{2,2}$ ,  $T_{e2,1}$ ,  $T_{e2,2}$ , and  $E_{1,1}^{(+)}$ ,  $E_{1,2}^{(+)}$  are weak enough to be determined from the linear approximation. The presented above equations will be used to describe the heating of electrons and the excitation of plasma instabilities in the  $F$  region under the action of powerful X-mode wave.

### 3. The Thermal Self-Focusing Instability in the $F$ Region of the Ionosphere

The self-focusing instability in nonlinear optics is a well-known phenomenon. This instability takes place in the media, which have self-focusing properties (the nonlinear correction to the dielectric permeability due to high intensity of the wave causes such modification in the media that the beam becomes more narrow and more intensive along its trajectory). As a result the beam tends to be stratified. The  $F$  layer of the ionosphere is a focusing medium. Indeed, the action of a powerful EM beam causes the heating of electrons and expansion of plasma from the heated volume. Due to this, the propagating beam is concentrated near the local minimum of plasma density that provides stronger heating of electrons and additional expansion of plasma. In reality, both the anisotropy (due to strong magnetic field) and nonlocality of the medium (due to high electron thermal conductivity along the magnetic field line) make the instability more complicated. These factors and the interaction of the upward propagating and the reflected waves were taken into account while discussing the action of strong HF O-mode wave on the ionosphere (Gurevich, 1978).

It is important that not only powerful O-mode but also X-mode cause the excitation of the self-focusing instability in the ionosphere. Moreover, the ohmic heating of electrons by X-mode wave is stronger than by O-mode wave of the same intensity and frequency. Below we present some well-known results concerning the excitation of the thermal self-focusing instability in the  $F$  region by the propagating powerful HF radio wave.

We assume that the pump X-mode wave propagates into the ionosphere along the magnetic field line. Its propagation can be described by the parabolic equation, which follows from equation (10) if we present  $E_1^{(+)}$  in the form  $E_1^{(+)} = E_{1,\alpha}^{(+)}(z_1) \exp(-ik_0 \int^{z_1} q(z_2) dz_2)$ :

$$-i2k_0 q(z_1) \frac{\partial E_{1,\alpha}^{(+)}}{\partial z_1} - k_0^2 p^2 \tilde{E}_{1,\alpha}^{(+)} = k_0^2 \frac{\delta v(N_{2,\alpha})}{1 - \sqrt{u}} \tilde{E}_0^{(+)}, \quad (12)$$

where  $\alpha = 1, 2$ . Note that we have used a parabolic equation because the pump wave is a HF wave and its wavelength is much smaller than the typical size of plasma variation along the magnetic field line. It can be

seen that the right-hand side of equation (12) depends on the plasma perturbation  $N_{2,\alpha}$  which in turn is caused by the perturbation of the electron temperature  $T_{e2,\alpha}$ . Suppose that all perturbations  $E_{1,\alpha}^{(+)}$ ,  $N_{2,\alpha}$ , and  $T_{e2,\alpha}$  vary along the  $z_1$  and the  $y_1$  axes as follows:

$$\begin{aligned} E_{1,1}^{(+)} &= \tilde{E}_{1,1}^{(+)} F_1(z_1, x), N_{2,1} = \tilde{N}_{2,1} F_1(z_1, x), T_{e2,1} = \tilde{T}_{e2,1} F_1(z_1, x) \\ E_{1,2}^{(+)} &= \tilde{E}_{1,2}^{(+)} F_2(z_1, x), N_{2,2} = \tilde{N}_{2,2} F_2(z_1, x), T_{e2,2} = \tilde{T}_{e2,2} F_2(z_1, x), \end{aligned} \quad (13)$$

where  $F_1 = \exp(\gamma z_1 + ik_0 p x)$  and  $F_2 = \exp(\gamma z_1 - ik_0 p x)$ . To find the amplitudes (marked by tilde), equations (2) and (4) must be linearized and the time derivative should be omitted (because the self-focusing instability develops in space along the trajectory of the beam). As a result we arrive at the system of equations that consists of equation (12) and the following equations:

$$\begin{aligned} (\delta v_e - \gamma^2 D_{Te}^{(II)}) \tilde{T}_{e2,1} &= \frac{e^2 \tilde{E}_0^{(+)} (\tilde{E}_{1,1}^{(+)} + \tilde{E}_{1,2}^{(+)*}) v_e}{3m(\omega_0^2 - \omega_{He}^2)} \\ (\delta v_e - \gamma^2 D_{Te}^{(II)}) \tilde{T}_{e2,2} &= \frac{e^2 \tilde{E}_0^{(+)} (\tilde{E}_{1,1}^{(+)*} + \tilde{E}_{1,2}^{(+)}) v_e}{3m(\omega_0^2 - \omega_{He}^2)}, \tilde{N}_{2,\alpha} = -\frac{D_{Ta}^{(L)}}{D_a^{(L)}} \tilde{T}_{e2,\alpha}. \end{aligned} \quad (14)$$

In the first two equation (14) we have neglected the transverse electron thermal diffusion and in the last equation the longitudinal ambipolar diffusion and the longitudinal thermal diffusion. Equation (12) and (14) determine the threshold and the growth rate of the thermal self-focusing instability in the ionosphere. The threshold gives the link between the pump field and the transverse wave number of the instability  $p_{th}$ :

$$p_{th}^2 = \frac{v}{1 - \sqrt{u}} \frac{k_T}{T_{e0} + T_{e1}} \frac{2e^2 |\tilde{E}_0^{(+)}|^2}{3m(\omega_0^2 - \omega_{He}^2) \delta}, \quad (15)$$

where  $k_T = \frac{(T_{e0} + T_{e1}) D_{Ta}^{(L)}}{(N_0 + N_1) D_a^{(L)}}$ . For rather small values  $\gamma^2 \leq \frac{\delta v_e}{D_{Te}^{(II)}}$  the growth rate of instability is determined by

$$\gamma^2 \approx \frac{k_0^2 p^2}{4q} (p_{th}^2 - p^2). \quad (16)$$

It is seen that the growth rate tends to 0 for very small transverse wave numbers ( $p^2 \rightarrow 0$ ) and also the instability disappears for large enough wave numbers  $p > p_{th}$  due to stabilization effect of the transverse diffusion. So the maximum growth rate corresponds to some intermediate values  $p^2 \approx 0.5 p_{th}^2$ . For typical parameters of the ionosphere and the pump wave the generated artificial irregularities have transverse sizes of the order of hundreds of meters.

The reflection of the pump wave in the  $F$  region causes the interaction between two waves propagating in opposite directions. This interaction changes the nature of instability (the perturbations begin to grow in time in contrast with the case discussed above). Analytical approach to theory of the thermal self-focusing instability in the vicinity of the reflection height can be found in Gurevich (1978). It is important that the growth rate depends not only on the intensity of the pump wave but also on the vertical gradient of the ionosphere. If the reflection takes place near the critical height of the  $F$  region (small vertical gradient), the instability becomes more intensive. Since the self-focusing in the  $F$  region is connected with the redistribution of plasma in a large volume by means of the ambipolar diffusion, this process takes significant duration of time. To achieve the stationary state, it requires the period of the order of 1 min.

The development of the thermal self-focusing instability in the  $F$  region was investigated not only analytically but also numerically. It was shown that the action of strong HF radio wave on the ionosphere, at the nonlinear stage, provides the formation of strong perturbations of plasma concentration and the electron temperature across the magnetic field line (up to 10%; see, e.g., Guzdar et al., 1989). Despite that the simulations were carried out in 2-D approximation, it is clear that in the plane across the magnetic field line there is no preferable direction. Therefore, the gradients of the electron temperature and plasma concentration should appear along both axes ( $x$  and  $y_1$ ).

#### 4. Excitation of the Temperature Gradient-Drift Instability in the *F* Region of the Ionosphere

It is known that if the plasma is inhomogeneous across the magnetic field line, several types of drift instabilities can be excited. One such instability is the temperature gradient-drift instability that has been investigated in a completely ionized plasma in kinetics (Kadomtsev, 1965) and in two-fluid hydrodynamics (Hudson & Kelley, 1976). This instability was attributed to explaining the excitation of plasma irregularities in some parts of the auroral (or subauroral) ionosphere (Greenwald et al., 2006; Hudson & Kelley, 1976). It was argued by Greenwald et al. (2006) that the temperature gradient-drift instability can provide the formation of decametric plasma irregularities in subauroral ionosphere even in case of very small drift velocities.

In distinction with the papers mentioned above in our case plasma gradients are associated with the artificial plasma irregularities caused by the development of the thermal self-focusing instability. Plasma gradients created by this instability usually are stronger than plasma gradients of the natural ionosphere. To discuss the temperature gradient-drift instability, we need to use more accurate system of equations than equations (2) and (4).

We start with the linearized system of hydrodynamic equations, which consists of the continuity equations:

$$\frac{\partial n_\alpha}{\partial t} + \nabla(N\mathbf{v}_\alpha) = 0, \quad (\alpha = i, e) \quad (17)$$

equation of the motion of electrons

$$\frac{d\mathbf{v}_e}{dt} = -\frac{e\mathbf{E}}{m} - \frac{e}{mc}[\mathbf{v}_e \times \mathbf{H}_0] - \frac{\nabla p_e}{mN} - v_{en}\mathbf{v}_e - v_{ei}(\mathbf{v}_e - \mathbf{v}_i) + \mathbf{e}_{z_1}R_{T,z_1}, \quad (18)$$

equation of the motion of ions

$$\frac{d\mathbf{v}_i}{dt} = \frac{e\mathbf{E}}{M} + \frac{e}{Mc}[\mathbf{v}_i \times \mathbf{H}_0] - \frac{\nabla p_i}{MN} - v_{in}\mathbf{v}_i - v_{ie}(\mathbf{v}_i - \mathbf{v}_e), \quad (19)$$

equation of quasi-neutrality  $n_i = n_e$ , and the heat equation for electrons

$$\frac{3}{2} \frac{d\delta T_e}{dt} + T_e \nabla \cdot \mathbf{v}_e = -\nabla \cdot \mathbf{q}_e + Q. \quad (20)$$

Here  $v_{ie}$ ,  $v_{in}$  are the collision frequencies of ions with electrons and neutrals,  $M$  is a mass of ions,  $p_e$  is the perturbed pressure of electrons  $p_e = nT_e + N\delta T_e$ ,  $n$  and  $\delta T_e$  are the small variations of plasma concentration and the electron temperature associated with the excitation of the temperature gradient-drift instability, and  $p_i = nT_i$  is the perturbation of the ion pressure. The thermal force along the magnetic field line is  $R_{T,z_1} = -c_t \frac{\partial \delta T_e}{m \partial z_1}$ . The linearized heat flux for electrons has significant component along the magnetic field line:

$$q_{\parallel} = c_t T_e v_{e,z_1} - c_{\parallel} \frac{\partial \delta T_e}{m v_e \partial z_1} \quad (21)$$

and also the component across the magnetic field line

$$q_{\perp} = -c_{\perp} \left( \frac{NT_e}{m\omega_{He}} [\mathbf{e}_{z_1} \times \nabla \delta T_e] + \frac{nT_e + N\delta T_e}{m\omega_{He}} [\mathbf{e}_{z_1} \times \nabla T_e] \right). \quad (22)$$

Here  $\mathbf{e}_{z_1}$  is a unit vector along the magnetic field line, the source  $Q$  describes the heating of electrons in plasma perturbations by the pump wave and the losses of the electron energy due to collisions with heavy particles (ions and molecules)  $Q = Q_{cl} + Q_E$ , where

$$Q_E = \frac{e^2 v_e (E_0^{(+)} E_n^* + E_0^{(+)*} E_n)}{3m(\omega_0^2 - \omega_{He}^2)}.$$

Here  $E_n$  is a small perturbation of the electric field caused by the interaction of the pump wave with the excited plasma irregularities  $n$ . In the ionospheric plasma nonelastic collisions give the main input to losses of the electron energy  $Q_{cl} \approx -\mu v_e \delta T_e$ , where  $\mu \approx 10^{-3}$  for thermal electrons; see Gurevich (1978). Exact values of

the coefficients  $c_t$ ,  $c_{\parallel}$ , and  $c_{\perp}$  can be obtained only for completely ionized plasma with one sort of ions. In such case according to Braginskii (1965),  $c_t = 0.71$ ,  $c_{\parallel} = 3.16$ , and  $c_{\perp} = 2.5$ . However, the ionospheric plasma is partly ionized and contains different types of ions. That is why, in the general case, the exact values of the coefficients  $c_t$ ,  $c_{\parallel}$ , and  $c_{\perp}$  cannot be obtained as it is done for a completely ionized plasma. Nevertheless, it may be assumed that the equations presented above can be applied for the ionospheric plasma at certain heights since the collision frequency of electrons with ions exceeds the collision frequency with neutral molecules in the  $F$  region from about 250 km in daytime and from 300 km in nighttime.

As it can be seen below the exact values of the coefficients  $c_t$ ,  $c_{\parallel}$ , and  $c_{\perp}$  are not very important for crude estimates of the growth rate of the instability. Note that due to self-focusing instability, the background plasma concentration and the electron temperature are inhomogeneous across the magnetic field (the terms  $N_2$  and  $T_{e2}$ ). The total plasma concentration can be presented as the sum of  $N + n$ , where  $N = N_0 + N_1 + N_2$  determines the background concentration for the instability.

It follows from equations (18) and (19) that in the stationary condition ( $\frac{d}{dt} = 0$ ), we have

$$N(T_e + T_i) + \frac{H_0^2}{8\pi} = \text{const.} \quad (23)$$

Applying the derivative  $d/dy_1$  to equation (23), we find

$$\kappa_T \approx -\kappa_N, \quad (24)$$

where  $\kappa_T = dT_e/T_e dy_1$ ,  $\kappa_N = dN/N dy_1$ . Such gradients, created by the self-focusing instability, are required for the excitation of the temperature gradient instability within the heated volume in the  $F$  region. If we neglect the electric field in the auroral electrojet (i.e., considering quiet conditions), then only the polarization electric field associated with the instability is present  $\mathbf{E} = -\nabla\varphi$ , where  $\varphi$  is the potential.

We shall investigate the linear stage of the temperature gradient-drift instability. First, we consider the case when the heating term  $Q_E$  in equation (20) is neglected (the intensity of the pump wave is not too high). Assume that the perturbations vary in space and in time as  $\propto f(y_1) \exp[i(\omega t - k_x x - k_{z_1} z_1)]$ . The linearized equations for electrons consist of the continuity equation

$$i\omega \frac{n}{N} + i\omega_N^* \frac{e\varphi}{T_e} - ik_{z_1} v_{e,z_1} = 0, \quad (25)$$

equation of the motion of electrons along the magnetic field

$$ik_{z_1} v_{e,z_1} = k_{z_1}^2 D_{\parallel} \left[ -(1 + c_t) \frac{\delta T_e}{T_e} - \frac{n}{N} + \frac{e\varphi}{T_e} \right], \quad (26)$$

and equation for the perturbed electron temperature

$$\left[ \frac{3}{2} i(\omega - \omega_N^*) + c_{\parallel} k_{z_1}^2 D_{\parallel} \right] \frac{\delta T_e}{T_e} - i(c_{\perp} - 1) \omega_{Te}^* \frac{n}{N} = (1 + c_t) ik_{z_1} v_{e,z_1}. \quad (27)$$

Here

$$\omega_N^* = \frac{\kappa_N k_x V_{Te}^2}{\omega_{He}}, \quad \omega_{Te}^* = \frac{\kappa_T k_x V_{Te}^2}{\omega_{He}}, \quad D_{\parallel} = \frac{T_e}{m\nu_e}.$$

From equations for ions we find

$$\frac{n}{N} = -\frac{\omega_N^*}{\omega} - i \frac{k_x^2 T_e \nu_{in}}{M \omega_{Hi}^2 \omega}, \quad (28)$$

where  $M$  is the mass and  $\omega_{Hi} = \frac{eH_0}{Mc}$  is the Larmour frequency of ion. Assume that the longitudinal diffusion of electrons is very high:

$$k_{z_1}^2 D_{\parallel} \gg |\omega|, |\omega_N^*|, |\omega_{Te}^*|, \quad (29)$$

while the transverse ambipolar diffusion is small:

$$\frac{k_x^2 T_e v_{in}}{M \omega_{Hi}^2} \ll |\omega|, |\omega_N^*|, |\omega_{Te}^*|. \quad (30)$$

Equating relative perturbations of plasma concentration for electrons and ions, we arrive at the following approximate equation describing the temperature gradient-drift instability:

$$\omega = -\omega_N^* + i \frac{k_x^2 T_e v_{in}}{M \omega_{Hi}^2} + i \eta \frac{\omega_{Te}^* \omega_N^*}{k_{z1}^2 D_{\parallel}}, \quad (31)$$

where the coefficient  $\eta$  is of the order of unity. It follows from equation (31) that the instability is excited if the following conditions are fulfilled. First, the gradients of plasma concentration and the electron temperature should have opposite signs. This result is in accordance with the previous investigations (Hudson & Kelley, 1976; Kadomtsev, 1965). Second, the last term in equation (31) should exceed the term describing the transverse ambipolar diffusion:

$$\eta \frac{|\omega_{Te}^* \omega_N^*|}{k_{z1}^2 D_{\parallel}} > \frac{k_x^2 T_e v_{in}}{M \omega_{Hi}^2}. \quad (32)$$

This relation means that the excited perturbations must be elongated:

$$k_{z1}^2 < \frac{m}{M} \frac{v_e}{v_{in}} |\kappa_N \kappa_T| \eta \quad (33)$$

If the condition (33) holds the temperature gradient-drift instability comes into play and its growth rate increases with the diminishing of the transverse scale of perturbations (growth of  $k_x$  numbers). Note that the approximation of two-fluid hydrodynamics can be applied only for transverse scales much larger than the Larmour radius of ions  $k_x^2 \rho_{Hi}^2 \ll 1$ . The growth rate of this instability as a function of transverse size was investigated in kinetics by Eltrass et al. (2014). It follows from this analysis that in case  $T_e \gg T_i$  the maximum growth rate corresponds to transverse wave numbers  $k_x \approx 0.7 \rho_{Hi}^{-1}$ . We shall estimate the growth rate for somewhat larger scales  $k_x \sim 0.25 \rho_{Hi}^{-1}$  utilizing two-fluid hydrodynamics. For Larmour radius of ions  $\rho_{Hi} = 7.10^2$  cm, the inhomogeneity of plasma produced by the self-focusing instability  $|\kappa_T| \approx |\kappa_N| \sim 5.10^{-6}$  cm<sup>-1</sup>, collision frequencies of electrons  $\nu_e \approx 10^2$  s<sup>-1</sup> and ions  $\nu_{in} \approx 2$  s<sup>-1</sup> we find from equation (32) that the instability exists if  $k_{z1} < k_{\parallel}^{(0)} \approx 3.10^{-7}$  cm<sup>-1</sup>. For  $k_{z1} = 2.10^{-7}$  the growth rate of the instability is  $\gamma \approx 0.05$  s<sup>-1</sup>. This means that the instability is rather fast and the decametric perturbations should appear in the *F* region soon after the development of the self-focusing instability. The results obtained above are based on a well-known theory of the temperature gradient-drift instability. Only there are two distinctions. First, in our case the gradients of plasma caused by the development of the self-focusing instability are much larger than in the nondisturbed ionosphere and due to this the instability develops rather fast. Second, we have included the transverse dissipation of perturbations and found out that only strongly elongated plasma irregularities can be excited. We have discussed only the linear stage of the temperature drift instability. This is because our aim is to confirm that decametric plasma irregularities can be excited due to the action of the HF X-mode wave on the ionosphere. More detailed analysis including the nonlinear stage of instability will be presented later. Note that without heating by HF radio wave condition (33) can only be satisfied for extremely elongated perturbations because the gradients  $\kappa_N$  and  $\kappa_T$  are much smaller. Due to this, according to our analysis, to excite the temperature gradient instability in quiet conditions in the ionospheric *F* region without strong HF heating is very difficult.

The interaction of the pump wave with the plasma irregularities excited by the temperature drift instability was neglected in the above discussion. Now we need to take into account such interaction and to estimate its magnitude. This problem is discussed in the next section.

## 5. Interaction of the Pump Wave With the Excited Plasma Perturbations

The scattering of the pump wave by the excited plasma irregularities modifies the usual approach to the temperature gradient-drift instability. To take into account this process, we present the perturbations of plasma concentration and the electron temperature in the following way:

$$\begin{aligned} n &= \tilde{n}_1 \exp[i(\omega t - k_x x)] + \tilde{n}_2 \exp[-i(\omega t - k_x x)] \\ \delta T_e &= \delta \tilde{T}_{e,1} \exp[i(\omega t - k_x x)] + \delta \tilde{T}_{e,2} \exp[-i(\omega t - k_x x)]. \end{aligned} \quad (34)$$

In the scattered electric field we single out the phase factor of the propagating pump wave:

$$E_n = \tilde{E}_{n,1} \exp[i(\omega_0 - k_0 \int^{z_1} q(z_2) dz_2 - k_x x)] + \tilde{E}_{n,2} \exp[i(\omega_0 - k_0 \int^{z_1} q(z_2) dz_2 + k_x x)].$$

Here  $\tilde{n}_1, \tilde{n}_2, \delta\tilde{T}_{e,1}, \delta\tilde{T}_{e,2}, \tilde{E}_{n,1}$ , and  $\tilde{E}_{n,2}$  are the amplitudes of perturbations and  $\omega_0$  and  $k_0 q(z_1)$  are the frequency of the pump wave and its wave vector along the magnetic field line. Interaction of the pump wave with the plasma perturbations  $\tilde{n}_1$  and  $\tilde{n}_2$  results in formation of the scattered waves  $\tilde{E}_{n,1}$  and  $\tilde{E}_{n,2}$ , which are described by equations

$$\begin{aligned} \left(-i2k_0 q(z_1) \frac{\partial}{\partial z_1} - k_x^2\right) \tilde{E}_{n,1} &= \frac{\omega_{pe}(N)^2}{c^2} E_0^{(+)} \frac{\tilde{n}_1}{N} \exp[i(\omega t - k_x x)] \\ \left(-i2k_0 q(z_1) \frac{\partial}{\partial z_1} - k_x^2\right) \tilde{E}_{n,2} &= \frac{\omega_{pe}(N)^2}{c^2} E_0^{(+)} \frac{\tilde{n}_2}{N} \exp[-i(\omega t - k_x x)]. \end{aligned} \quad (35)$$

The waves  $\tilde{E}_{n,1}, \tilde{E}_{n,2}$  contribute to the source  $Q_E$  in equation (20) for the electron temperature and produce the mixing between the perturbations  $\tilde{n}_1, \tilde{n}_2, \delta\tilde{T}_{e,1}$ , and  $\delta\tilde{T}_{e,2}$ . This makes the analysis of the temperature gradient-drift instability in general case more complicated. Here we consider the most simple case. Taking into account that the right-hand sides of equation (35) change very slowly along the  $z_1$  axis because the perturbations  $n_1$  and  $n_2$  according to equation (33) are strongly elongated, we present approximate solutions of equation (35) as follows:

$$\begin{aligned} \tilde{E}_{n,1} &\approx -\frac{\omega_{pe}(N)^2}{c^2 k_x^2} E_0^{(+)} \frac{\tilde{n}_1}{N} \exp[i(\omega t - k_x x)] \\ \tilde{E}_{n,2} &\approx -\frac{\omega_{pe}(N)^2}{c^2 k_x^2} E_0^{(+)} \frac{\tilde{n}_2}{N} \exp[-i(\omega t - k_x x)]. \end{aligned} \quad (36)$$

As a result we have two terms  $Q_{E,1}$  and  $Q_{E,2}$  that vary with time as  $\propto \exp(i\omega t)$  and  $\propto \exp(-i\omega t)$  correspondingly:

$$\begin{aligned} Q_{E,1} &= -\frac{e^2 v_e}{3m(\omega_0^2 - \omega_{He}^2)} \frac{\omega_{pe}(N)^2}{c^2 k_x^2} |E_0^{(+)}|^2 \left(\frac{\tilde{n}_2^*}{N} + \frac{\tilde{n}_1}{N}\right) \exp[i(\omega t - k_x x)] \\ Q_{E,2} &= -\frac{e^2 v_e}{3m(\omega_0^2 - \omega_{He}^2)} \frac{\omega_{pe}(N)^2}{c^2 k_x^2} |E_0^{(+)}|^2 \left(\frac{\tilde{n}_1^*}{N} + \frac{\tilde{n}_2}{N}\right) \exp[-i(\omega t - k_x x)]. \end{aligned} \quad (37)$$

It is seen from these expressions that the variable  $\delta\tilde{T}_{e,1}$  is coupled with  $\tilde{n}_2^*$  and  $\tilde{n}_1$  and  $\delta\tilde{T}_{e,2}$  is coupled with  $\tilde{n}_2$  and  $\tilde{n}_1^*$ , thus presenting a more complicated system of equations than before. Let us compare the term  $Q_{E,1}$  (or  $Q_{E,2}$ ) with the main term in equation (20) for the electron temperature  $k_{z1}^2 D_{||} \delta\tilde{T}_{e,1}, (k_{z1}^2 D_{||} \delta\tilde{T}_{e,2})$ . It follows from equations (25)–(27) that the relative variations of the electron temperature are rather small in case of strong longitudinal thermodiffusion:

$$k_{z1}^2 D_{||} \frac{\delta T_e}{T_e} \sim \omega_N^* \frac{n}{N}. \quad (38)$$

The source terms (37) provide small input to the temperature gradient drift instability if

$$|\omega_N^*| > \frac{e^2 v_e}{3m(\omega_0^2 - \omega_{He}^2)} \frac{\omega_{pe}(N)^2}{c^2 k_x^2} \frac{|E_0^{(+)}|^2}{T_e}. \quad (39)$$

This condition can be rewritten in another way:

$$k_x > k_{cr} = \left( \frac{V_E^2 \omega_{pe}(N)^2 v_e \omega_{He}}{3c^2 V_{Te}^4 \kappa_N} \right)^{1/3}, \quad (40)$$

where  $V_E = \frac{eE_0^{(+)}}{m\omega_0}$  is the amplitude of the electron velocity in the electric field of the pump wave. For strong electric field  $E_0 \approx 1$  V/m of the pump wave with the frequency  $\omega_0 = 3.10^7$  s<sup>-1</sup> inequality (40) in the  $F$  region means that  $k_x \geq 10^{-4}$  cm<sup>-1</sup>. According to relation (40), for the excitation of decametric plasma irregularities the source terms (37) can be neglected.

Note that the waves  $\tilde{E}_{n,1}$  and  $\tilde{E}_{n,2}$  were introduced in a similar manner as those that determine the thermal self-focusing instability ( $\tilde{E}_{1,1}^{(+)}$ ,  $\tilde{E}_{1,2}^{(+)}$ ); see section 3. But in contrast with section 3 the waves  $\tilde{E}_{n,1}$  and  $\tilde{E}_{n,2}$  sometimes play insignificant role; see equations (39) and (40). More general case corresponding to larger plasma irregularities  $k_x < k_{cr}$  will be discussed in a separate paper.

## 6. The Gradient-Drift and the Current-Convective Instabilities

Besides the temperature gradient-drift instability two other drift instabilities (the gradient-drift and the current-convective instabilities) are often attributed for the explanation of plasma irregularities generated in the ionospheric  $F$  region (Chaturvedi & Chaturvedi, 1981; Guzdar et al., 1998; Huba, 1984; Ossakow & Chaturvedi, 1979; Tsunoda, 1988). In this section we would like to investigate if these instabilities contribute to the formation of artificial decametric irregularities in the auroral  $F$  region illuminated by high power HF radio waves with X-mode polarization.

Let us take into account the electrojet that exists in the auroral ionosphere. Assume that its electric field  $E_{jet}$  is directed along the  $x$  axis. In such case the drift speed of ions and electrons in the  $F$  region along the  $y_1$  axis approximately are equal to each other:

$$V_{e,y_1}^{(0)} = V_{i,y_1}^{(0)} = V_d = -\frac{c}{H_0} E_{jet}. \quad (41)$$

At the same time there is a finite component of the drift speed of ions along the  $x$  axis:

$$V_{i,x}^{(0)} = \frac{v_{in}}{\omega_{Hi}} \frac{c}{H_0} E_{jet}, \quad (42)$$

while the drift of electrons along this axis is much smaller.

In the system of coordinates moving along the  $y_1$  axis with the speed of electrons and ions we have the same equations for electrons as in the previous section. But for ions there exists a small distinction compared with equation (28)

$$\frac{n}{N} = -\frac{\omega_N^*}{\omega - k_x V_{i,x}^{(0)}} - i \frac{k_x^2 T_e v_{in}}{M \omega_{Hi}^2 (\omega - k_x V_{i,x}^{(0)})}. \quad (43)$$

After the substitution of equation (43) into equation of quasi-neutrality we arrive at the following result

$$\omega = -\omega_N^* + k_x V_{i,x}^{(0)} + i \frac{k_x^2 T_e v_{in}}{M \omega_{Hi}^2} + i \frac{\omega_N^*}{k_{z_1}^2 D_{\parallel}} \left( \eta \omega_{Te}^* + k_x V_{i,x}^{(0)} \right). \quad (44)$$

It is seen from equation (44) that for small electric field  $E_{jet}$  the frequency and the growth rate of instability coincide with the result (31). The drift of ions influences significantly the growth rate if

$$|V_d| \geq \eta \frac{|\kappa_T| T_e}{M v_{in}}. \quad (45)$$

For ionospheric parameters given at the end of section 4 this inequality means that  $|V_d|$  should exceed the speed  $|V_d| \geq 3, 3 \cdot 10^4$  cm/s (electric field in the electrojet  $E_{jet} \geq 14$  mV/m). Such speeds are reported in auroral  $F$  region in disturbed conditions. The sign of the last term in square brackets in equation (45) is very important. The transverse drift of ions increases the growth rate of instability only if  $\kappa_N V_{i,x}^{(0)} < 0$ . In the opposite case the transverse drift of ions diminishes the growth rate and can even suppress the instability. Hence, the gradient-drift instability can contribute to the formation of decametric plasma irregularities only in disturbed conditions.

Now we would like to check if the current-convective instability could generate decametric plasma irregularities under the X-mode heating. In contrast with the instability discussed above the current-convective instability is excited by a longitudinal current (Kadomtsev, 1965). To take into account the longitudinal current produced by moving electrons, we need to insert in equations (25)–(27) for electrons the Doppler shift  $\omega \rightarrow \omega - k_{z_1} V_{e,z_1}^{(0)}$ , or (which is equivalent) in equations for ions to include the motion with the opposite speed

$\omega \rightarrow \omega + ik_{z_1} V_{e,z_1}^{(0)}$ . As a result we arrive at the final expression describing the excitation of irregularities in the  $F$  region due to the ohmic heating of plasma by strong HF radio wave:

$$\omega = -\omega_N^* + k_x V_{i,x}^{(0)} - k_{z_1} V_{e,z_1}^{(0)} + i \frac{k_x^2 T_e v_{in}}{M \omega_{Hi}^2} + i \frac{\omega_N^*}{k_{z_1}^2 D_{\parallel}} \left( \eta \omega_{Te}^* + k_x V_{i,x}^{(0)} - k_{z_1} V_{e,z_1}^{(0)} \right) \quad (46)$$

Equation (46) describes three different drift instabilities. The first term in brackets in the right-hand side corresponds to the temperature drift instability, the second term relates to the gradient-drift instability, while the third term in brackets corresponds to the current-convective instability. Equation (46) is valid both for O-mode and X-mode heating. But in case of O-mode pump wave much faster and more efficient mechanism-transformation into UHR oscillations plays the main role in creation of decametric artificial irregularities in the  $F$  region, which can be detected by oblique sounding (e.g., Cutlass radar). The estimates based on plasma parameters presented at the end of section 4 and equation (46) show that to excite the decametric irregularities, the longitudinal velocity of electrons should exceed a high value  $V_{e,z_1}^{(0)} > V_{e,cr} \approx 2.10^5$  cm/s.

Let us estimate the real values of the longitudinal electron velocities in the auroral  $F$  region. Under the action of high power HF radio wave on auroral electrojet the perturbation of electric conductivity  $\delta\sigma$  appears in the  $E$  region within the heated region. This perturbation causes significant local modification of the current system. As a result 3-D current system is formed, which consists of electric currents closed in the horizontal plane and electric currents propagating along the magnetic field line (Borisov & Stubbe, 1997). To estimate the magnitude of the longitudinal current due to the local modification of electric conductivity by HF heating, we apply the same approach that was used for a natural current system (Gurevich et al., 1976).

For this purpose we proceed with the continuity equation of the electric current, which contains in the right-hand side the source of perturbation:

$$\frac{\partial}{\partial z_1} \sigma_{\parallel} \frac{\partial \Phi}{\partial z_1} + \sigma_p \Delta_{\perp} \Phi = \nabla(\delta\hat{\sigma}_{\perp} \mathbf{E}_{jet}). \quad (47)$$

Here  $\Phi$  is electric potential describing the electric field of polarization  $\mathbf{E}_p = -\nabla\Phi$ ,  $\sigma_{\parallel}$  is the nondisturbed electric conductivity along the magnetic field line,  $\sigma_p$  is the nondisturbed Pedersen electric conductivity across the magnetic field line,  $\delta\hat{\sigma}_{\perp}$  is the tensor of the transverse conductivity associated with the perturbation of the plasma concentration, and  $\mathbf{E}_{jet}$  is the electric field in the auroral electrojet. Electrons and ions give input to the Pedersen electric conductivity in the  $E$  region:

$$\sigma_p = \frac{\omega_{pe}^2 v_{en}}{4\pi\omega_{He}^2} + \frac{\omega_{pi}^2 v_{in}}{4\pi(\omega_{Hi}^2 + v_{in}^2)}, \quad (48)$$

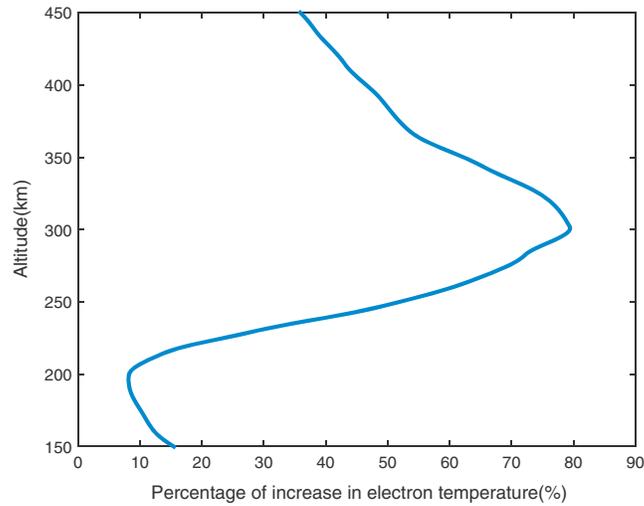
where  $\omega_{pi} = \sqrt{\frac{4\pi e^2 N}{M}}$  is the ion plasma frequencies. Note that  $\sigma_p$  in the presented form (48) can be used only in the lower ionosphere where the main part of the transverse conductivity is concentrated. The tensor  $\delta\hat{\sigma}_{\perp}$  consists of two components  $\{\delta\sigma_p$  and  $\delta\sigma_H\}$  (the Pedersen and the Hall components). It should be mentioned that in the  $E$  region for slowly changing in time perturbations the main input to the source in the right-hand side of equation (47) give variations of plasma concentration  $N_1$  (not the electron collision frequency):

$$\delta\sigma_p \approx \frac{eN_1 c}{H_0} \frac{\omega_{Hi} v_i}{\omega_{Hi}^2 + v_i^2}, \quad \delta\sigma_H \approx \frac{eN_1 c}{H_0} \left( \frac{\omega_{He}^2}{\omega_{He}^2 + v_e^2} - \frac{\omega_{Hi}^2}{\omega_{Hi}^2 + v_i^2} \right). \quad (49)$$

In the  $E$  region there is an increase of plasma concentration ( $N_1 > 0$ ) due to the recombination process; see equation (9).

It is possible to find an approximate solution of equation (47) assuming that the transverse perturbations are large scale and the polarization current is closed in the conjugate ionosphere. As the longitudinal electric conductivity determined by electrons is very high, the electric potential can be presented as a series  $\Phi = \Phi^{(1)} + \Phi^{(2)} + \dots$ . In the first approximation the electric potential does not depend on the longitudinal coordinate  $z_1$ :

$$\Delta_{\perp} \Phi^{(1)} \approx \frac{\int_0^{\infty} \nabla(\delta\hat{\sigma}_{\perp} \mathbf{E}_{jet}) dz_1}{\Sigma_p}, \quad (50)$$



**Figure 2.** Percentage of increase in  $T_e$  under the action of X-mode heating. During the European Incoherent Scatter Scientific Association heating experiment at Tromsøen 19 October 2012, European Incoherent Scatter Scientific Association measurements of electron temperature and electron density for the period before heater being turned on have been used as background plasma parameters, whereas the neutral parameters are from NRLMSISE-00.

where  $\Sigma_p = \Sigma_p^{(N)} + \Sigma_p^{(S)}$  is the sum of the height-integrated Pedersen conductivities in both ionospheres (the northern and the southern). In the next approximation we find the longitudinal electric field as

$$\sigma_{\parallel} \frac{\partial \Phi^{(2)}}{\partial z_1} = \int_0^{z_1} [\nabla(\delta \hat{\sigma}_{\perp} \mathbf{E}_{\text{jet}}) - \sigma_p(z_2) \Delta_{\perp} \Phi^{(1)}] dz_2, \quad (51)$$

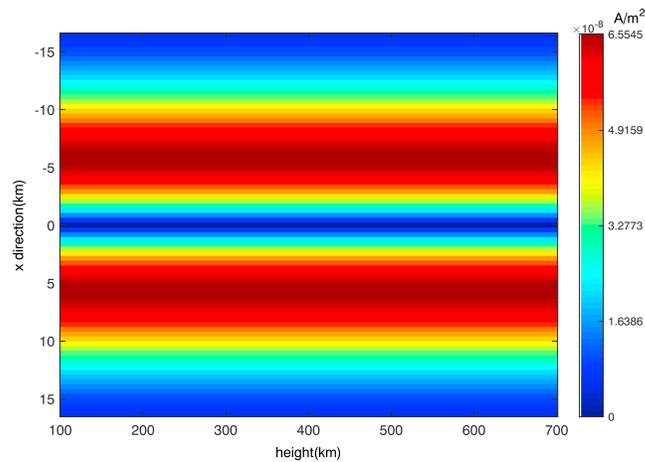
where  $\Delta_{\perp} \Phi^{(1)}$  is determined by equation (50). The increase of the electron concentration  $N_1$  in the heated region caused by the recombination process takes place only in the lower ionosphere (approximately up to the height  $\sim 200$  km) where the recombination is stronger than the diffusion process (Gurevich, 1978). So the longitudinal electric current given by equation (51) in the  $F$  region is presented as

$$j_{\parallel} \approx \left( 1 - \frac{\int_0^{z_1} \sigma_p(z) dz}{\Sigma_p} \right) \nabla(\delta \hat{\Sigma} \mathbf{E}_{\text{jet}}), \quad (52)$$

where the tensor  $\delta \hat{\Sigma}$  determines the perturbation of the height-integrated Pedersen and Hall conductivities. It is seen that the longitudinal current decreases with the height in the ionosphere remains almost constant in the upper ionosphere and the magnetosphere (where  $\sigma_p \approx 0$ ) and tends to 0 in the conjugate ionosphere. Note that this current divided by the electric charge and the concentration of electrons  $eN_0$  gives the hydrodynamic velocity of electrons along the magnetic field line  $V_{e,z_1}^{(0)} = j_{\parallel} / eN_0$ . The magnitude of this velocity is very important because it determines the threshold of the current-convective instability. Therefore, it is important to know the typical magnitude of this velocity in the  $F$  region (approximately from the heights 250 km and higher) under the action of X-mode heating.

First, we need to calculate numerically how the electron temperature and plasma concentration are modified in the ionosphere under HF X-mode heating. For this purpose data from the experiment carried out on 19 October 2012 by Blagoveshchenskaya and her team have been utilized. The effective radiated power of the pump wave in this experiment was 458 MW and the frequency of its operation was chosen as 6.2 MHz. European Incoherent Scatter Scientific Association measurements of electron temperature and electron density for the period before heater being turned on have been used as background plasma parameters, whereas the neutral parameters are from NRLMSISE-00. Figure 2 illustrates the modeled stationary electron temperature enhancement employing equation (8). It is seen that the maximum growth of the electron temperature (up to 80%) corresponds to the height of about 300 km. At the same time in the  $E$  region where the main perturbation of the Pedersen conductivity exists the increase is only about 10–15%.

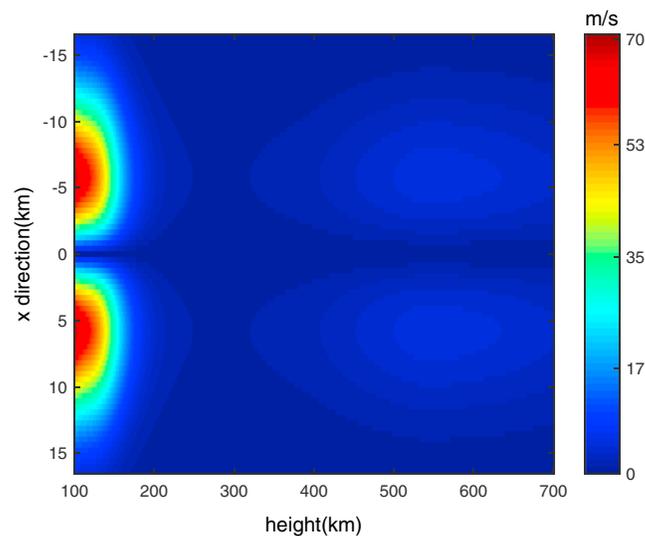
The obtained perturbations of the electron temperature  $T_{e1}$  makes it possible to find with the help of equation (9) the perturbation of plasma concentration  $N_1$  in the heated volume that determines the variations of the



**Figure 3.** Height variation of the longitudinal electric current during the X-mode heating experiments on 19 October 2012. The electron temperature and electron density for the period heater being turned on are used as initial parameters, the data of ion densities and ion temperature are from the IRI2012, and the neutral molecule parameters are from NRLMSISE-00.

Pedersen conductivity; see equation (49). The variation of the longitudinal electric current can be simulated by using equation (52). As it follows from equation (52), this electric current depends on the electric field in the electrojet  $E_{jet}$  and the magnitudes of different conductivities (perturbation of the Pedersen conductivity, the background Pedersen conductivity, and the height integrated Pedersen conductivity) and decreases with an increase in height. In Figure 3 the longitudinal electric current as a function of height for the experiment carried out on 19 October 2012 is depicted. The electric current is presented in  $A/m^2$ , the horizontal axis represents ionosphere height, the longitudinal axis is calculated from the center of the heated spot, and the range is  $[-2a, 2a]$ , where  $a$  is the radius of the heated spot.

Figure 4 shows the corresponding velocity of electrons, which decreases with height (below the  $F$  maximum), but increases at larger height due to diminishing of the plasma concentration. The meaning of the horizontal axis and the longitudinal axis is the same as that in Figure 3.



**Figure 4.** Height variation of the longitudinal velocity of electrons during the X-mode heating experiments on 19 October 2012. The electron temperature and electron density for the period heater being turned on are used as initial parameters, the data of ion densities and ion temperature are from the IRI2012, and the neutral molecule parameters are from NRLMSISE-00.

It is clear from this figure that the typical longitudinal electron velocities in the  $F$  region are of the order of  $V_{e,z_1}^{(0)} \sim (20-50)$  m/s, which are much smaller than those required for the excitation of the current-convective instability  $V_{e,z_1}^{(0)} \approx 2$  km/s.

## 7. Discussion and Conclusion

In our paper we have discussed the physical mechanisms that can explain the excitation of artificial plasma irregularities under the action of a powerful HF X-mode wave on the auroral ionosphere. The suggested mechanism requires as a first step the excitation of the self-focusing instability. This instability is developed approximately in 1 min after the heater is turned on and provides the formation of plasma irregularities with the characteristic transverse sizes much smaller than the heated spot in the  $F$  region. The instability becomes stronger with the increase of the applied HF power and the ionospheric vertical scale in the vicinity of the reflection height (Gurevich, 1978). So the situation is most favorable if the reflection of the pump wave takes place near the maximum of the  $F$  layer. At the same time the self-focusing instability develops also if the frequency of the pump wave is high enough and the wave penetrates above the  $F$  maximum without any reflection. Only the manifestation of the instability is somewhat different in two cases (it develops in time near the reflection height and in space if there is no reflection; see Gurevich, 1978). Also for a given frequency and the intensity of the HF radiation the X-mode pump wave produces stronger ohmic heating and hence stronger perturbations. According to our analysis in the presence of large-scale artificial inhomogeneities (generated by the self-focusing of a pump wave), the temperature gradient-drift instability can be excited in the auroral  $F$  region. This instability is responsible for plasma perturbations of various sizes (up to decametric irregularities). Experiment confirms that such perturbations indeed appear in the  $F$  region under the strong X-mode heating (Blagoveshchenskaya et al., 2011). Note that one of the main distinctions with the artificial decametric irregularities excited due to O-mode heating is the characteristic period of the development. In the case of O-mode heating small-scale artificial irregularities (striations) appear much faster (up to a few seconds) compared with the case of X-mode heating (on the average within a few minutes). Such distinction is connected with different physical mechanisms. In the case of O-mode heating the conversion of the EM pump wave into electrostatic (the UHR) oscillations provides very efficient and fast formation of striations. In the case of X-mode heating such mechanism does not operate and the generation of large-scale artificial irregularities (due to the self-focusing) is required as a first step for the development of the temperature gradient-drift instability in the heated volume.

According to our analysis the generation of decametric plasma irregularities in the  $F$  region under X-mode heating takes place in overdense and underdense plasma because both instabilities (the self-focusing and the temperature gradient-drift instability) operate in these cases. This conclusion is in accordance with the experimental results (Blagoveshchenskaya et al., 2014). Also, since the excitation of self-focusing instability is more efficient when the pump wave is launched along the local magnetic field, our theoretical approach confirms the experimental observations.

We have discussed how electric field of auroral electrojet  $E_{\text{jet}}$  influences the growth rate of the drift instability. It was shown that for strong electric fields  $|E_{\text{jet}}| \geq 14$  mV/m the temperature drift instability transforms into the gradient-drift instability. For smaller electric fields the transverse drift causes only correction to the growth rate of the temperature gradient-drift instability. Also, we have analyzed the role of the current-convective instability in the generation of decametric irregularities in the  $F$  region under the action of HF X-mode heating. The lack of strong longitudinal currents in the  $F$  region confirmed in our numerical simulation has ruled out the current-convective instability as a potential mechanism for the generation of such irregularities. So our main conclusion is that in quiet conditions the only instability responsible for the generation of the decametric artificial irregularities in the auroral  $F$  region under X-mode heating is the temperature gradient-drift instability. But in highly disturbed conditions the gradient-drift instability and the current-convective instability should influence the formation of decametric irregularities. The detailed analysis of such situation will be presented in a separate paper. The predicted time of development of decametric irregularities in our theory corresponds to the averaged characteristic times measured in experiments. This means that our theoretical model can be considered as a first approximation to the explanation of the real situation.

**Acknowledgments**

EISCAT is an international scientific association supported by research organizations in China (CRIRP), Finland (SA), Japan (NIPR and STEL), Norway (NFR), Sweden (VR), and the United Kingdom (NERC). The data of background plasma parameters used in this paper are available through the EISCAT Madrigal database (<http://www.eiscat.se/madrigal/>). The data of ion densities and ion temperature are available through the website ([https://omniweb.gsfc.nasa.gov/vitmo/iri2012\\_vitmo.html](https://omniweb.gsfc.nasa.gov/vitmo/iri2012_vitmo.html)). The data of neutral parameters used in this paper are available through the website (<https://ccmc.gsfc.nasa.gov/modelweb/models/nrlmsise00.php>). Nikolay Borisov is grateful to the Lancaster University for the opportunity to visit and to discuss the problems related to this paper. Farideh Honary acknowledges fruitful discussions of the results from this research with colleagues at the International Space Science Institute, Bern, Switzerland. Haiying Li thanks the support from Natural Science Basic Research Plan in Shaanxi Province of China under grants 2016JQ4015, the 111 Project (B17035), and the overseas training program for young backbones teachers sponsored by China Scholarship Council (CSC) and Xidian University.

**References**

Blagoveshchenskaya, N. F., Borisova, T. D., Kosch, M., Sergienko, T., Brändström, U., Yeoman, T. K., & Häggström, I. (2014). Optical and ionospheric phenomena at EISCAT under continuous X mode HF pumping. *Journal of Geophysical Research: Space Physics*, 119, 10,483–10,498. <https://doi.org/10.1002/2014JA020658>

Blagoveshchenskaya, N. F., Borisova, T. D., Yeoman, T. K., Rietveld, M. T., Ivanova, I. M., & Baddeley, L. J. (2011). Artificial small-scale field-aligned irregularities in the high latitude F-region of the ionosphere induced by an X-mode HF heater wave. *Geophysical Research Letters*, 38, L08802. <https://doi.org/10.1029/2011GL046724>

Borisov, N., & Stubbe, P. (1997). Excitation of longitudinal (field-aligned) currents by modulated HF heating of the ionosphere. *Journal of Atmospheric and Solar-Terrestrial Physics*, 59(15), 1973–1989. [https://doi.org/10.1016/S1364-6826\(97\)00019-9](https://doi.org/10.1016/S1364-6826(97)00019-9)

Braginskii, S. I. (1965). Transport processes in a plasma. *Reviews of Plasma Physics*, 1, 205.

Chaturvedi, P. K., & Chaturvedi, S. L. (1981). The current convective instability as applied to the auroral ionosphere. *Journal of Geophysical Research*, 86(A6), 4811–4814. <https://doi.org/10.1029/JA086iA06p04811>

Eltrass, A., Mahmoudian, A., Scales, W. A., Larquier, S., Ruohoniemi, J. M., et al. (2014). Investigation of the temperature gradient instability as the source of midlatitude quiet time decametric scale ionospheric irregularities: 2. Linear analysis. *Journal of Geophysical Research: Space Research Letters*, 119, 4882–4893. <https://doi.org/10.1002/2013JA019644>

Ginzburg, V. L. (1970). *The propagation of electromagnetic waves in plasmas*, Edition, 2. Publisher: Pergamon Press.

Greenwald, R. A., Okasavik, K., Erickson, P. J., Lind, F. D., Rouhoniemi, J. M., Baker, J. B. H., & Gjerloev, J. W. (2006). Identification of the temperature gradient instability as the source of decameterscale ionospheric irregularities on plasmopause field lines. *Geophysical Research Letters*, 33, L18105. <https://doi.org/10.1029/2006GL026581>

Gurevich, A. V. (1978). *Nonlinear phenomena in the ionosphere*. New York: Springer-Verlag.

Gurevich, A. V., Krylov, A. L., & Tsedilina, E. E. (1976). Electric fields in the Earth's magnetosphere and ionosphere. *Space Science Reviews*, 19, 59–160. <https://doi.org/10.1007/BF00215629>

Guzdar, P. N., Chaturvedi, P. K., Papadopoulos, K., & Ossakow, S. L. (1989). The thermal self-focusing instability near the critical surface in the high-latitude ionosphere. *Journal of Geophysical Research*, 103(A2), 2231–2237. <https://doi.org/10.1029/97JA03247>

Guzdar, P. N., Goncharenko, N. A., Chaturvedi, P. K., & Basu, S. (1998). Three dimensional nonlinear simulations of the gradient drift instability in the high-latitude ionosphere. *Radio Science*, 33(6), 1901–1913. <https://doi.org/10.1029/98RS01703>

Huba, J. D. (1984). Long wavelength limit of the current convective instability. *Journal of Geophysical Research*, 89(A5), 2931–2935. <https://doi.org/10.1029/JA089iA05p02931>

Hudson, M. K., & Kelley, M. C. (1976). The temperature gradient drift instability at the equatorward edge of the ionospheric plasma trough. *Journal of Geophysical Research*, 81(22), 3913–3918. <https://doi.org/10.1029/JA081i022p03913>

Kadomtsev, B. B. (1965). *The plasma turbulence*. New York: Academic Press.

Kelley, M. C. (1989). *The Earth's ionospheric plasma physics and electrodynamics*. San Diego, CA: Academic Press.

Ossakow, S. L., & Chaturvedi, P. K. (1979). Current convective instability in the diffuse aurora. *Geophysical Research Letters*, 6(4), 332–335. <https://doi.org/10.1029/GL006i004p00332>

Robinson, T. R. (1989). The heating of the high latitude ionosphere by high power radio waves. *Geophysical Research Letters*, 17(2-3), 79–209. [https://doi.org/10.1016/0370-1573\(89\)90005-7](https://doi.org/10.1016/0370-1573(89)90005-7)

Stubbe, P. (1996). Review of ionospheric modification experiments at Tromsø. *Journal of Atmospheric and Terrestrial Physics*, 58(1-4), 349–368. [https://doi.org/10.1016/0021-9169\(95\)00041-0](https://doi.org/10.1016/0021-9169(95)00041-0)

Stubbe, P., & Hagfors, T. (1997). The Earth's ionosphere: A wall-less plasma laboratory. *Surveys in Geophysics*, 18(1), 57–127. <https://doi.org/10.1023/A:1006583101811>

Tsunoda, R. T. (1988). High latitude F-region irregularities: A review and synthesis. *Reviews of Geophysics*, 26(4), 9171. <https://doi.org/10.1029/RG026i004p00719>

Vas'kov, V. V., & Ryabova, N. A. (1998). Parametric excitation of high frequency plasma oscillations in the ionosphere by a powerful extraordinary radio wave. *Advances in Space Research*, 21(5), 697–700. [https://doi.org/10.1016/S0273-1177\(97\)01006-5](https://doi.org/10.1016/S0273-1177(97)01006-5)

Wang, X., Cannon, P., Zhou, C., Honary, F., Ni, B., & Zhao, Z. Y. (2016). A theoretical investigation of the parametric instability excited by X-mode polarized electromagnetic wave at Tromsø. *Journal of Geophysical Research: Space Physics*, 121, 3578–3591. <https://doi.org/10.1002/2016JA022411>