Endogenous efficiency of the dynamic profit maximization in intertemporal production models on venture behavior

Abstract

How do ventures manage adjustment costs, input elasticities and productivity growth? We draw on the intertemporal production decisions of ventures that are quasi-fixed, costly to adjust, and endogenous. Using a modified version of the Bayesian Exponentially Tilted Empirical Likelihood (BETEL) adjusted for the presence of dynamic latent variables the proposed moment-based multiple-equation estimation system incorporates dynamic and static optimality conditions derived from a firm's intertemporal expected profit maximization. Using reliable tax information data from a sample of 72,035 Portuguese ventures founded between 2010 and 2017, we find that the most important inputs are labor, equity, and inventories. However, the technical change is small, and so is productivity growth 1.1%. Adjustment costs for efficiency vary from 15 to 30%, however, adjustments to capital stocks are much higher (ranging from 24 to 30%). Ventures have a high degree of labor input elasticity (0.64) but much lower elasticities for equity (0.108), inventories (0.063), capital (0.044), and advertising (0.048). The findings provide an understanding of the intertemporal behavior of ventures in managing adjustment costs, input elasticities and productivity growth where adjustments to efficiency is 'cheaper' than adjustments to capital, only labor elasticity is much higher, and productivity growth remains small.

1. Introduction

Both practitioners and academics have widely documented the challenges faced by ventures (Voigt and Cambell 2017). According to the widely accepted estimates, about half the ventures fail within the first five years, and some figures are as high as 78% (Song et al. 2008). With a perennial interest in understanding the venture development process (Josefy et al. 2017), past works have highlighted the role of legitimacy (Zimmerman and Zeitz 2002), resource scarcity (Cooper et al. 1991), performance metrics (Cooper et al. 1994), environmental conditions (Ensley et al. 2006), and stakeholder relationships (Hiatt et al. 2018). The primary theoretical components that the research has proposed relate to the newness and smallness of liabilities and the limited legitimacy and challenges to developing and sustaining exchanges with stakeholders (Aldrich and Auster 1986; Aldrich 2008; Aldrich and Pfeffer 1976).

In the studies on venture development, intertemporal changes in efficiency in production remains largely absent. Consideration of intertemporal changes to efficiency is essential to converging towards a stable operational core (Bourgeois III 1985). Efficiency represents stable operational routines undergirded by stable conversion processes and routines. In the broader operations literature, efficiency is measured using a production frontier or deviation of a firm from the efficient frontier. There are two main streams on modeling efficiency—data envelopment analysis (DEA) and stochastic frontier analysis (SFA). Data Envelopment Analysis (DEA), a non-parametric approach, was introduced by Charnes et al. (1984) and extended by Banker et al. (1984). The DEA approach is widely used in studies on multi-input and multi-output. Two other perspectives added to the DEA were the details on whether there is no noise in the data or whether all possible realizations are attributed to production possibility sets (Davtalab-Olyaie et al. 2019). Banker (1993) proposed a maximum likelihood approach to estimate production functions under certain conditions, with later works providing asymptotic distribution of DEA for single input and output (Gijbels et al. 1999), or with multiple inputs and outputs (Kneip et al. 1998). Thereafter, Simar and Wilson (1998) and Simar and Wilson (2000) proposed the bootstrap method, and future developments focused on double smooth bootstrap (Kneip et al. 2011), two-stage estimation method (Simar and Wilson 2007). Simar and Wilson (2015) provide a detailed review of these methods. Aigner et al. (1977) and Meeusen and van Den Broeck (1977) proposed a stochastic frontier approach for cross-sectional data followed by Battese and Coelli (1995) proposing stochastic frontier analysis for panel data, where non-negative technical inefficiency is driven by firm-level variables and time effects. Kumbhakar and Lovell (2003) review theoretical and practical concerns in applying the stochastic frontier approach.

However, time-varying inputs in a fledgling venture require two added considerations in modeling efficiency—intertemporal choices driving adjustment costs, input elasticities and productivity growth. First, The intertemporal choice to maximize profits requires transition from static formulations of efficiency that may be amenable to more stable and established firms facing fewer operational challenges (for reviews refer to Kumbhakar, Parmeter, Zelenyuk, 2017, Parmeter, Kumbhakar, 2014). Ventures experiment with a variety of tools, tasks, processes and operational relationships. As its products are commercialized and scaled, relative to firms with established operations, input adjustments are required—calling for consideration of dynamic implications of production decision in ventures. Equally important is a consideration that due to liabilities of smallness and newness ventures may not be able to readily adjust their inputs. In adapting their intertemporal choices, ventures face adjustment costs, including attracting and retaining employees, installing and tuning production and supply chain lines—a set of quasi-fixed costs in the short run.

Second, related to nature of efficiency conversion models, static models do not account for adjustments to dynamic inputs. Silva and Stefanou (2003) and Silva and Stefanou (2007) are among the early works proposing the dynamic optimization model based on intertemporal cost minimization with stickier inputs in the short-term. Building on Silva and Stefanou (2003) and Silva and Stefanou (2007), other researchers have focused on nonparametric dynamic (technical) efficiency (see Kapelko and Oude Lansink, 2017), dynamic duality model to manage intertemporal cost minimization framework (Rungsuriyawiboon and Stefanou (2007), primal directional-distance-function-based representation of production technology (Sengupta (1995) and Nemoto, Goto, 1999, Nemoto, Goto, 2003), decomposition of dynamic efficiency (Kapelko, Oude Lansink, & Stefanou, 2014) and measure "dynamic" productivity growth (Oude Lansink, Stefanou, & Serra, 2015). These methods inferred dynamic production functions from the implied distance functions but did not use information in intertemporal economic behavior, thereby limiting consideration of dynamic evolution of efficiency—a key consideration for ventures where efficiency is more likely to dynamically evolve over the course of venture life-cycle. Answering this call, Tsionas, Malikov, and Kumbhakar (2019) introduced explicitly and endogenously determined conceptualization of efficiency in firm's intertemporal production decisions. Tsionas et al. (2019) used a modified version of a Bayesian Exponentially Tilted Empirical Likelihood finding that modeling for potential intertemporal endogenous adjustments produces significantly higher estimates of technical efficiency.

Specific to the context of dynamic efficiency in ventures, we extend the approach in Tsionas et al. (2019). Although ventures are resource-constrained, have less developed routines, and have legitimacy concerns, the dynamic profit maximization models provide "nuts and bolts" understanding of early-stage profit maximization. We construct a profit maximization model by using the Bayesian Exponentially Tilted Empirical Likelihood (BETEL) that is adjusted for the presence of dynamic latent variables. The model is a moment-based multiple-equation estimation system that incorporates dynamic and static optimality conditions that are derived from the firm's intertemporal expected profit maximization (Tsionas et al. 2019). This study extends the popular approach of using the dynamic directional distance function that assumes freely varying inputs (Silva and Lansink 2013). This function is based on a firm's intertemporal cost-minimization problem under the Hamilton-Jacobi-Bellman conditions (Kapelko et al. 2016; Minviel and Sipiläinen 2018). Based on Tsionas et al. (2019) the estimate is

> ...restrictive because it (i) does not explicitly account for dynamics inefficiency itself as well as does not allow for the costly firm-controlled efficiency change, (ii) neglects the likely possibility that past dynamic efficiency is a part of the information set based upon which the firm optimizes, (iii) makes no use of information about economic behavior in the estimation beyond an indirect appeal to duality despite seeking a deeply structural 'dynamic' interpretation of efficiency, and (iv) suffers from the endogeneity problem due to simultaneity of the variable input and investment decisions. (p. 314)

We test our model using a sample of 72,035 Portuguese ventures (formed between 2010 to 2017, representing 164,538 venture-year observations) to estimate the proposed model that includes variable-input-oriented technical efficiency and quasi-fixed factor distortions with a second-order vector autoregression in which the choice of order is based on Euler's equation. We provide the following contributions. First, we model for dynamic evolution of efficiency to account for endogenization of that efficiency and adjustment costs. For firms that engage in a variety of production activities and that lack the baseline routines to improve performance, such assumptions may be strong. We extend Tsionas, Malikov, and Kumbhakar's (2019) cost minimization approach by using a profit maximization procedure. Specifically, we extend Tsionas et al. (2019) who develop a structural conceptualization of dynamic efficiency by proposing explicit and endogenously determined intertemporal and dynamic production decisions. Allowing for these dynamic latent variables of variable-input-oriented inefficiency and factor-specific distortions in quasi-fixed inputs, we propose a moment-based multiple-equation estimation system to incorporate the variable cost function and intertemporal minimization of a discounted stream of future costs. This system addresses the endogeneity of the inputs to gain a meaningful interpretation of the endogenous dynamic technical efficiency that evolves along an optimal path that is consistent with the firm's intertemporal cost-minimizing objective. Our approach is unique in providing a full intertemporal optimization problem that endogenizes inefficiency. Compared with other approaches to technical efficiency, the proposed model takes into account the information about the intertemporal economic behavior and allows for the dynamic evolution of profit maximization efficiency as optimized by the entrepreneur.

Second, entrepreneurship research has focused on the conditions (legitimacy or the environment) or the outcome (survival). Our study adds to this research by focusing on the intertemporal development of efficiency in the firm that allows us to assess the components of *how*. Though the dynamic evolution of profit maximization remains less understood, our approach is the first attempt to explore the elements that enhance firm profitability through the intertemporal lens of dynamic profit maximization. Focusing on intertemporal profit maximization efficiency is essential for practicing entrepreneurs facing resource scarcity (Cooper et al. 1991), and although cash flows are critical to firms, the primary step towards improving the internal availability of resources is to induce an intertemporal increase in the profitability of firms. Due to limited scale and scope along with less developed routines, an explicit focus on cost efficiency may not be desired, but profit maximization provides a more encompassing intertemporal choice for improving sales, or lowering costs, or both. Although we do not discount the value of the components of profit (including cost efficiency, inventory management, accounts receivables, and payables management), we aim to provide a preliminary comprehensive assessment of the intertemporal choices in managing profit maximization. We further do not discount the value of survival as an outcome, but survival as an event does not allow for a deeper understanding of the intertemporal choices. Considering the centrality of survival in entrepreneurship research we also test for the influence of profit maximization and efficiency on a firm's exit.

Third, a focus on the efficiency of profit maximization is central to building it in early-stage firms. Although the romanticized view of gazelles (high-growth firms) is widely regarded among practitioners and academics, most of the firms start and stay small, and very few realize growth. We focus on the value of managing operational inputs and levers that explain the elemental factors that drive the profit maximization efforts of ventures.

The paper is organized as follows: Section 2 provides the preliminaries, and Section 3 presents the model of dynamic production decisions aimed at profit maximization under endogenous efficiency. Section 4 presents the data, and Section 5 presents the empirical results. Section 6 provides discussion, conclusion and limitations.

2. Background and Model setup

At inception and during the early years of its operations a venture faces liabilities of newness and smallness (Bruderl and Schussler 1990; Aldrich and Auster 1986). Liabilities of newness stems from limited availability of operational routines and processes. Development and crystallization of these baseline operational routines are further limited by the limited availability of resources to ventures. As such, a vast number of ventures face a greater threat to survival. Survival strategies include improving legitimacy, increasing the resource base, and pursuing performance growth (Josefy et al. 2017). However, the broader question on the actual "nuts and bolts" of how entrepreneurs make intertemporal decisions to dynamically improve efficiency that are informed by past performance is a critical consideration in developing a deeper understanding of the evolution of dynamic profit maximization in ventures.

We propose an intertemporal profit maximization model that incorporates inefficiency, uncertainty, and quasi-fixed inputs as well as the endogeneity in profit maximization. The uncertainty arises from limited foresight on product design; resource availability; stakeholder cooperation; and evolving input costs, prices, and sales. We also presume that the entrepreneur is risk-neutral to allow for expectations for future outcomes and without requiring the need to consider the optimization of utility functions that are less reliable for early-stage ventures. Although risk-neutrality is a restrictive assumption that allows for greater tractability in modeling intertemporal behavior because of the Euler expectation conditions that undergird the estimations of moment conditions. The set up allows for greater flexibility in estimating dynamic efficiency as compared to recent works that assume forward-looking firms with perfect oversight (Serra et al. 2011; Silva and Stefanou 2003, 2007). As such, our criteria fits with the fundamental challenge for entrepreneurs—making ex-ante costly investments in dynamic factor inputs under uncertainty.

Consider the standard cost minimization problem of the firm:

$$C(\boldsymbol{w}, y) = \min_{x \in \Re_{+}^{J}} \ \boldsymbol{w}'\boldsymbol{x}, y = f(\boldsymbol{x}), \tag{1}$$

in which $\boldsymbol{w}$ is the vector of factor prices, $\boldsymbol{x}$ represents factor demands (inputs), y is the single output, and $f(\boldsymbol{x})$ represents the production function. We have J factors of production so, $\boldsymbol{w} \in \Re_{++}^{J}$, and $\boldsymbol{x} \in \Re_{+}^{J}$. The cost function is well-known to completely characterize the technology of the firm. Defining the Lagrange function $\mathfrak{L} = \boldsymbol{w}'\boldsymbol{x} + \lambda^{*}[y - f(\boldsymbol{x})]$, the first-order conditions are:

$$\begin{gathered} w_j = \lambda^{*} \frac{\partial f(\boldsymbol{x})}{\partial x_j} \ \forall j = 1, \dots, J, \\ y = f(\boldsymbol{x}). \end{gathered} \tag{2}$$

To eliminate λ^{*} (whose interpretation is that it is the marginal cost), we have:

$$\frac{\partial f(\boldsymbol{x})/\partial x_j}{\partial f(\boldsymbol{x})/\partial x_1} = \frac{w_j}{w_1} \forall j = 1, \dots, J, \tag{3}$$

and the production function in the second equation in (2). These conditions can be written as follows:

$$\frac{\partial \ log \ f(\boldsymbol{x})/\partial x_j}{\partial \ log \ f(\boldsymbol{x})/\partial x_1} = \frac{w_j x_j}{w_1 x_1} \ \forall j = 1, \dots, J. \tag{4}$$

Define the elasticities $\varepsilon_j(\boldsymbol{x}) = \frac{\partial \ log \ f(x)}{\partial \ log \ x_j}$ that are positive by assumption. Then we can rewrite these equations as follows:

$$\log x_j - \log x_1 + \omega_j = \log \varepsilon_j(\boldsymbol{x}) - \log \varepsilon_j(\boldsymbol{x}) \ \forall j = 1, \dots, J, \tag{5}$$

in which $\omega_j = -\log(w_j/w_1)$. If data exists for the log relative factor prices (ω_j), factor demands (x_j), and output (y), then an estimation of the system of J equations in (5) and the production function can be done. The endogenous variables are input demands, while the output is predetermined; a requirement that comes directly from the nature of the cost minimization problem. To make things more explicit, suppose we have panel data so that we can re-write (5) as follows:

$$\begin{aligned} \log x_{j,it} - \log x_{1,it} + \omega_{j,it} = \log \varepsilon_j(\boldsymbol{x}_{it}) - \log \varepsilon_j(\boldsymbol{x}_{it}) + v_{j,it} \ \forall j \\ = 1, \dots, J, i = 1, \dots, n, t = 1, \dots, T, \end{aligned} \tag{6}$$

that assumes we have n firms and T time periods. And $v_{j,it}$ are error terms. We also need to rewrite the production function to include an error term:

$$\log y_{it} = \varphi(\log \boldsymbol{x}_{it}; \beta) + v_{0,it}, i = 1, \dots, n, t = 1, \dots, T, \tag{7}$$

in which $\varphi(\boldsymbol{x}_{it}; \beta)$ is a functional form (the translog being the most popular), β is a vector of unknown parameters, and $v_{0,it}$ represents an error term. However, we do not estimate the cost function directly, because we instead rely on the so-called primal approach that estimates the first-order conditions in (6) and the production function in (7). The reason is that to estimate the cost function we need input prices that are lacking in our case. Seemingly, we do not solve this problem as the primal approach also depends on log relative prices. However, this is not so because we can still use the first-order conditions under reasonable assumptions regarding the relative prices. For example, in (6) we assume that:

$$\omega_{j,it} = \alpha_{j,i} + \gamma_{j,t} \ \forall j = 1, \dots, T, \tag{8}$$

in which $\alpha_{j,i}, \gamma_{j,t}$ represent, respectively, input-specific firm and time effects. Therefore, the primal approach can be used to estimate the technology even when relative prices are not available. In fact, the primal approach provides a solution to an old problem: How to estimate production functions when the inputs are endogenous. This estimation can be easily taken into account in our framework as long as the vector error term $[v_{0,it}, v_{1,it}, \dots, v_{J,it}]'$ follows a zero-mean multivariate normal distribution with a certain covariance matrix. This problem dates back to (Marschak and Andrews 1944) and has gained considerable attention in the literature (see *inter alia* Blundell and Bond (2000), Olley and Pakes (1996), Levinsohn and Petrin (2003), and Wooldridge (2009)). We mention this point because it is important in the next section where we consider a dynamic model in which, again, factor relative prices are not available.

3. Model

The dynamic stochastic profit maximization of a firm is as follows:

$$\max_{x_t \in \Re_+^J, k_t \geq 0} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ p_t f(\boldsymbol{x}_t, k_t) - \boldsymbol{w}'_t \boldsymbol{x}_t - r_t k_t - p_t G\left(\underbrace{k_t - (1-\delta) k_{t-1}}_{\iota_t} \right) \right\}, \quad (9)$$

in which $\beta \in (0,1)$ is the discount factor, $\boldsymbol{x}_t \in \Re_+^J$ is the vector of variable factors of production whose prices are $\boldsymbol{w}_t \in \Re_+^J$, k_t is a capital stock whose user cost is r_t, $f(\boldsymbol{x}_t, k_t)$ is the production function that uses variable inputs and capital to produce a single product (y_t) whose price is p_t, ι_t is an investment, $\delta \in (0,1)$ denotes the capital depreciation rate, and $G(\iota_t)$ is the adjustment costs. We adopt the popular formulation $G(\iota) = \frac{1}{2}\gamma_k \iota^2$, where $\gamma_k > 0$ is an unknown parameter. Here, the adjustment costs are expressed in terms of the product price, p_t. As is customary, we assume that $k_0 > 0$ is given.
The first-order conditions are:

$$\begin{gathered} \frac{\partial f(\boldsymbol{x}_t, k_t)}{\partial x_{tj}} = \frac{w_{tj}}{p_t} \ \forall j = 1, \dots, J, \\ p_t \frac{\partial f(\boldsymbol{x}_t, k_t)}{\partial k_t} - r_t - p_t G'(\iota_t) = \beta(1-\delta)\mathbb{E}_t G'(\iota_{t+1}) p_{t+1}. \end{gathered} \quad (10)$$

The first set of equations corresponds to cost minimization as we have:

$$\frac{\partial f(\boldsymbol{x}_t, k_t)/\partial x_{tj}}{\partial f(\boldsymbol{x}_t, k_t)/\partial x_{t1}} = \frac{w_{tj}}{w_{t1}} \ \forall j = 2, \dots, J, \quad (11)$$

from which it follows that:

$$\frac{\partial \log f(\boldsymbol{x}_t, k_t)/\partial \log x_{tj}}{\partial \log f(\boldsymbol{x}_t, k_t)/\partial \log x_{t1}} = \frac{w_{tj} x_{tj}}{w_{t1} x_{t1}} \ \forall j = 2, \dots, J. \quad (12)$$

If we define $\varepsilon_j(\boldsymbol{x}_t, k_t) = \frac{\partial \log f(x_t, k_t)}{\log x_{tj}}$, then these equations, after taking logs, have the following form:

$$\omega_{tj} + \log x_{tj} - \log x_{t1} = \log \varepsilon_j(\boldsymbol{x}_t, k_t) - \log \varepsilon_1(\boldsymbol{x}_t, k_t), \forall j = 2, \dots, J, \quad (13)$$

in which $\omega_{tj} = \log \frac{w_{tj}}{w_{t1}}$ ($\forall j = 2, \dots, J$). From the second condition in (10) we have:

$$\frac{\partial f(\boldsymbol{x}_t, k_t)}{\partial k_t} - \varrho_t - G'(\iota_t) = \beta(1-\delta)\mathbb{E}_t G'(\iota_{t+1}) \pi_{t+1}, \quad (14)$$

in which $\varrho_t = \frac{r_t}{p_t}$, and $\pi_{t+1} = \frac{p_{t+1}}{p_t}$. Using our functional form for $G(\iota)$ whose derivative is $G'(\iota) = \gamma_k \iota$, we obtain:

$$\frac{\partial f(\boldsymbol{x}_t, k_t)}{\partial k_t} - \varrho_t - \gamma_k \iota_t = \beta\gamma(1-\delta)\mathbb{E}_t \iota_t \pi_{t+1}. \quad (15)$$

To use logs we find that the above expression is equivalent to:

$$\varepsilon_k(\boldsymbol{x}_t, k_t) \frac{f(\boldsymbol{x}_t, k_t)}{k_t} - \varrho_t - \gamma_k \iota_t = \beta\gamma_k(1-\delta)\mathbb{E}_t \iota_t \pi_{t+1}, \quad (16)$$

in which $\frac{f(x_t, k_t)}{k_t} = \frac{y_t}{k_t}$ is the output per unit of capital or capital productivity, and $\varepsilon_k(\boldsymbol{x}_t, k_t) = \frac{\partial \log f(x_t, k_t)}{\partial \log k_t}$.

The model with inefficiency and productivity is presented in detail in Appendix A where we also present an estimation in which we use Bayesian techniques as well as our benchmark prior.

4. Data

4.1. Sample

We draw on the annual reports of Portuguese firms. All private firms in Portugal, irrespective of their size, are required to file their annual reports after being certified by a public accountant. As such, the data provide reliable information on the annual performance of these young firms.

Specifically, data for the analysis was obtained from *INFORMA D&B* by using the IES (*Informação Empresarial Simplificada (IES)*) Form that contains certified yearly financial and performance information. Our initial dataset comprised the entire population of new ventures from all industries established between 2010 and 2017. We excluded the ventures that reported zero sales in the first year of the activity or had reported a temporary suspension of activity or inactivity. Thereafter, we excluded the ventures that reported no advertising during the period (although we examine here an alternative scenario where more observations are obtained through omitting advertising). Finally, we excluded the acquired ventures during the period and those that had evidence of some errors, such as negative values for inventories or assets, or missing values. The final dataset comprises 164,538 venture-year observations, representing a total of 72,035 ventures.

Table 1a presents the sample description across industries and Table 1b shows the correlations among the variables. Among the 72,035 established venture, the largest share is in retail trade, except for motor vehicles, (industry code 47) at 22.1% of the sample (N=36,348). It is followed by food and beverage services (industry code 56) at 13.7% of the sample (N=22,491) and wholesale trade, except motor vehicles and motorcycles, (industry code 46) at 10.8% of the sample (N=17,836). The smallest share is in Insurance, reinsurance and pension funding, except compulsory social security, (industry code 65) (N=4), manufacture of coke and refined petroleum products (industry code 19) (N=12), air transport (industry code 51) (N=37), manufacture of basic pharmaceutical products and pharmaceutical preparations (industry code 21) (N=42), and programming and broadcasting activities (industry code 60) (N=63). All are at less than 0.1% of the sample.

It is important to note that our estimates are adjusted for the industry effects, or for other time in-variant effects (e.g., firm age or region). Specifically, equations (15) and (16) contain such effects, see also part A1 of Technical Appendix A.

4.2. Variables

4.2.1. Output variables. The outcome variable is sales revenues defined as the total operating revenue that the venture obtains from the sale and/or services offered in its main business operations.
4.2.2. Input variables. Equity represents the book value of the venture assets and is obtained from the difference between total assets and total liabilities of the firm. However, for small and young ventures the formal accounting-based measures may not be available and therefore the precise measures of book value may not be available. Although basic, the net asset measure of book value reflects the level of equity in the business. In the event where a firm sustains losses, the losses are deducted from the equity value of the business. As such, for these young firms with less formalized systems, the subtraction of assets from liabilities provides a viable measure of their value.

Related to the remaining inputs, labor is defined as the total number of employees of the venture. Inventories are the goods and materials the venture holds to sell as the main object of its business. It represents the main source of revenue obtained from the normal activity of the venture and is expected to be translated into revenue within one year, hence it is classified as a current asset. Services represent the operating revenues obtained by the venture from the services rendered. Advertising is the operating expenses of activities such as media and marketing, and others such as business cards, brochures, or web pages that are incurred with the intent to promote the business. Capital is the fixed tangible assets held by the venture. It is represented by the total value of plant, property, and equipment that includes buildings, factories, machinery, vehicles, and office equipment.

5. Empirical results

Some results are shown in Figure 1. We find that $\frac{\partial \log f(x,k)}{\partial \tau}$ (whose sample distribution is reported in panel (b)) is a measure of technical change. The returns to scale can be estimated as $RTS = \frac{1}{\partial \log C(w,t,y)/\partial \log y}$. We also present sample distributions of $e_j(\boldsymbol{x},k) = \frac{\partial \log f(x,k)}{\partial \log x_j}$ in panel (c). These quantities should be positive to satisfy neoclassical properties. The sample density of technical efficiency is reported in panel (a). It ranges from about 82% to 93%. The returns to scale in panel (d) range from about 0.85 to slightly over 1.07, and for most ventures, they are less than one with decreasing results to scale.

In Figure 2 we report the marginal posterior densities of some important structural parameters of the model that include the discount factor β in panel (a), depreciation rate δ in panel (b), and the adjustment cost parameters for capital and efficiency (γ_k and γ_u) in panel (c). Based on (Maccini 2016) adjustment costs refer to the costs incurred when decision variables are changed. The discount factors are fairly close to 0.90, and the depreciation rate ranges from 2% to slightly over 20% with an average value of 12.4 (posterior standard deviation of 0.027, see Table 3). The marginal posterior densities of adjustment cost parameters are non-normal (showing that asymptotically-based inferences would be misleading in this instance). The capital adjustment costs average 0.268 (posterior standard deviation 0.012) and adjustment costs for efficiency average 0.223 (posterior standard deviation 0.022). From a visual inspection of their marginal posterior densities in panel (c) of Figure 2, it turns out that the costs for efficiency adjustments are much more heterogeneous compared to the costs for capital adjustments. Specifically, the costs for efficiency adjustments range from a low of 15% to nearly 30% that shows some ventures find it easier to adjust their efficiency levels rather than capital stock, which ranges from 24% to slightly above 30% (this distribution is also bimodal with two distinct models near 26% and 29% that ventures are heterogeneous in this aspect).

In panel (a) of Figure 3, we present the sample distribution of productivity growth in a model that includes advertising. The inclusion of advertising is motivated by strong prior evidence (Hansen et al. 2019; Sadiku-Dushi et al. 2019) and is strongly favored by the data as shown by the Bayes factors that support this model. The results are reported in panel (f) and are obtained through randomly omitting B observations (where B is randomly selected from $10, 20, \ldots, 100$) 10,000 times. Ignoring advertising produces a sample distribution of productivity growth that is much more dispersed, ranging from approximately -3% to nearly 6%. Including advertising produces estimates in the range of -3% to 2% (from Table 3 the average value is 1.1% with a standard deviation of 1.4%). The sample distribution of productivity growth has a considerable left tail with negative values that are understated by the sample distribution of productivity growth.

In panels (b), (c), (d), and (e) of Figure 2 illustrate, respectively, the marginal posterior densities for ρ_Ω, ρ_π, ρ_ϱ, and ρ_u. According to panel (d), inefficiency is highly persistent as ρ_u averages 0.911 (posterior s.d 0.015). The reason is that the costs for efficiency adjustments produce, in turn, inefficient behavior that is persistent over time. The dynamic coefficient of productivity growth averages 0.503 (posterior s.d 0.055); its density in panel (b) is highly non-normal, bimodal, and skewed as are all other marginal posteriors shown in panels (b), (c), (d), and (e) of Figure 2. Therefore, productivity is somewhat persistent, and its coefficient ranges from about 0.3 to 0.65. Prices and ϱ are also persistent in their coefficients and average 0.904 and 0.905, respectively. Again, their distributions are non-normal that indicates the sampling theory-based inferences might be misleading in this instance.

In panel (f), we report the sample distributions of the Bayes factors (BF) in favor of the model with advertising and against the model without advertising. To obtain these sample distributions we randomly omit all observations for B firms (where B is randomly selected in $\{10, 20, \ldots, 100\}$), and we reestimate the two models. This is performed 10,000 times. The BF is defined as:

$$BF_{12} = \frac{p(\boldsymbol{y}|\mathcal{M}_1)}{p(\boldsymbol{y}|\mathcal{M}_2)},$$

in which $p(\boldsymbol{y}|\mathcal{M}_1)$ is the probability of observing the data under model $\mathcal{M}_1$ (the model that includes advertising as a variable factor of production), and $p(\boldsymbol{y}|\mathcal{M}_2)$ is the probability of observing the data under model $\mathcal{M}_2$ that ignores advertising. Apparently, the data strongly favor the model with advertising. Further, endogeneity of advertising is explicitly taken into account through the first-order conditions for profit maximization (Geweke 1991). The BFs are obtained using the procedure in (Perrakis et al. 2014). The reestimation of models is performed using Sampling-Importance-Resampling (SIR, (Rubin 1987, 1988)) to avoid computationally expensive MCMC[1].

Returns to scale refer to proportionality of changes in output after the amounts of all inputs in production have been changed by the same factor (Elsner et al. 2015). The returns to scale are 0.956 on average with a posterior standard deviation of 0.028. The input elasticity is the degree of substitutability of inputs given a change in the marginal productivity of an input (Miller et al. 2019). Input elasticities

[1]For SIR we use a random subsample of length 10,000 from the original MCMC sample.

are all positive as they should be to satisfy the monotonicity conditions of the production function. The most important input is labor (elasticity 0.64) followed by equity (0.108), and inventories (0.063). The capital, service, and advertising elasticities are much lower. The sample distributions of all these quantities are reported in Figure 1. Efficiency is, on average, 0.875 and ranges from 0.82 to 0.93. Technical change (panel (b) of Figure 1) is rather small and averages -0.0004. Sample distributions of productivity growth reported in panel (a) of Figure 3 range from -4% to roughly 5.5% and averages 1.1%. In the model without advertising, productivity growth ranges from -3% to 4%. Both models indicate a bimodal sample distribution of productivity growth. In the model with advertising one model is close to zero and the other is close to 2% that indicates heterogeneity in the firm for productivity growth.

The marginal posterior densities of the parameters reported in the other panels of Figures 2 and 3 are highly non-normal, which indicates that in this instance, the inferences from the asymptotic theory might be misleading. The discount factor (β) is estimated to be 0.969 with a posterior standard deviation of 0.033. The adjustment costs of capital and inefficiency are, on average, 0.268 and 0.223, respectively (posterior standard deviations of 0.012 and 0.022). These are expressed as units of product price.

Productivity growth is persistent (ρ_{Ω} has a posterior mean of 0.503 and posterior standard deviation of 0.033). The posterior mean of ρ_o is fairly close to one (0.982 with posterior s.d. 0.012) that indicates that anchoring on economy-wide inflation is important. In other words, economy-wide inflation and product prices are significantly correlated. Other ρs also show persistence.

5.1. Firm exit

Next, we examine ex-post the relation among efficiency, productivity, and failure. To appreciate the significance of our (posterior mean estimates of) efficiency and productivity, we report density estimates of the log sales per employee ratio in panel (a) of Figure 4 for failed and surviving firms. Surviving firms have a more inflated left tail that indicates they have smaller sales per employee ratio. In other words, failed firms are somewhat too large relative to surviving ventures.

The marginal posterior densities of efficiency and productivity are reported in panels (b) and (c) of Figure 4. The interesting aspects of this analysis are two: successful ventures are more efficient and, ii) they are more productive. In panels (b) and (c), the densities of efficiency and productivity, respectively, are shifted to the right relative to failed ventures. The distribution of productivity growth for failed ventures is bimodal and certain ventures are productive (with posterior mean productivity growth that roughly ranges up to 2%): One model of the distribution (the dominant one) is near zero but the other model is, on average, close to -4% per year, which shows a lack of productivity in unsuccessful ventures. Moreover, for surviving ventures efficiency is higher and has a less pronounced mode around 89%. For unsuccessful ventures, average efficiency is close to 80% (compared to over 86% for successful ventures).

5.2. Summary of results. As a summary of results, we start with the adjustment costs. Adjustment costs are pivotal to ventures, especially during their formative years. We find that adjustment costs for efficiency vary from 15 to 30%, implying that some ventures incur relative lower costs. However, adjustments to capital stocks are much higher (ranging from 24 to 30%). These two estimates imply that ventures may be better off focusing on efficiciency adjustments instead of adjusting capital stocks. Input elasticities, or the degree of substitutability of inputs given a change in the marginal productivity or price of an input, show that ventures have a high degree of labor elasticity (0.64) with much lower elasticities for equity (0.108), inventories (0.063), capital (0.044) and advertising (0.048). Productivity growth range from -4% to roughly 5.5% and averages 1.1% (Figure 3, panel A). For firm exit related analysis, the productivity growth dominant distribution is close to zero with an average of -4% for unsuccessful ventures. Surviving ventures have a higher average efficiency by about 9%. Overall, adjustment costs to efficiency are lower than capital adjustment costs, and ventures maintain significant labor substitution elasticity, however, input elasticities for equity, inventories, capital and advertising are much lower. The productivity growth remains very small, however, surviving ventures have a higher productivity growth, but by a small amount in absolute terms.

6. Discussion

The results provide three main inferences. First, in the choice for adjusting efficiency or capital, the results indicate that efficiency related changes are less costlier than from capital adjustments. The potential reason for this finding could be two-fold. Ventures have less developed routines that could make adjsutments to newer capital more challenging, resulting in higher adjustment costs. Adjustments costs related to efficiency may be lower as ventures may intertemporally move down the learning curve to lower operational costs. As ventures build on operational routines, efficiency may be easier to adjust than the capital costs.

Second, labor input elasticity is higher, and equity, inventory, capital and advertising input elasticities are much smaller. Plausible reasons are that labor can be changed more easily as an input, however, equity is in short supply for newer ventures. Ventures are either funded by owners and investors and because additional equity inflows are contingent on performance, equity elasticity is limited. During their early years, ventures focus on a set of products at the core of their value proposition () and due to the idiosyncratic nature of their products fulfilling unique value proposition the associated could only be liquidated at a significant discount. Alternatively, due to less developed supply chain relationships inventory may move slowly for ventures facing lower legitimacy from suppliers and buyers. Similarly, capital and advertisement elasticities could be lower due to lower flexilbity in adjusting costly capital and intangibles associated with advertisements. As such, labor elasticity is much higher than other modes of elasticity.

Third, technical change is small for ventures and the average productivity growth is only 1.1%, perhaps indicating the challenges faced by ventures with limited operational capabilities in pursuing productivity growth. The findings highlight an important point that productivity growth may not be the mainstay for ventures. Consistent with past work, maintaining liquidity and ensuring survival may not lead to focus on productivity growth improvements. In terms of difference in labor productivity, surviving firms had a slightly lower productivity perhaps indicating the need for slack necessary to adjust to variegated demands.

6.1. Theoretical Implications

We aim to provide important extensions to the entrepreneurship literature. From the perspective of operations research, we extend the dynamic efficiency model of cost minimization by Tsionas et al. (2019) to an intertemporal profit-maximization model. We go beyond the works that do not allow for the dynamic evolution of efficiency by making a strong assumption of the exogenous evolution of efficiency. Thus, our proposed method provides an important extension that focuses on a venture's intertemporal costly and endogenously determined production decisions. The measure of efficiency is based on the evolution of two dynamically latent variables--variable-input-oriented inefficiency and factor-specific distortions in quasi-fixed inputs. The profit maximization methods based on moment-based multiple-equation estimation system incorporates a variable cost function and both the dynamic and static optimality conditions that are derived from the firm's decisions on intertemporal expected profit maximization. The proposed nonparametric BETEL is implemented in a sample of Portuguese firms.

These results are relevant to resource allocation and underline the importance of efficiency and productivity growth for new ventures. Traditionally, efficiency and productivity growth are seldom considered in venturing context due to their "interrupted" development. Ventures face interrupted resource and sales flow with many closing during the early years. Such an uneven flow of resources could render the value of productivity and efficiency growth as less meaningful. As such, much research has focused on outcome and survival, while others focus on growth in performance. However, in parallel, the population ecology theory calls to improve relationships in the task environment, that is, sustained exchanges with stakeholders. Although survival is a distal outcome and growth a more aggregate outcome, continued growth in productivity and efficiency could be the undergirding metrics central to dynamic intertemporal decision-making in ventures. We expect the focus on productivity and efficiency growth could be the primer of the ensuing outcomes, including survival.

The second important extension is our contribution to the entrepreneurship literature. Knowledge of how ventures dynamically improve profit maximization is the first important step to understanding how ventures allocate resources. A critical element here is that we assume that inputs are quasi-fixed and endogenously adjusting over time. This approach provides a novel mechanism to understand the so-called "black box" of evolution in ventures. Although studies in operations management have generally focused on statistical approaches, the proposed profit maximization-based approach is critical to understanding the *how* of resource allocation in ventures that thus provides the necessary depth in dynamically interpreting the evolution of their profit maximization.

The third important extension is based on recent work in the operations research on using analytics as a dynamic capability (Conboy et al. 2020). Although studies remain qualitative or focus on a higher-order construct of dynamic capability (Côrte-Real et al. 2019; Mikalef et al. 2019) or theoretical development (Conboy et al. 2020), our work informs this growing line of research by providing the baseline analytics that entrepreneurs can focus on to improve performance. The study is among the first to disaggregate the profit maximization path on a census of firms to develop a deeper understanding of the micro-foundations of performance in firms, a type less explored in the broader operations research.

Our paper draws on and improves upon recent works on intertemporal tradeoff framework by Tsionas et al. (). These advancements contribute to the ongoing need to move from static efficiency frameworks to intertemporal tradeoffs in efficiency considerations. The application of the proposed framework provides a nuanced and relevant approach to develop efficiency measures for ventures and firms that typically face significant intertemporal challenges in adjusting inputs and managing efficiency growth. The proposed methodology highlights the need to focus on the Bayesian modeling to develop efficiency related estimates, a necessity for firms as they update their efficiency through intertemporal decision making.

6.2. Managerial implications

Our empirical results provide an important set of guidelines for entrepreneurs. As entrepreneurs attempt to assemble and leverage resources for operational activities, the adjustment costs, elasticities, and productivity growth are essential elements to consider intertemporally. Our results based on Bayesian learning show that the decisions are not straightforward. Adjusting efficiency is cheaper than adjusting capital. Labor elasticity is much higher than capital, inventory or advertisement elasticity. Productivity growth is limited. The findings indicate that entrepreneurs may operate in survival mode as they tend to be more elasticity with labor, but cannot exercise greater adjustment flexibility with costlier capital, performance contingent equity, or intangilility based advertising expenditures.

With entrepreneurs better off improving efficiency instead of adjusting capital, the implication is significant for entrepreneurs seeking to refresh their capital base. Focusing on improving efficiency from an existing capital base, instead of investing in newer capital, is an important take away from the current study. The relatively small productivity growth is not surprising in this context. Limited operational templates to draw on, demand volatility, and unstable stakeholder relationship may lead to limited productivity growth. Our findings show that surviving ventures differ systematically from non-surviving in terms of small productivity growth. Though productivity growth should be the focus on entreprneeurs its influence on survival is limited. Entrepreneurs must intertemporally manage adjustment costs, elasticities, and productivity growth in their operational decisions.

6.3. Limitations and Future Research Directions

This research is not without limitations. As mentioned, the constructed sample of firms is from Portugal, and as such the findings cannot be generalized beyond Portugal. More importantly, as expected detailed financial statements are not available, we are unable to include more micro-financial or micro-operational dynamics. However, this is not a severe limitation as to the widely accepted and used financial items are available in the sample to draw inferences. We use secondary data. Though the nature of the data is amenable to the typical operations research related empirical studies, we acknowledge the limitations of such data. Secondary data do not provide the necessary contextual richness. The intertemporal decision making may be driven by a variety of factors associated with the entrepreneur and the context. The necessary richness from qualitative data could help further understand how entreprneeurs make such choices, but may also explain the relative variations in elasticities, adjustment costs, and productivity growth. Our analysis controls for both the endogeneity and quasi-fixed inputs in the intertemporal processes of a venture. As such, our study focuses on the structural aspects of the provided financial line items and therefore does not consider the microdynamics of the decision process of the entrepreneur. Future studies could consider different decision-making heuristics and long-term survival outcomes of the venture.

As entrepreneurs consider supply chain relationships and intertemporal tradeoffs in operational activities, lower capital and inventory elasticity, greater ability to change efficiency instead of changing capital, or supply chain relationships may be difficult to develop and sustain. Limited productivity growth coupled with the limited ability to gain from capital investments, could lead to poorer supply chain relationships. Future research could focus on how ventures develop and sustain relationships with supply chain members under the intertemporal tradeoffs that hinder operational gains for ventures.

Though we focus on statistical modeling with Bayesian learning the internally fused nature venture activities may not fully separate operations from the non-operations functions. Though not observable directly in the data, the diffused roles and responsibilities in a venture may confound the identified relationships. Multiple optimization decisions made by entrepreneurs could be explored in future research. Research focused on allocations, elasticities, and inefficiencies in ventures could focus on relative tradeoffs in the cross-functional settings.

Despite the potential gains from intertemporal tradeoffs, future research could also focus on the nature of operational learning in ventures. Future research could focus on the nature of knowledge and learning processes that entrepreneurs leverage in making intertemporal tradeoffs. Greater intertemporality results in reinforcing operational routines and processes systems that may increase consideration of joint intertemporality in operational and supply chain activities.

6.4. Conclusion

This paper supports and helps inform the basic rule of management “what cannot be measured cannot be managed”. A significant body of work in entrepreneurship has focused on the general terms that afflict firms—legitimacy, resource scarcity, and liabilities of age and size—and others have focused on an aggregate outcome such as survival. The current paper is among the first that provides a relevant and direct dynamic of intertemporal efficiency to improve operational decisions for ventures. Informed by the dynamic optimization model in Bayesian thinking, the findings of the paper provide early impetus to start focusing on the intertemporal operational changes over time. The productivity growth is very low, adjustment costs to capital stocks are much higher than for costs for efficiency adjustments. The findings indicate that labor elasticity is higher than capital, inventory or advertisement elasticity. The findings paint a sobering picture—lower productivity growth, limited flexibility in leveraging input elasiticies, and higher adjustment costs—of the challenges that ventures face as they intertemporally manage adjustment costs, elasticities, and productivity growth.

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Table 1
Sample Descriptives
Table 1(a): Sample description across industries

Industry code		Number of firms	Revenues from Sales Mean	Revenues from Sales SD	Equity Mean	Equity SD	Labor Mean	Labor SD
10	Manufacture of food products	2,801	638,305.8	2,776,898.0	118,702.1	755,684.0	9.8	33.3
11	Manufacture of beverages	644	383,498.4	1,938,317.0	283,802.2	1,865,652.0	2.7	5.1
13	Manufacture of textiles	601	1,368,047.0	4,451,759.0	313,108.2	972,490.4	16.6	36.6
14	Manufacture of wearing apparel	1,319	737,325.5	2,815,545.0	73,540.0	447,668.3	15.9	32.8
15	Manufacture of leather and related products	742	939,311.3	1,833,067.0	103,448.9	501,041.6	18.3	22.2
16	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials	713	1,636,620.0	10,000,000.0	655,585.7	4,679,760.0	12.4	42.7
17	Manufacture of paper and paper products	153	7,200,161.0	39,700,000.0	3,383,672.0	23,300,000.0	11.5	21.0
18	Printing and reproduction of recorded media	528	232,011.9	458,569.8	48,243.5	159,364.5	3.9	4.8
19	Manufacture of coke and refined petroleum products	12	63,000,000.0	212,000,000.0	6,737,045.0	10,400,000.0	11.3	12.6
20	Manufacture of chemicals and chemical products	314	894,076.2	3,059,947.0	552,652.7	2,654,832.0	5.8	8.3
21	Manufacture of basic pharmaceutical products and pharmaceutical preparations	42	1,700,670.0	2,657,302.0	363,949.1	761,834.8	8.4	8.7
22	Manufacture of rubber and plastic products	312	1,233,351.0	3,225,778.0	333,913.5	753,036.6	11.2	17.0
23	Manufacture of other non-metallic mineral products	511	497,811.2	1,293,085.0	194,135.8	1,152,627.0	8.6	15.2
24	Manufacture of basic metals	123	2,139,297.0	4,374,963.0	453,742.0	1,178,779.0	9.7	14.5
25	Manufacture of fabricated metal products, except machinery and equipment	2,679	488,574.3	1,083,337.0	102,403.4	350,557.3	7.9	16.0
26	Manufacture of computer, electronic and optical products	140	1,617,047.0	4,231,765.0	529,918.3	1,433,135.0	15.0	30.6
27	Manufacture of electrical equipment	259	497,293.1	1,157,087.0	118,369.6	341,042.8	6.5	10.2
28	Manufacture of machineryand equipment n.e.c.	470	697,116.0	1,199,257.0	148,184.3	394,529.5	7.1	8.5
29	Manufacture of motor vehicles, trailers and semi-trailers	168	2,769,083.0	8,122,870.0	577,376.1	1,736,492.0	24.0	40.0
30	Manufacture of other transport equipment	119	2,461,196.0	7,971,513.0	535,230.4	1,320,883.0	19.5	43.0
31	Manufacture of furniture	972	607,989.4	1,582,818.0	138,227.4	542,471.0	10.1	15.5
32	Other manufacturing	669	222,878.5	788,559.3	51,567.6	281,111.5	4.4	6.1
33	Repair and installation of machinery and equipment	1,336	475,010.4	1,654,232.0	113,489.1	857,132.7	6.7	19.5
41	Construction of buildings	6,314	446,385.6	1,298,779.0	76,346.5	383,464.6	9.0	21.7
42	Civil engineering	649	4,820,669.0	43,000,000.0	2,297,535.0	24,900,000.0	42.7	379.3
43	Specialised construction activities	6,409	284,919.3	768,704.4	47,526.8	312,280.7	5.8	18.2
45	Wholesale and retail trade and repair of motor vehicles and motorcycles	10,609	454,804.7	1,945,607.0	32,748.8	206,704.9	3.2	4.9
46	Wholesale trade, except of motor vehicles and motorcycles	17,836	901,908.7	4,275,310.0	116,715.7	1,560,502.0	3.7	7.0
47	Retail trade, except of motor vehicles and motorcycles	36,348	445,849.9	1,323,635.0	44,865.4	438,537.9	4.3	10.7
49	Land transport and transport via pipelines	2,778	519,134.8	1,794,067.0	103,545.2	293,940.8	6.5	19.3
50	Water transport	160	2,588,992.0	8,421,588.0	170,045.9	2,125,096.0	7.5	12.3

51	Air transport	37	5,790,746.0	8,140,199.0	3,018,958.0	10,500,000.0	6.2	9.8
52	Warehousing and support activities for transportation	754	2,403,578.0	15,900,000.0	- 382,018.5	11,100,000.0	9.2	22.3
53	Postal and courier activities	150	368,024.2	976,867.1	44,218.1	95,664.4	7.0	11.9
55	Accommodation	4,440	380,175.6	1,754,686.0	183,424.4	1,888,802.0	7.5	34.3
56	Food and beverage service activities	22,491	208,944.7	420,770.1	- 1,773.6	133,880.6	6.2	10.9
58	Publishing activities	944	310,695.1	994,573.4	6,251.0	1,704,594.0	4.7	12.6
59	Motion picture, video and television programme production, sound recording and music publishing activities	701	304,976.0	981,979.4	47,116.5	666,054.3	2.9	8.7
60	Programming and broadcasting activities	63	376,219.8	966,708.5	161,391.2	349,081.2	2.9	3.0
61	Telecommunications	337	738,324.7	4,189,422.0	422,923.9	4,994,171.0	5.1	5.7
62	Computer programming, consultancy and related activities	4,279	380,175.3	2,462,886.0	98,536.8	832,046.0	6.1	18.7
63	Information service activities	630	187,103.8	818,363.6	52,349.8	784,075.1	5.7	31.0
64	Financial service activities, except insurance and pension funding	392	398,644.2	1,369,637.0	8,354,001.0	70,300,000.0	3.9	11.7
65	Insurance, reinsurance and pension funding, except compulsory social security	4	21,536.8	25,366.0	- 18,004.1	60,981.1	2.3	1.0
66	Activities auxiliary to financial services and insurance activities	2,917	167,698.9	1,869,785.0	57,165.9	523,957.5	2.3	2.6
69	Legal and accounting activities	3,593	78,408.6	183,496.0	21,232.2	65,070.7	3.1	5.1
70	Activities of head offices; management consultancy activities	5.870	281,951.0	1,621,208.0	- 3,359,513.0	102,000,000.0	4.2	19.6
71	Architectural and engineering activities; technical testing and analysis	3,692	287,175.6	1,368,731.0	63,048.8	516,740.0	3.9	10.5
72	Scientific research and development	348	160,301.2	566,546.5	373,062.4	1,407,893.0	5.8	25.6
73	Advertising and market research	2,183	218,928.7	584,014.3	27,646.6	280,831.3	3.0	4.5
74	Other professional, scientific and technical activities	3,593	211,523.6	1,006,148.0	51,484.2	577,799.3	2.6	6.2
75	Veterinary activities	862	132,647.3	295,701.9	65,177.1	541,829.7	3.2	7.0
77	Rental and leasing activities	1,219	352,539.8	1,146,283.0	111,973.8	419,295.0	3.3	6.3
78	Employment activities	489	1,609,758.0	3,343,786.0	195,624.1	795,900.2	95.8	263.4
79	Travel agency, tour operator reservation service and related activities	2,092	617,069.8	2,610,664.0	24,606.6	247,015.9	2.8	4.8
80	Security and investigation activities	340	594,266.2	2,812,400.0	145,182.1	796,953.4	28.9	86.1
81	Services to buildings and landscape activities	1,905	163,419.4	462,897.4	19,864.0	154,925.7	9.9	39.6
82	Office administrative, office support and other business support activities	3,483	287,603.0	1,110,376.0	57,257.2	456,316.1	5.8	32.0
	Total	164,538	485,393.9	4,233,883.0	- 22,596.4	19,700,000.0	5.8	32.8

Notes: N = 164.538 firm-year observations, representing a total of 72,035 new firms established between 2010 and 2017 and followed until 2017

	Industry code	Inventories		Services		Advertising		Capital	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
10	Manufacture of food products	48,262.0	198,149.7	65,073.9	224,270.3	5,492.6	84,126.5	248,532.3	1,050,628.0
11	Manufacture of beverages	110,291.4	257,832.1	15,672.2	59,977.2	6,250.1	22,185.9	279,344.7	1,442,684.0
13	Manufacture of textiles	216,362.0	897,657.8	305,662.7	887,980.6	4,122.7	14,429.4	403,772.0	1,517,289.0
14	Manufacture of wearing apparel	91,459.6	448,373.4	153,156.0	360,710.7	2,393.0	10,945.5	106,584.3	474,657.9
15	Manufacture of leather and related products	113,003.9	295,773.0	138,918.2	272,975.1	5,389.2	25,374.0	168,183.6	418,822.2
16	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials	1,017,053.0	9,167,445.0	101,373.2	340,293.9	2,297.3	14,326.8	461,901.6	1,816,649.0
17	Manufacture of paper and paper products	438,293.7	1,505,301.0	104,162.3	775,443.7	49,071.4	240,987.6	4,290,729.0	23,500,000.0
18	Printing and reproduction of recorded media	10,906.5	30,195.3	75,685.8	191,778.9	1,466.7	10,203.0	59,427.4	139,655.7
19	Manufacture of coke and refined petroleum products	1,377,494.0	4,587,916.0	1,044,351.0	1,612,611.0	2,378.1	4,044.6	12,900,000.0	9,959,354.0
20	Manufacture of chemicals and chemical products	91,748.3	234,325.2	62,671.6	294,685.7	6,885.7	23,597.8	768,317.6	3,068,597.0
21	Manufacture of basic pharmaceutical products and pharmaceutical preparations	258,104.9	649,817.3	24,681.9	56,482.4	12,745.8	25,890.2	366,528.3	673,703.3
22	Manufacture of rubber and plastic products	180,749.2	591,334.0	52,490.4	172,848.3	5,404.6	28,780.2	710,201.8	1,915,200.0
23	Manufacture of other non-metallic mineral products	98,414.7	425,126.6	83,041.0	325,101.7	3,814.4	17,671.0	327,692.8	1,745,495.0
24	Manufacture of basic metals	357,953.5	1,141,649.0	83,411.3	252,158.6	49,071.4	240,987.6	4,290,729.0	23,500,000.0
25	Manufacture of fabricated metal products, except machinery and equipment	49,301.6	206,111.6	176,537.1	573,452.5	1,466.7	10,203.0	59,427.4	139,655.7
26	Manufacture of computer, electronic and optical products	191,436.8	568,034.9	272,644.1	492,564.9	2,378.1	4,044.6	12,900,000.0	9,959,354.0
27	Manufacture of electrical equipment	71,881.2	155,402.9	108,481.1	255,500.2	6,885.7	23,597.8	768,317.6	3,068,597.0
28	Manufacture of machineryand equipment n.e.c.	99,714.5	233,750.1	115,121.4	352,372.4	12,745.8	25,890.2	366,528.3	673,703.3
29	Manufacture of motor vehicles, trailers and semi-trailers	252,537.9	531,376.9	135,489.4	282,869.9	5,404.6	28,780.2	710,201.8	1,915,200.0
30	Manufacture of other transport equipment	250,408.7	884,606.3	1,170,985.0	4,105,846.0	3,814.4	17,671.0	327,692.8	1,745,495.0
31	Manufacture of furniture	96,250.4	320,336.5	51,926.3	239,580.6	7,279.4	31,650.8	145,491.7	492,491.1
32	Other manufacturing	38,881.8	107,023.4	35,401.9	148,614.6	2,268.7	7,663.6	167,369.0	1,500,651.0
33	Repair and installation of machinery and equipment	29,424.7	79,027.2	288,722.4	1,209,051.0	1,292.8	5,203.9	56,093.2	198,963.6
41	Construction of buildings	101,104.3	1,087,420.0	404,506.4	1,090,620.0	1,059.7	4,716.5	58,441.0	476,620.1
42	Civil engineering	235,213.7	2,115,134.0	4,415,725.0	41,500,000.0	3,808.0	37,619.8	1,229,133.0	13,000,000.0
43	Specialised construction activities	15,575.7	46,168.3	223,395.4	608,916.9	845.1	3,805.8	26,367.0	66,404.4
45	Wholesale and retail trade and repair of motor vehicles and motorcycles	100,294.0	359,239.1	56,501.1	213,822.4	1,740.4	11,110.0	31,234.8	136,174.1
46	Wholesale trade, except of motor vehicles and motorcycles	95,697.0	626,093.9	43,264.0	259,465.1	8,931.1	93,269.0	61,929.9	1,589,451.0
47	Retail trade, except of motor vehicles and motorcycles	62,476.1	183,004.8	17,175.0	178,893.9	3,386.5	32,850.6	46,844.8	218,060.3
49	Land transport and transport via pipelines	6,632.6	54,417.3	442,502.6	1,213,676.0	1,474.1	11,385.0	121,945.8	404,699.2
50	Water transport	25,472.0	100,104.6	2,558,872.0	8,347,657.0	6,084.3	30,175.4	1,233,157.0	3,435,051.0
51	Air transport	7,939.0	35,153.0	5,790,746.0	8,140,199.0	7,582.0	11,068.9	7,923,317.0	19,500,000.0
52	Warehousing and support activities for transportation	9,778.3	86,844.0	2,372,739.0	15,900,000.0	3,235.0	13,860.4	189,264.3	1,116,700.0
53	Postal and courier activities	4,341.0	22,378.3	360,156.4	975,612.0	1,959.9	7,165.6	30,561.1	63,277.5

55	Accommodation	24,628.1	392,394.8	360,163.0	1,729,594.0	4,195.9	23,503.9	536,275.2	2,378,444.0
56	Food and beverage service activities	8,518.9	22,283.0	173,512.9	365,735.6	1,983.0	11,037.4	55,769.7	177,450.1
58	Publishing activities	46,297.2	319,217.1	175,598.3	699,004.7	12,104.9	63,124.5	18,875.1	86,965.5
59	Motion picture, video and television programme production, sound recording and music publishing activities	1,426.3	8,241.6	259,540.6	800,226.5	28,119.5	232,326.2	75,299.6	335,639.4
60	Programming and broadcasting activities	971.6	3,588.5	371,098.5	961,688.5	5,810.2	13,655.9	46,416.1	110,496.1
61	Telecommunications	4,146.7	11,856.9	717,714.2	4,191,302.0	1,729.2	7,577.5	56,232.6	634,578.5
62	Computer programming, consultancy and related activities	7,227.8	79,839.1	284,135.4	1,061,515.0	6,070.2	38,964.2	31,371.0	336,292.1
63	Information service activities	3,465.4	30,164.8	183,232.0	817,718.3	5,031.3	22,797.0	137,597.5	2,791,120.0
64	Financial service activities, except insurance and pension funding	9,404.0	61,958.9	282,110.8	1,003,998.0	5,268.3	22,080.4	206,299.4	1,689,938.0
65	Insurance, reinsurance and pension funding, except compulsory social security	-	-	21,536.8	25,366.0	699.5	748.1	3,685.6	5,313.0
66	Activities auxiliary to financial services and insurance activities	426.7	6,949.9	166,866.9	1,869,817.0	1,075.9	5,099.9	12,730.5	31,262.8
69	Legal and accounting activities	662.1	8,613.3	76,987.3	182,968.1	775.9	3,156.5	16,085.6	48,282.9
70	Activities of head offices; management consultancy activities	79,586.0	1,926,238.0	226,152.0	1,157,326.0	6,783.6	83,588.6	40,633.7	370,530.6
71	Architectural and engineering activities; technical testing and analysis	14,280.9	121,973.6	218,953.4	981,415.1	1,779.4	8,724.7	43,566.2	307,458.8
72	Scientific research and development	19,590.0	63,842.5	128,802.9	560,433.8	2,117.6	5,621.2	179,075.5	1,020,869.0
73	Advertising and market research	2,243.2	9,135.0	191,660.7	568,680.3	13,803.5	147,140.1	19,256.6	66,315.8
74	Other professional, scientific and technical activities	12,001.1	77,693.0	134,225.3	747,135.6	2,334.9	14,304.9	19,501.3	70,650.2
75	Veterinary activities	13,731.9	24,653.0	70,190.2	237,065.7	988.0	3,341.7	57,869.6	147,582.6
77	Rental and leasing activities	26,603.0	144,159.1	264,753.2	990,158.5	5,429.7	35,767.5	281,838.8	1,342,657.0
78	Employment activities	1,133.7	9,478.4	1,608,794.0	3,344,081.0	3,417.5	8,302.4	25,233.8	56,580.2
79	Travel agency, tour operator reservation service and related activities	1,317.2	17,041.9	602,882.2	2,611,657.0	7,482.0	49,775.6	21,428.1	48,306.0
80	Security and investigation activities	29,584.8	286,081.9	542,180.5	2,818,307.0	11,139.8	116,091.2	62,274.1	267,567.3
81	Services to buildings and landscape activities	3,435.2	28,281.5	153,780.9	460,276.4	1,040.2	10,734.1	18,022.4	38,847.7
82	Office administrative, office support and other business support activities	6,368.1	42,450.6	251,262.7	1,025,572.0	3,988.5	20,384.1	49,876.4	340,944.6
	Total	54,778.3	802,381.5	181,446.1	2,966,975.0	3,973.8	48,607.9	92,890.7	1,410,789.0

Notes: N = 164.538 firm-year observations, representing a total of 72,035 new firms established between 2010 and 2017 and followed until 2017

Table 1(b): Mean, SD, and pairwise correlations

		Mean	SD	1	2	3	4	5	6	7
1	Sales Revenues	485,393.9	4,233,883.0	1						
2	Equity	-22,596.4	19,700,000.0	0.057***	1					
3	Labor	5.8	32.8	0.506***	0.018***	1				
4	Inventories	54,778.3	802,381.5	0.352***	-0.304***	0.196***	1			
5	Services	181,446.1	2,966,975.0	0.726***	0.060***	0.602***	0.148***	1		
6	Advertising	3,973.8	48,607.9	0.168***	0.011***	0.086***	0.092***	0.058***	1	
7	Capital	92,890.7	1,410,789.0	0.587´***	0.080***	0.433***	0.228***	0.504***	0.069***	1

Notes: N = 164.538 firm-year observations, representing a total of 72,035 new firms established between 2010 and 2017 and followed until 2017

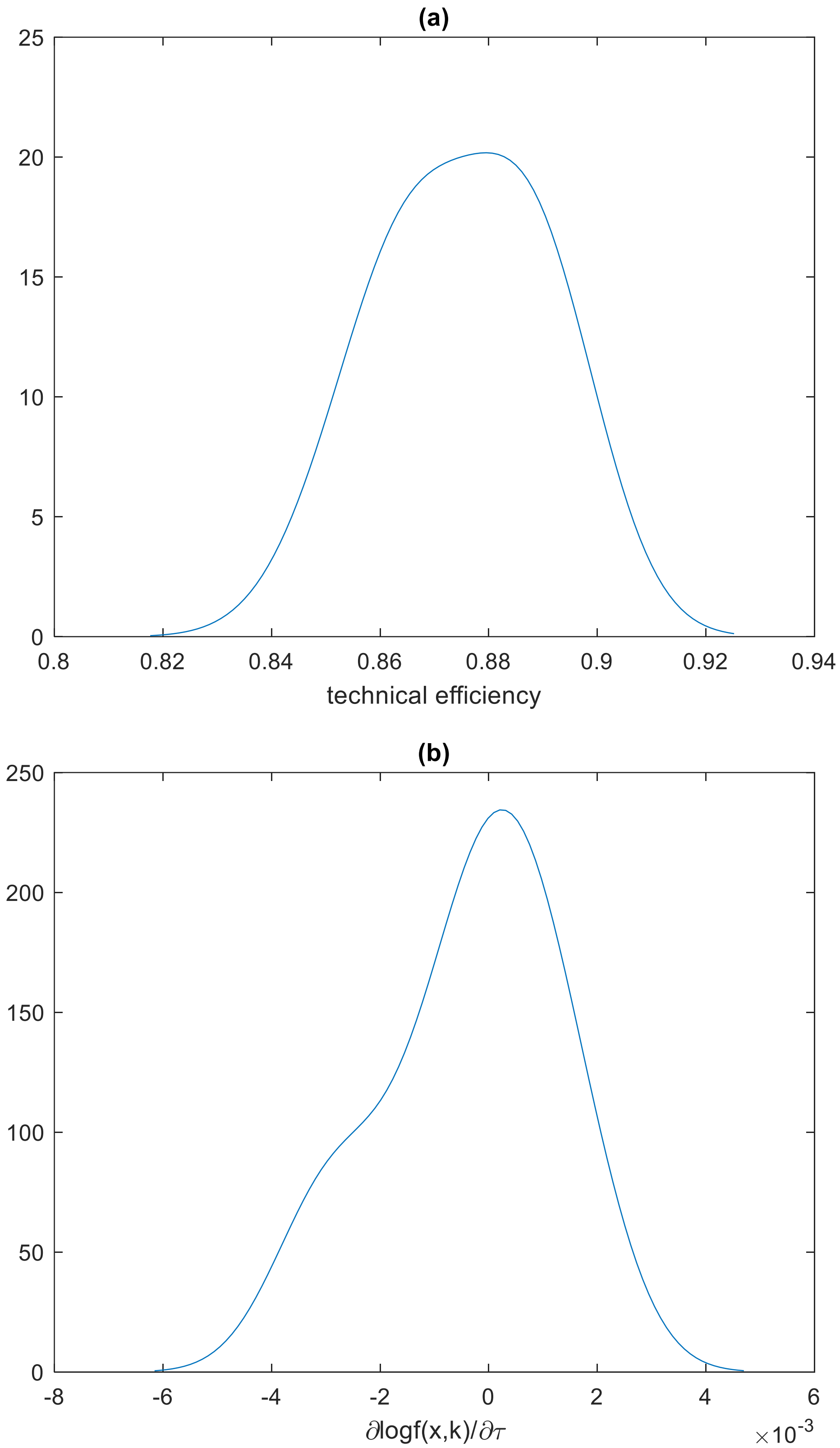
* p<0.10, ** p<0.05, ***p<0.01

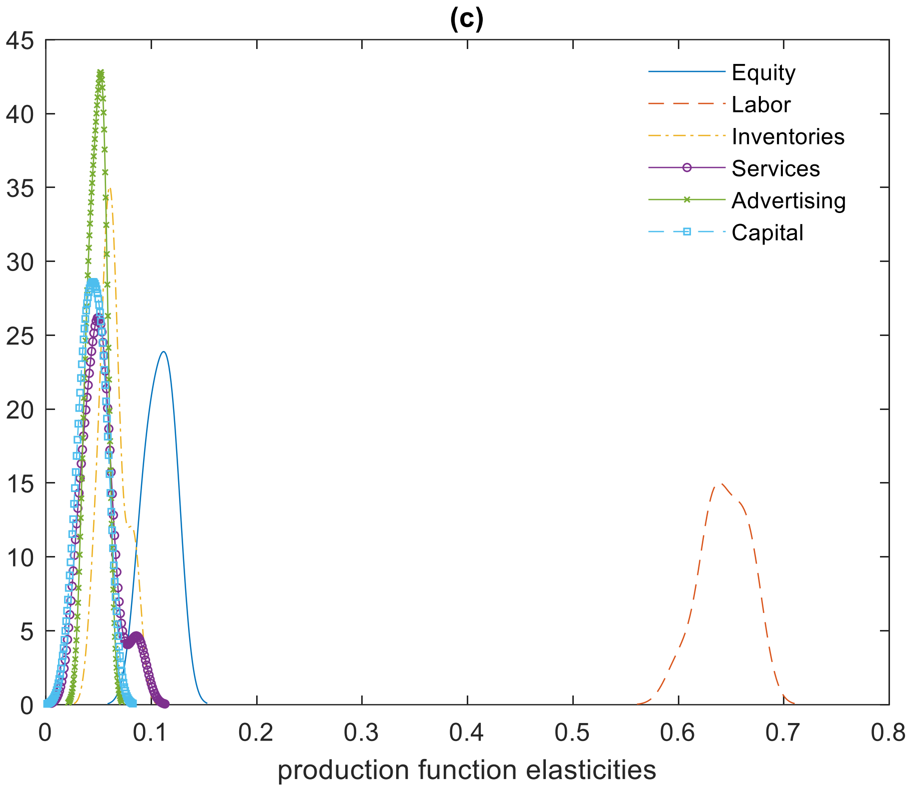
Table 2. Empirical results for posterior moments

	post. mean	post. median	post. s.d.
efficiency	0.875	0.874	0.014
technical change	-0.0004	0.0001	0.0016
production function elasticities			
equity	0.108	0.109	0.012
labor	0.640	0.642	0.022
inventories	0.063	0.061	0.012
services	0.051	0.050	0.016
advertising	0.048	0.049	0.007
capital	0.044	0.044	0.011
returns to scale	0.956	0.957	0.028
β	0.969	0.979	0.033
δ	0.124	0.126	0.027
γ_k	0.268	0.263	0.012
γ_u	0.223	0.214	0.022
productivity growth	0.011	0.015	0.014
ρ_Ω	0.503	0.518	0.055
ρ_π	0.904	0.901	0.023
ρ_ϱ	0.705	0.691	0.044
ρ_u	0.911	0.908	0.015
BF	1.60 10^3	1.65 10^3	0.17 10^3

Notes: Returns to scale are estimated as $RTS = \sum_{j=1}^{J} \frac{\partial \log f(\boldsymbol{x},k)}{\partial \log x_j} + \frac{\partial \log f(\boldsymbol{x},k)}{\partial \log k}$. Productivity growth is estimated as $PG = \log \frac{\Omega_{it}}{\Omega_{i,t-1}}$. Technical change is $TC = \frac{\partial \log f(\boldsymbol{x},k)}{\partial \tau}$. BF is the Bayes factor in favor of the model with advertising and against the model without advertising.

Figure 1. Technical efficiency and other aspects of the model




(c)
45
40
35
30
25
20
15
10
5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
production function elasticities
Equity
Labor
Inventories
Services
Advertising
Capital

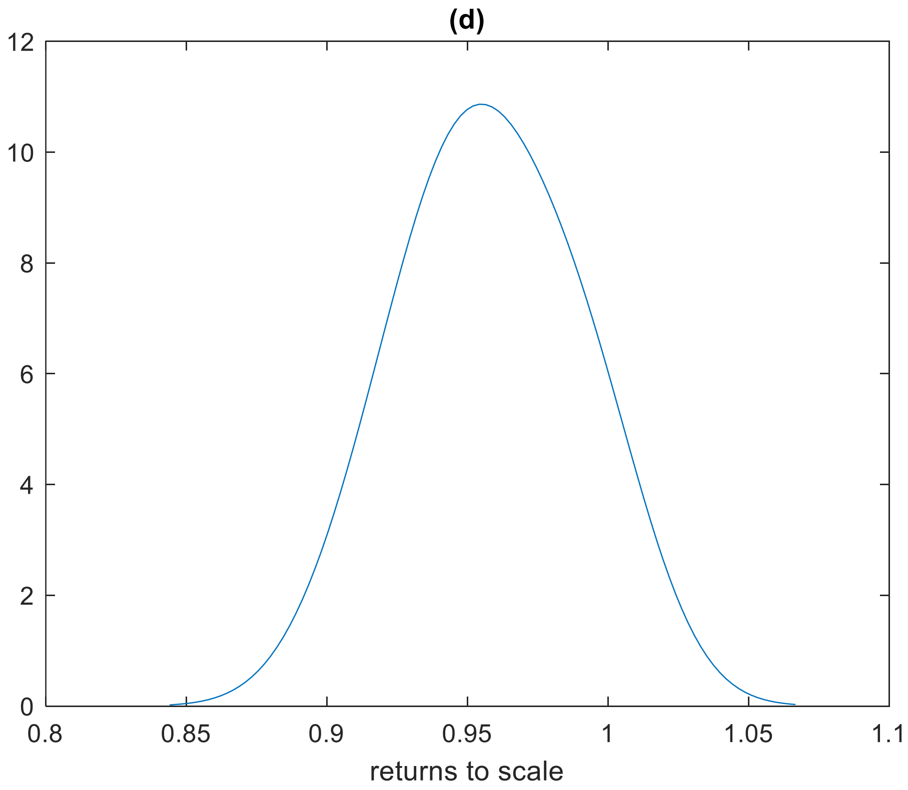
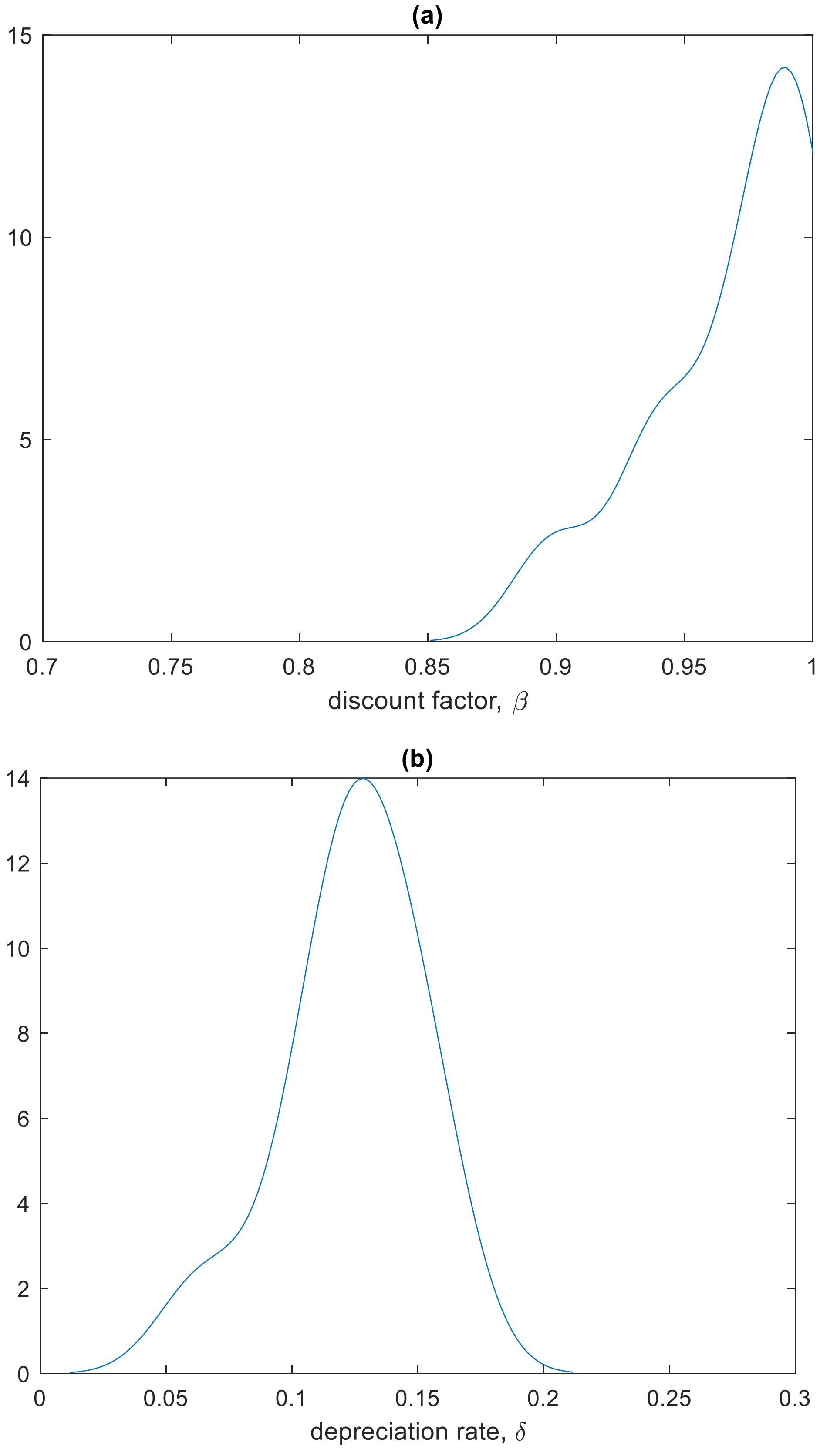
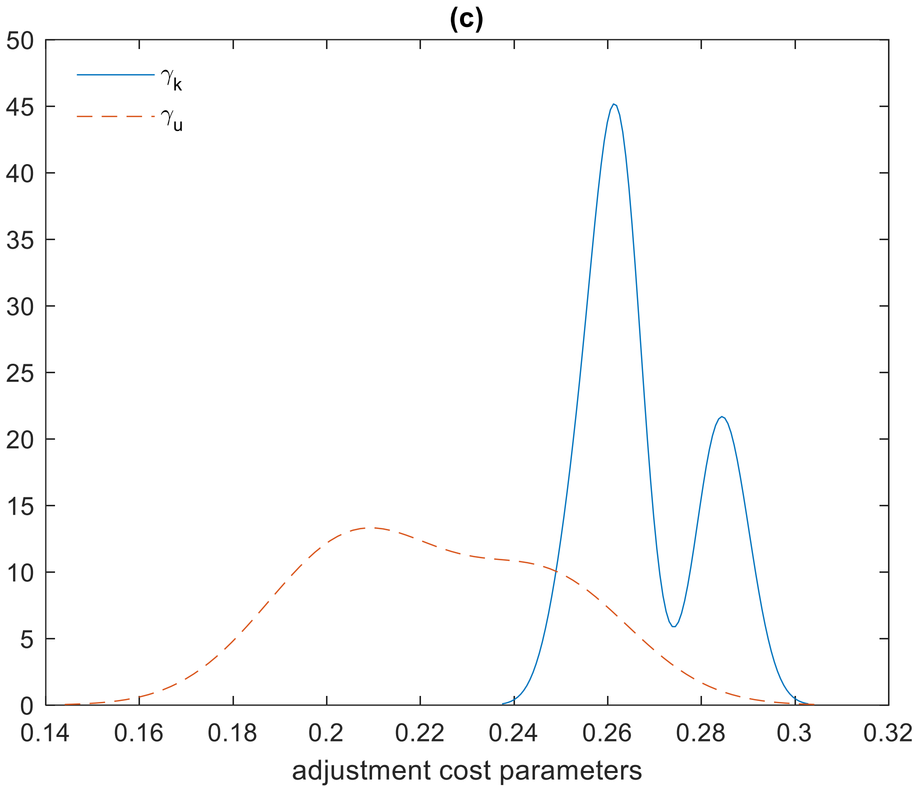

(d)
12
10
8
6
4
2
0
0.8
0.85
0.9
0.95
1
1.05
1.1
returns to scale

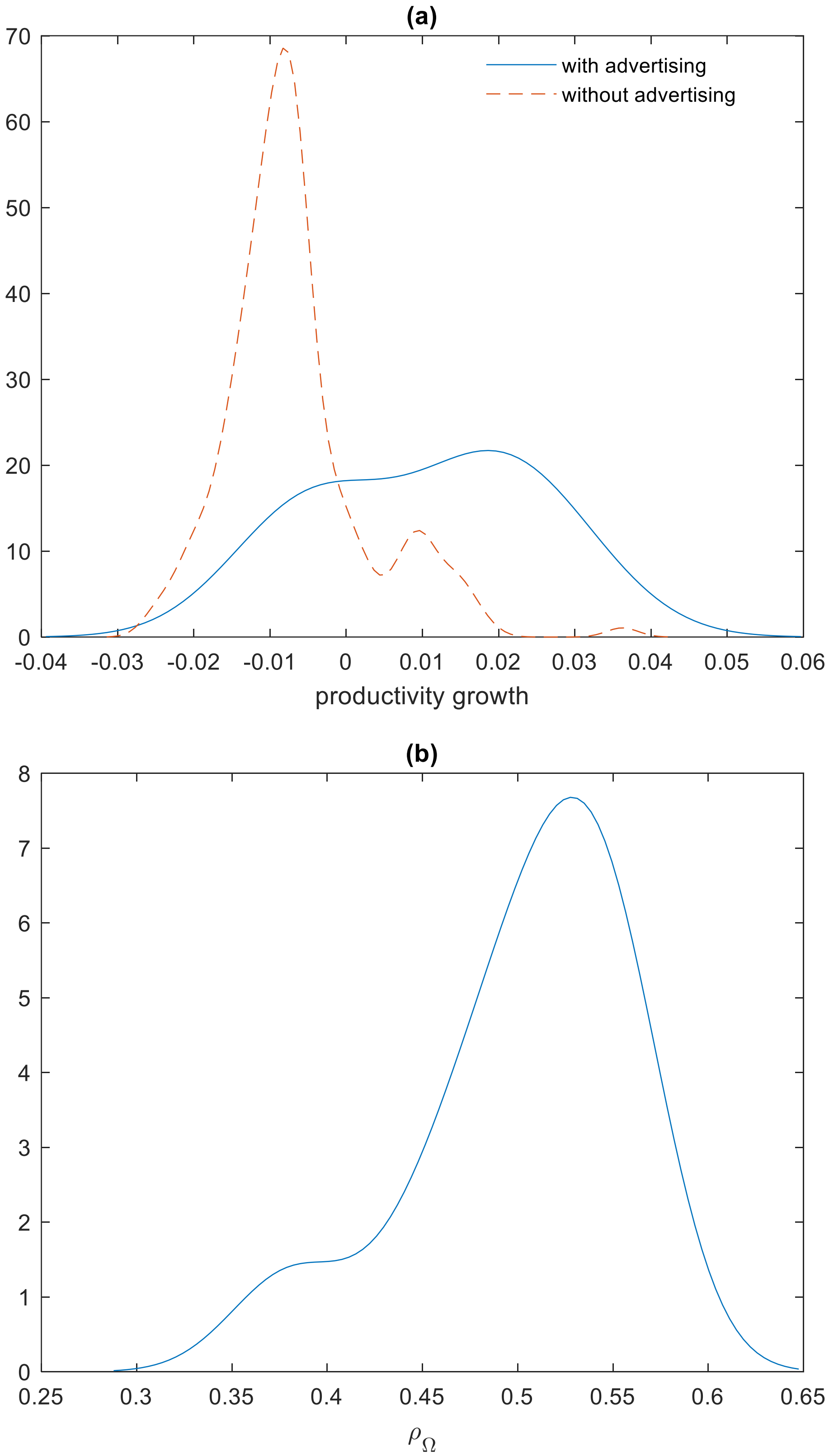
Figure 2. Marginal posterior densities

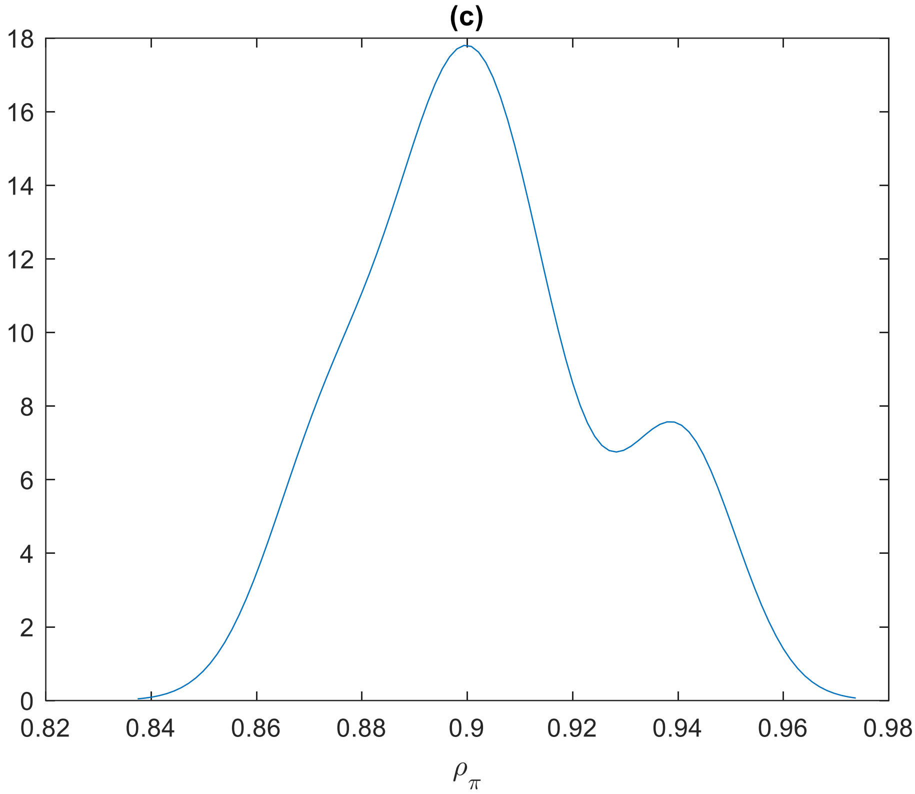


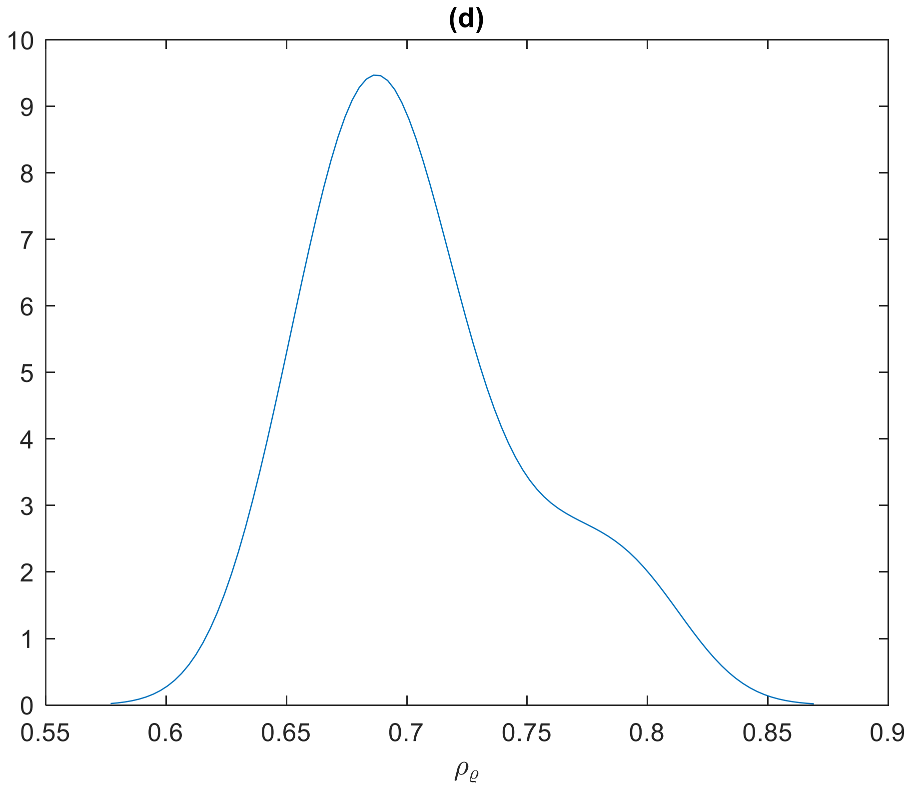


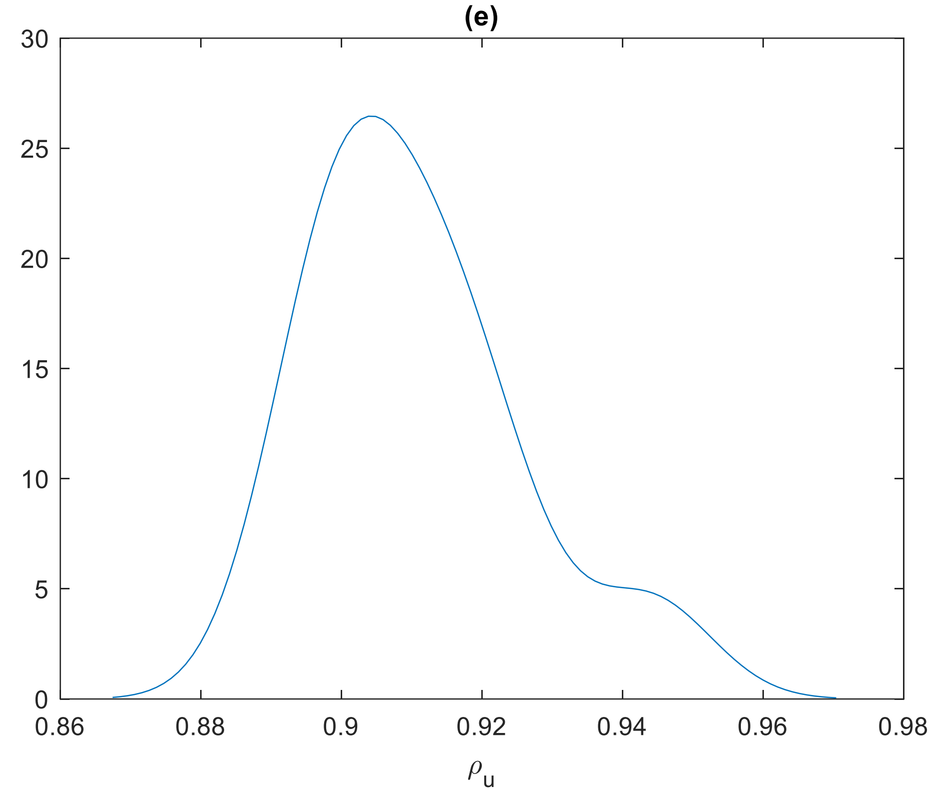
Notes: Productivity growth is estimated as firm-specific posterior mean of $\log \frac{\Omega_{it}}{\Omega_{i,t-1}}$.

Figure 3. Marginal posterior densities, II

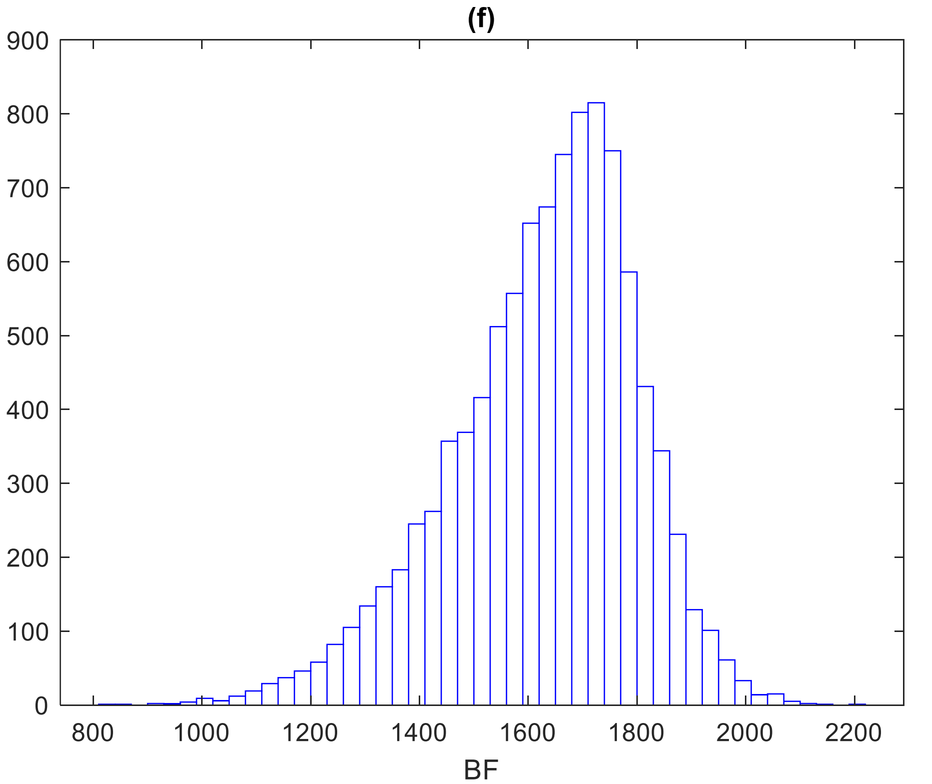



(c)
18
16
14
12
10
8
6
4
2
0
0.82
0.84
0.86
0.88
0.9
0.92
0.94
0.96
0.98
ρ_π


(d)
10
9
8
7
6
5
4
3
2
1
0
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
ρ_ϱ

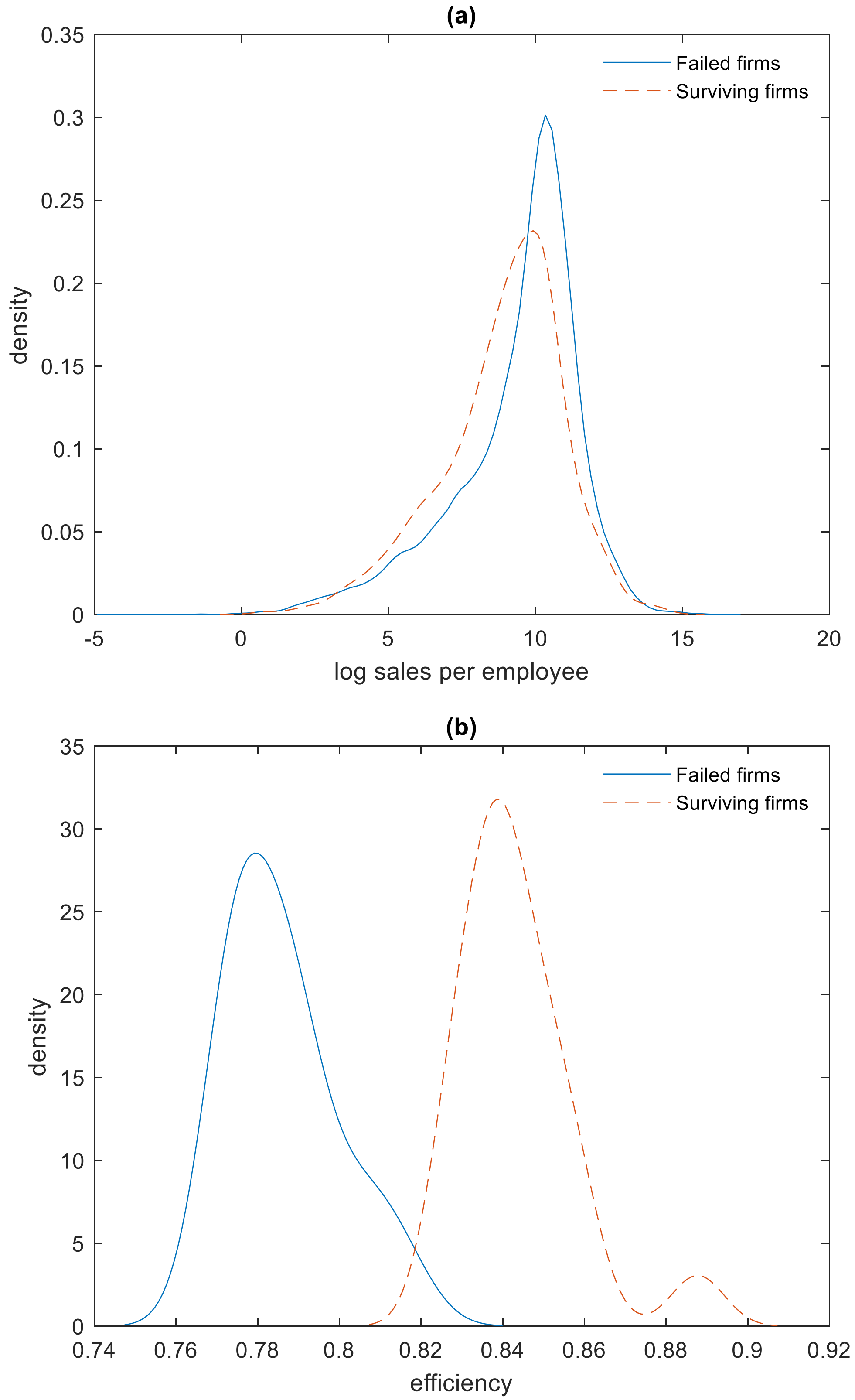


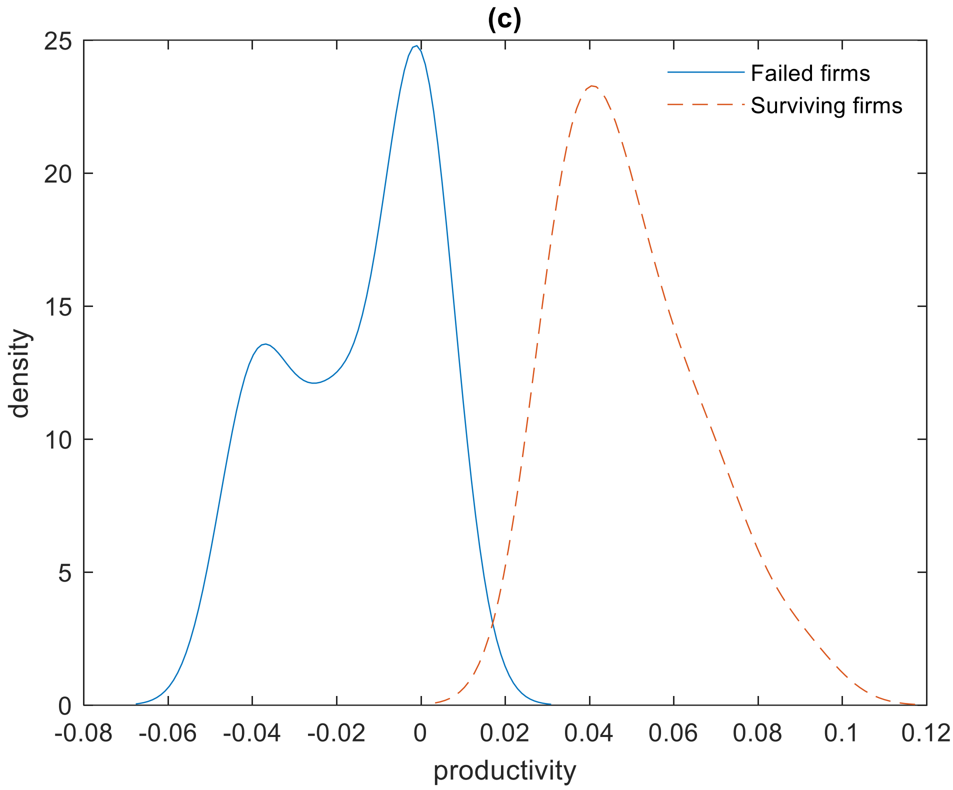
(e)
30
25
20
15
10
5
0
0.86
0.88
0.9
0.92
0.94
0.96
0.98
ρ_u



(f)
900
800
700
600
500
400
300
200
100
0
800
1000
1200
1400
1600
1800
2000
2200
BF

Figure 4. Aspects of the model in terms of efficiency and productivity




(c)
Failed firms
Surviving firms
density
productivity
25
20
15
10
5
0
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1
0.12

Appendix A

The model with inefficiency and productivity

To introduce inefficiency, we modify the production function as

$$y_t = f(\boldsymbol{x}_t, k_t)e^{\Omega_t - u_t}, \tag{A1}$$

where $u_t \geq 0$ represents technical inefficiency, and $\Omega_t \in \Re$ is productivity (Olley and Pakes, 1996, Levinsohn and Petrin, 2003). We assume that Ω_t is exogenously given to the firm but inefficiency is not. Efficiency can be improved and its cost of adjustment is $\gamma_u > 0$ per unit of the product price. The cost of efficiency adjustment is $\frac{1}{2}\gamma_u p_t(e^{-u_t} - e^{-u_{t-1}})^2$. The problem of the firm becomes:

$$\begin{aligned} \max_{x_t \in \Re^J_+, k_t \geq 0, u_t \geq 0} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & p_t f(\boldsymbol{x}_t, k_t)e^{\Omega_t - u_t} - \boldsymbol{w}'_t \boldsymbol{x}_t - r_t k_t \\ & - p_t G\Big(\underbrace{k_t - (1-\delta)k_{t-1}}_{\iota_t}\Big) \\ & - p_t \frac{1}{2}\gamma_u (e^{-u_t} - e^{-u_{t-1}})^2 \Bigg\}. \end{aligned} \tag{A2}$$

Expectations are taken with respect to all future prices as well as the law of motion of productivity:

$$\begin{gathered} \Omega_t = \varphi_\Omega + \rho_\Omega \Omega_{t-1} + \xi_t, \xi_t \sim \text{i.i.d}\, \mathcal{N}(0, \sigma_\xi^2), t = 1,2,\ldots, \\ \Omega_0 \sim \mathcal{N}\left(\frac{\varphi_\Omega}{1-\rho_\Omega}, \frac{\sigma_\xi^2}{1-\rho_\Omega^2}\right), \rho_\Omega \in (0,1). \end{gathered} \tag{A3}$$

All first-order conditions have to be modified in view of introducing productivity and costly efficiency. However, the cost-minimizing conditions remain the same as in (13). The Euler equation for capital in (16) becomes:

$$e_k(\boldsymbol{x}_t, k_t)\frac{f(\boldsymbol{x}_t, k_t)}{k_t} e^{\Omega_t - u_t} - \varrho_t - \gamma_k \iota_t = \beta\gamma_k(1-\delta)\mathbb{E}_t \iota_t \pi_{t+1}. \tag{A4}$$

The Euler equation for inefficiency becomes:

$$\begin{aligned} f(\boldsymbol{x}_t, k_t)e^{\Omega_t} - \gamma_u(e^{-u_t} - e^{-u_{t-1}}) + \beta\gamma_u e^{-u_t}\mathbb{E}_t \pi_{t+1}(e^{-u_{t+1}} - e^{-u_t}) \\ = 0. \end{aligned} \tag{A5}$$

Finally, Bellman's equation for the problem is:

$$\begin{gathered} V(k, u, \Omega) = \left\{ \max_{x \in \Re^J_+, k \geq 0, u \geq 0} \quad pf(\boldsymbol{x}, k) - \boldsymbol{w}'\boldsymbol{x} - rk - pG(\iota) \right\} \\ +\beta \int V(k', u', \Omega')\, \mathrm{d}F(\Omega'|\Omega), \\ k' = (1-\delta)k + \iota, \\ \Omega' = \varphi_\Omega + \rho_\Omega \Omega, \end{gathered} \tag{A6}$$

where $F(\Omega'|\Omega)$ denotes the distribution function of Ω' conditional on Ω.

Estimation and inference

1. Assumptions

Our system of equations consists of a set of conditional moments in (13), (A4), and (A5). Both (A4) and (A5) contain sector, firm, region, and age effects to capture time-invariant effects. In addition, log relative prices ω_{tj} ($j = 2, \ldots, J$), the relative user cost of capital (ϱ_t) and inflation (π_{t+1}) are unknown. Using the formulation in section 2, we assume

$$\omega_{it,j} = \alpha_{j,i} + \gamma_{j,t}\ \forall j = 2, \ldots, J, \tag{A7}$$

where i and t denote the firm and time ($i = 1, \ldots, n, t = 1, \ldots, T$), $\alpha_{j,i}$ denotes factor-specific firm effects, and $\gamma_{j,t}$ denotes factor-specific time effects. For productivity we assume the standard specification (Levinsohn and Petrin 2003; Olley and Pakes 1996):

$$\Omega_{it} = \varphi_{\Omega} + \rho_{\Omega}\Omega_{i,t-1} + \xi_{it}, \xi_{it} \sim \text{i.i.d}\,\mathcal{N}(0,\sigma_{\xi}^2), i = 1,\dots,n, t = 1,\dots,T. \tag{A8}$$

The initial condition is modeled as

$$\Omega_{i0} \sim \mathcal{N}\left(\frac{\varphi_{\Omega}}{1-\rho_{\Omega}}, \frac{\sigma_{\xi}^2}{1-\rho_{\Omega}^2}\right), i = 1,\dots,n, \rho_{\Omega} \in (0,1). \quad \text{(A9}$$

It remains to specify processes for the relative user cost of capital and inflation. We assume:

$$\begin{gathered}\log\,\pi_t = \varphi_{\pi} + \rho_{\pi}\log\,\pi_{t-1} + \rho_o\Pi_t^* + \xi_{\pi,t}, \xi_{\pi,t} \sim \text{i.i.d}\,\mathcal{N}(0,\sigma_{\pi}^2), \rho_{\pi} \in (0,1),\\ \log\,\pi_0 \sim \mathcal{N}\left(\frac{\varphi_{\pi}}{1-\rho_{\pi}}, \frac{\sigma_{\pi}^2}{1-\rho_{\pi}^2}\right).\end{gathered} \tag{A10}$$

Here, Π_t^* is the economy-wide inflation ratio (based on the consumer price index, CPI) which we take from the IMF data to help us anchor better inflation and relative prices.

For the relative user cost of capital we can, in turn, assume that it is related to inflation as follows:

$$\begin{gathered}\log\,\varrho_t = \varphi_{\varrho} + \rho_{\varrho}\log\,\varrho_{t-1} + \rho_{\pi\varrho}\log\,\pi_t + \xi_{\varrho,t}, \xi_{\varrho,t} \sim \text{i.i.d}\,\mathcal{N}(0,\sigma_{\varrho}^2),\\ \log\,\varrho_0 \sim \mathcal{N}\left(\frac{\varphi_{\varrho} + \rho_{\pi}\,log\,\pi_t}{1-\rho_{\varrho}}, \frac{\varrho_{\pi\varrho}^2 + \sigma_{\pi0}^2 + \sigma_{\varrho}^2}{1-\rho_{\varrho}^2}\right), \rho_{\varrho} \in (0,1),\end{gathered} \tag{A11}$$

where $\sigma_{\pi0}^2 = \frac{\sigma_{\pi}^2}{1-\rho_{\pi}^2}$.

2. Estimation

In the interest of brevity let us define the vector of log relative prices $\boldsymbol{\omega}_t = [\omega_{2,t},\dots,\omega_{J,t}]'$, and the other (dynamic) latent variables in the model, viz.

$$\boldsymbol{\lambda}_t = [\Omega_t, \varrho_t, \boldsymbol{u}_t, \pi_t], \tag{A12}$$

where $\Omega_t = [\Omega_{1t},\dots,\Omega_{nt}]', \boldsymbol{u}_t = [u_{1t},\dots,u_{nt}]'$. Although the firm optimizes with respect to inefficiency, this is unobserved by us. Therefore, we have to adopt a law of motion for this variable:

$$\begin{gathered}\log\,u_{it} = \varphi_u + \rho_u\log\,u_{i,t-1} + \xi_{u,it}, \xi_{u,it} \sim \text{i.i.d}\,\mathcal{N}(0,\sigma_u^2), i = 1,\dots,n, t = 1,\dots,T,\\ \log\,u_{i0} \sim \mathcal{N}\left(\frac{\varphi_u}{1-\rho_u}, \frac{\sigma_u^2}{1-\rho_u^2}\right), i = 1,\dots,n, \rho_u \in (0,1).\end{gathered} \tag{A13}$$

In addition, we specify our production function as the translog functional form which enjoys widespread popularity:

$$\begin{gathered}\log\,y_{it} = \beta_{i0} + \boldsymbol{\beta}_1{}'\log\,\boldsymbol{x}_{it} + \beta_k\log\,k_{it} + \beta_{\tau}\tau_{it}\\ +\frac{1}{2}\log\,\boldsymbol{x}_{it}{}'\boldsymbol{B}_1\log\,\boldsymbol{x}_{it} + \frac{1}{2}\beta_{kk}(\log\,k)_{it}^2 + \frac{1}{2}\beta_{\tau\tau}\tau_{it}^2 + \boldsymbol{\beta}_{\tau x}{}'\tau_{it}\log\,x_{it} + \boldsymbol{\beta}_{kx}{}'\log\,x_{it}\log\,k_{it}\end{gathered} \tag{A14}$$

where $\tau_{it} = t$ ($\forall i = 1,\dots,n, t = 1,\dots,T$) denotes a time trend. Moreover, β_{i0} denotes firm effects. The parameters of the translog production function are collectively denoted by $\boldsymbol{\beta} \in \Re^P$, where P denotes the dimensionality of $\boldsymbol{\beta}$.

To proceed with estimation, our estimating equations have a conditional moments structure and we can state them compactly as follows.

$$\mathbb{E}_t\mathfrak{F}(\boldsymbol{\lambda}_{t-1}, \boldsymbol{\lambda}_t, \boldsymbol{\lambda}_{t+1}, \boldsymbol{\omega}_t, \boldsymbol{Y}_t, \Theta) = \boldsymbol{0}_{M+J+1}, \tag{A15}$$

where $\mathfrak{F}(\cdot)$ is a vector function of observable data $\boldsymbol{Y}_t = [\log\,\boldsymbol{w}_t{}', \log\,y_t, k_t, \boldsymbol{x}_t]'$, $\boldsymbol{\omega}_t = [\omega_{jt}, j = 2,\dots,J]'$ is the vector of log factor prices, unknown parameters Θ (which contains β plus all ρ and φ parameters previously introduced) and dynamic latent variables and distortions $\boldsymbol{\lambda}_t$. The equations in (A15) could have been used in the context of estimation by the method of Generalized Method of Moments (GMM). In our case however, things are more complicated because the system contains unobserved dynamic latent variables ($\boldsymbol{\lambda}_t$) and the unknown log relative prices ($\boldsymbol{\omega}_t$). For details on computation using state of the art Markov Chain Monte Carlo (MCMC) methods related to (Gallant et al. 2017) (see also Gallant et al. (2018)), we refer the reader to Appendix A. In Appendix C we provide evidence on prior sensitivity as well as numerical performance of our MCMC.

3. Benchmark prior

The parameter vector is $\Theta \in \Re^P$. Our benchmark prior distribution is

$$\Theta \sim \mathcal{N}_P(\bar{\Theta}, \Sigma_{\Theta}), \tag{A16}$$

where $\mathcal{N}_P(\cdot)$ denotes the P-variate normal distribution, $\bar{\Theta} \in \Re^P$ is the prior mean, and $\Sigma_\Theta \in \Re^{P\times P}$ is the prior covariance matrix. To represent a state of knowledge of "knowing little" we set $\bar{\Theta} = \mathbf{0}$ (a $P \times 1$ zero vector), and $\Sigma_\Theta = \bar{h}^2 \boldsymbol{I}_P$, where $\boldsymbol{I}_P$ is the $P \times P$ identity matrix, and $\bar{h} > 0$ is a scalar parameter that measures prior uncertainty about the prior mean. For our benchmark prior we set $\bar{h} = 100$.

Appendix B

We estimate our model via a (nonparametric) Bayesian Exponentially Tilted Empirical Likelihood (BETEL) method proposed by Schennach (2005) as an alternative to fully parametric Bayesian methods which we modify to accommodate the presence of dynamic latent variables—namely, technical efficiency and the distortions in quasi-fixed factors—in the moment conditions.

To fix ideas, first, suppose that no latent variables are involved in the model and we have the moment conditions of the following form: $\mathbb{E}_t\mathcal{G}(\Xi_t, \boldsymbol{a}) = \mathbf{0}_{\dim(\mathcal{G})}\ \forall\, t = 1, \ldots, n$, where Ξ_t and $\boldsymbol{a}$ respectively represent data and unknown parameters. The Bayesian posterior corresponding to the BETEL is given by

$$p(\boldsymbol{a}|\Xi) \propto p(\boldsymbol{a}) \prod_{t=1}^{n} \omega_t^*(\boldsymbol{a}), \tag{B1}$$

where $p(\boldsymbol{a})$ is a prior and $\{\omega_t^*(\boldsymbol{a}), t = 1, \ldots, n\}$ are solutions to the following problem:

$$\max_{\{\omega_t\}_{t=1}^{n}} \quad -\sum_{t=1}^{n} \omega_t \log\, \omega_t \tag{B2}$$

$$\text{subject to} \quad \sum_{t=1}^{n} \omega_t = 1 \tag{B3}$$

$$\sum_{t=1}^{n} \omega_t\, \mathcal{G}(\Xi_t, \boldsymbol{a}) = \mathbf{0}_{\dim(\mathcal{G})}, \tag{B4}$$

provided that the interior of the convex hull of $\bigcup_{t=1}^{n}\{\mathcal{G}(\Xi_t, \boldsymbol{a})\}$ contains the origin.

Now suppose that the model contains dynamic latent variables $\boldsymbol{\lambda}_t$ and we have the moment conditions in (31): $\mathbb{E}_t\mathfrak{F}(\boldsymbol{\lambda}_{t+1}, \boldsymbol{Y}_t, \Theta) = \mathbf{0}\ \forall\, t = 1, \ldots, n$, where we have suppressed the dependence on $\boldsymbol{\lambda}_{t-1}$ and $\boldsymbol{\lambda}_t$ for notational simplicity. Also, assume that dynamic latent variables evolve according to some autoregressive process:

$$\boldsymbol{\lambda}_{t+1} = G(\boldsymbol{\lambda}_t, \boldsymbol{\pi}) + \varepsilon_t, \tag{B5}$$

where $G(\cdot)$ is a conditional mean function, $\boldsymbol{\pi}$ is a vector of parameters (corresponding to φs and ρs in main text), and ε_t is a random innovation.

Our objective is to reduce the estimation problem, which contains latent variables, into the more conventional BETEL problem so that the posterior result from above may also be used in our case. Thus, our posterior has the following form:

$$p(\Theta, \boldsymbol{\pi}, \boldsymbol{\lambda}|\boldsymbol{Y}) \propto p(\Theta)p(\boldsymbol{\pi}) \prod_{t=1}^{n} p\,(\boldsymbol{\lambda}_{t+1}|\boldsymbol{\lambda}_t, \boldsymbol{\pi}) \prod_{t=1}^{n} \omega_t^*(\Theta, \boldsymbol{\lambda}), \tag{B6}$$

where $\boldsymbol{\lambda} = \{\boldsymbol{\lambda}_t, t = 1, \ldots, n\}$, and $\omega_t^*(\Theta, \boldsymbol{\lambda})$ solves

$$\max_{\{\omega_t\}_{t=1}^{n}} \quad -\sum_{t=1}^{n} \omega_t \log\, \omega_t \tag{B7}$$

$$\text{subject to} \quad \sum_{t=1}^{n} \omega_t = 1 \tag{B8}$$

$$\sum_{t=1}^{n} \omega_t\, \mathcal{F}(\boldsymbol{\lambda}_{t+1}, \boldsymbol{Y}_t, \Theta) \otimes \boldsymbol{z}_t = \mathbf{0}, \tag{B9}$$

with $\boldsymbol{z}_t$ being a vector of instruments. These instruments are subsumed in conditional expectations $\mathbb{E}_t(\cdot)$ in main text and should, at least, include variables that are considered by the firm when making its decisions. Here, we assume that the instruments are lagged values of inputs, output, time, their squares and interactions and regional and firm dummies multiplied by all these variables.

Posterior in (A.6) depends on parameters Θ and $\boldsymbol{\pi}$ as well as the dynamic latent variables in $\boldsymbol{\lambda}$. While these latent variables are of fundamental interest in themselves, at the same time, they must also be integrated out of the posterior to perform statistical inference on the parameters:

$$p(\Theta, \boldsymbol{\pi}|\boldsymbol{Y}) \propto \int p\,(\Theta, \boldsymbol{\pi}, \boldsymbol{\lambda}|\boldsymbol{Y})d\boldsymbol{\lambda}, \tag{B10}$$

which, in general, is impossible to perform analytically.

Before proceeding further, we first need to specify the process in (A.5). We opt for a second-order vector autoregressive (VAR) specification with Gaussian innovations, i.e.,

$$\boldsymbol{\lambda}_t = \boldsymbol{\pi}_0 + \boldsymbol{\pi}_1\boldsymbol{\lambda}_{t-1} + \boldsymbol{\varepsilon}_t \quad \text{with} \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}_M, \Sigma_\varepsilon), \tag{B11}$$

where $M = \dim(\boldsymbol{\lambda}_t)$, $\boldsymbol{\pi}_0$, $\boldsymbol{\pi}_1$ are the $M \times 1$ and $M \times M$ respectively, and $\boldsymbol{\pi} = [\boldsymbol{\pi}_0', \text{vec}(\boldsymbol{\pi}_1)']'$. The choice of a VAR model is motivated by the dynamics associated with our optimization problem.

We use Markov Chain Monte Carlo (MCMC) methods to perform computations. Our MCMC involves two steps that are carried out for each MCMC iteration. In the first step, we use Sequential Monte Carlo (SMC), or Particle Filtering (PF), to provide draws for $\{\boldsymbol{\lambda}_t^{(i)}, i = 1, \dots, N\}$, where i indexes the MCMC simulation, and N is the total number of such simulations. In the second step, we draw parameters $\Theta^{(i)}$ and $\boldsymbol{\pi}^{(i)}$. Since standard Metropolis-Hastings algorithms may be quite computationally inefficient, we use the Girolami and Calderhead (2011) Langevin–Hamiltonian Monte Carlo method (hereafter, the GC algorithm). This technique is reliable, requires almost no tuning, and the MCMC draws that it provides have considerably less autocorrelation compared to other MCMC algorithms.

In what follows, we briefly describe the employed methodologies using generic notation.

1. Step 1.

The SMC/PF methodology is applied to state-space models of the following generic form:

$$y_T \sim p(y_t|x_t) \quad \text{and} \quad s_t \sim p(s_t|s_{t-1}), \tag{B12}$$

where s_t is a state variable. Given the data Y_t, the posterior distribution $p(s_t|Y_t)$ can be approximated by a set of (auxiliary) particles $\{s_t^{(i)}, i = 1, \dots, N\}$ with probability weights $\{w_t^{(i)}, i = 1, \dots, N\}$ such that $\sum_{i=1}^N w_t^{(i)} = 1$. With this, we can approximate the predictive density by

$$p(s_{t+1}|Y_t) = \int p\,(s_{t+1}|s_t)p(s_t|Y_t)ds_t \simeq \sum_{i=1}^N p\,(s_{t+1}|s_t^{(i)})w_t^{(i)}, \tag{B13}$$

with the final approximation for the filtering density being given by

$$\begin{aligned} p(s_{t+1}|Y_t) &\propto p(y_{t+1}|s_{t+1})p(s_{t+1}|Y_t) \\ &\simeq p(y_{t+1}|s_{t+1})\sum_{i=1}^N p\,(s_{t+1}|s_t^{(i)})w_t^{(i)}. \end{aligned} \tag{B14}$$

Then, the basic mechanism of particle filtering rests on propagating $\{s_t^{(i)}, w_t^{(i)}, i = 1, \dots, N\}$ to the next step, i.e., $\{s_{t+1}^{(i)}, w_{t+1}^{(i)}, i = 1, \dots, N\}$. Because parameters $\Theta \in \Re^P$ are often available, Andrieu and Roberts (2009), Flury and Shephard (2011) and Pitt et al. (2012) provide the Particle Metropolis-Hastings (PMCMC) technique which uses an unbiased estimator of the likelihood function $\hat{p}_N(Y|\Theta)$ since $p(Y|\Theta)$ is often unavailable in a closed form.

We use Sequential Monte Carlo / Particle Filtering (SMC/PF), where particles are simulated through the state density $p(s_t^{(i)}|s_{t-1}^{(i)})$ and then re-sampled with weights determined by the measurement density evaluated at the resulting particle, i.e., $p(y_t|s_t^{(i)})$. The latter is simple to construct and rests upon the following steps, for $t = 0, \dots, T-1$ given samples $s_t^k \sim p(s_t|Y_{1:t})$ with mass π_t^k for $k = 1, \dots, N$:

1. For $k = 1, \dots, N$, compute $\omega_{t|t+1}^{(k)} = g(y_{t+1}|s_t^{(k)})\pi_t^{(k)}$, $\pi_{t|t+1}^{(k)} = \omega_{t|t+1}^{(k)} / \sum_{i=1}^N \omega_{t|t+1}^{(i)}$.
2. For $k = 1, \dots, N$, draw $\tilde{s}_t^{(k)} \sim \sum_{i=1}^N \pi_{t|t+1}^{(i)} \delta_{s_t}^{(i)}(ds_t)$.
3. For $k = 1, \dots, N$, draw $u_{t+1}^{(k)} \sim g(u_{t+1}|\tilde{s}_t^{(k)}, y_{t+1})$ and set $s_{t+1}^{(k)} = h(s_t^{(k)}; u_{t+1}^{(k)})$.
4. For $k = 1, \dots, N$, compute
$$\omega_{t+1}^{(k)} = \frac{p(y_{t+1}|s_{t+1}^{(k)})p(u_{t+1}^{(k)})}{g(y_{t+1}|s_t^{(k)})g(u_{t+1}^{(k)}|\tilde{s}_t^{(k)}, y_{t+1})} \quad \text{and} \quad \pi_{t+1}^{(k)} = \frac{\omega_{t+1}^{(k)}}{\sum_{i=1}^N \omega_{t+1}^{(i)}}.$$
5.

Lastly, the estimate of likelihood from ADPF is $p(Y_{1:T}) = \prod_{t=1}^T \left(\sum_{i=1}^N \omega_{t-1|t}^{(i)}\right)\left(N^{-1}\sum_{i=1}^N \omega_t^{(i)}\right)$.

2. Step 2.

To update draws for the parameter vector of interest Θ, the GC algorithm uses local information about both the gradient and the Hessian of the log-posterior conditional on Θ at the existing draw. We use 50,000 iterations. Then, we run another 100,000 MCMC iterations to obtain the final results for posterior moments and densities of parameters and functions of interest.

Let $L(\Theta) = \log\, p\,(\Theta|Y)$ denote the log posterior of Θ. Also, define $G(\Theta) = \text{est.cov}\frac{\partial}{\partial\theta}\log\, p\,(Y|\Theta)$ to be the empirical counterpart of $G_o(\Theta) = -\mathbb{E}_{Y|\Theta}\frac{\partial^2}{\partial\theta\partial\theta'}\log\, p\,(Y|\Theta)$. The Langevin diffusion is provided by the stochastic differential equation below.

$$d\Theta(t) = \frac{1}{2}\tilde{\nabla}_\theta L\{\Theta(t)\}dt + dB(t), \tag{B5}$$

where

$$\tilde{\nabla}_{\Theta} L\{\Theta(t)\} = -G^{-1}\{\Theta(t)\} \cdot \nabla_{\Theta} L\{\Theta(t)\} \quad \text{(B16)}$$

is the"natural gradient" of the Riemann manifold generated by the log posterior. The elements of the Brownian motion, in this instance, are given as:

$$G^{-1}\{\Theta(t)\} dB_i(t)$$
$$= |G\{\Theta(t)\}|^{-1/2} \sum_{j=1}^{P} \frac{\partial}{\partial \Theta}\left[G^{-1}\{\Theta(tt)\}_{ij} |G\{\Theta(t)\}|^{1/2}\right] dt + \left[\sqrt{G\{\Theta(t)\}} dB(t)\right]_i . \text{(B17)}$$

The discrete form of the stochastic differential equation provides a proposal as follows:

$$\widetilde{\Theta}_i = \Theta_i^o + \frac{\epsilon^2}{2}\{G^{-1}(\Theta^o) \nabla_\theta L(\Theta^o)\}_i - \epsilon^2 \sum_{j=1}^{P} \left\{G^{-1}(\Theta^o) \frac{\partial G(\Theta^o)}{\partial \Theta_j} G^{-1}(\Theta^o)\right\}_{ij} +$$
$$\frac{\epsilon^2}{2} \sum_{j=1}^{P} \{G^{-1}(\Theta^o)\}_{ij} \operatorname{tr} \begin{cases} G^{-1}(\Theta^o) \dfrac{\partial G(\Theta^o)}{\partial \Theta_j} \mathring{\imath}ght\} + \left\{\epsilon \sqrt{G^{-1}(\Theta^o)} \xi^o\right\}_i \\ \equiv \mu(\Theta^o, \epsilon)_i + \left\{\epsilon \sqrt{G^{-1}(\Theta^o)} \xi^o\right\}_i, (A.18) \end{cases}$$

where Θ^o is the current draw, and the constant $\epsilon > 0$ is calibrated during the burn-in phase to maintain an acceptance rate close to 20% (which is optimal for a multivariate normal proposal as its dimensionality becomes large). The proposal density is

$$q\left(\widetilde{\Theta} | \Theta^o\right) = \mathcal{N}_P\left(\widetilde{\Theta}, \epsilon^2 G^{-1}(\Theta^o)\right), \quad \text{(B19)}$$

and convergence to the invariant distribution is ensured by using the standard-form Metropolis-Hastings probability:

$$\min\left\{1, \frac{p\left(\tilde{\Theta} | \cdot, Y\right) q\left(\Theta^o | \tilde{\Theta}\right)}{p(\Theta^o | \cdot, Y) q\left(\tilde{\Theta} | ol\Theta^o\right)}\right\}. \quad \text{(B20)}$$

Appendix C

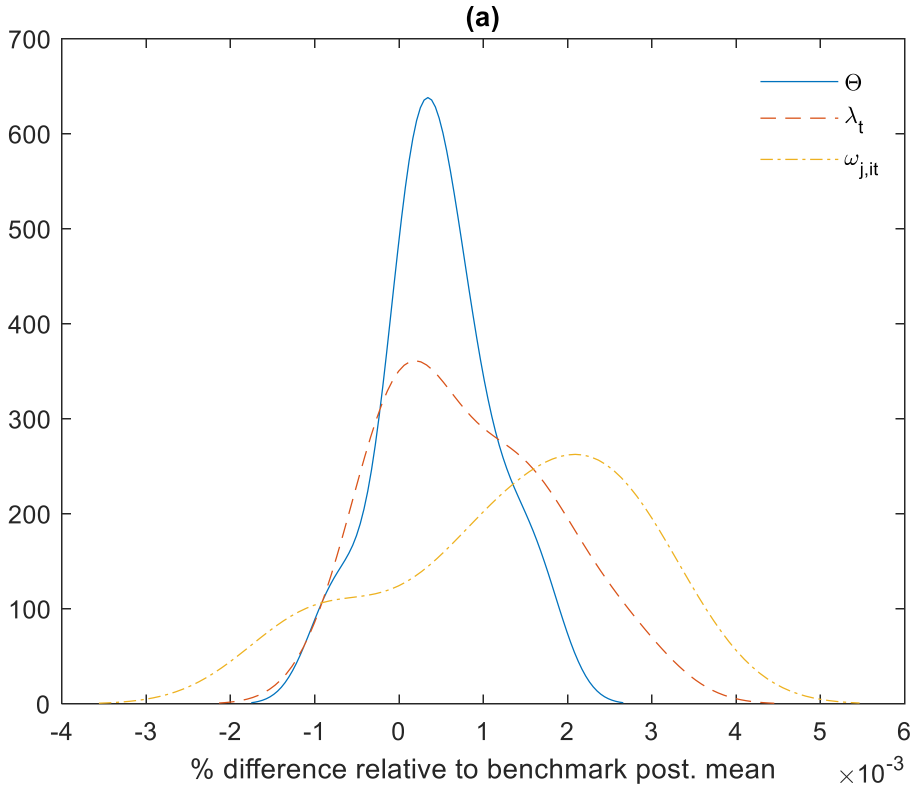
In this Appendix, we examine the behavior of MCMC as well as sensitivity to prior assumptions. We consider 10,000 priors obtained by varying the parameters of the benchmark prior (32) as follows:

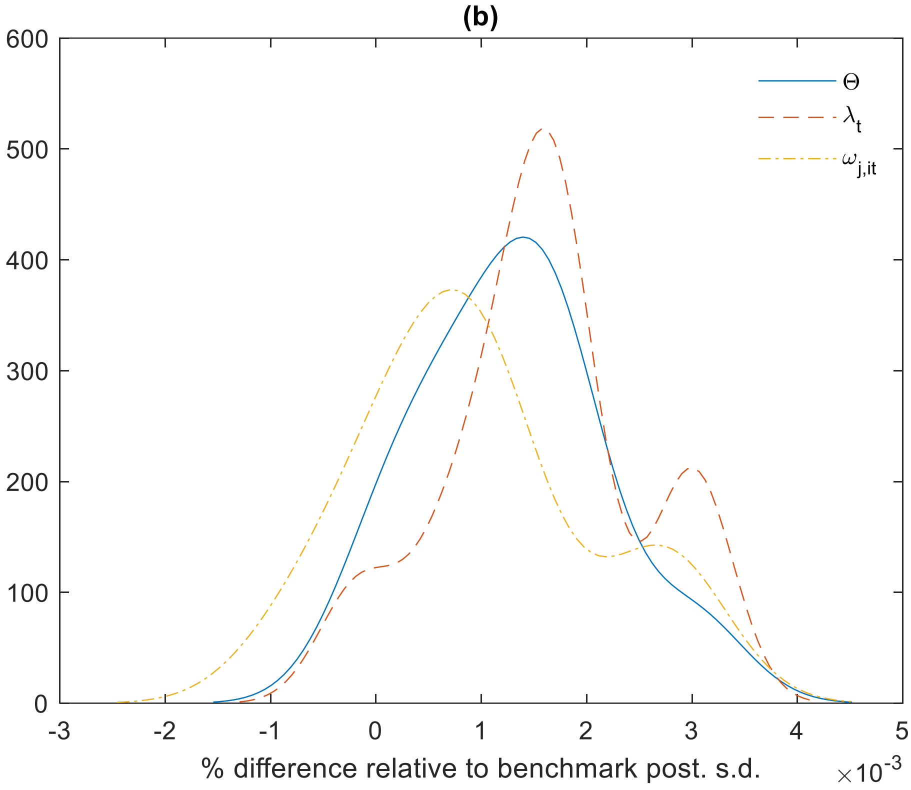
$$\begin{aligned} \bar{\Theta} &\sim \mathcal{N}_P(0, 10^3), \\ \Sigma_\Theta &= h\boldsymbol{A}'\boldsymbol{A}, \\ \log h &\sim \mathcal{N}(0, 10^2), \end{aligned} \tag{C1}$$

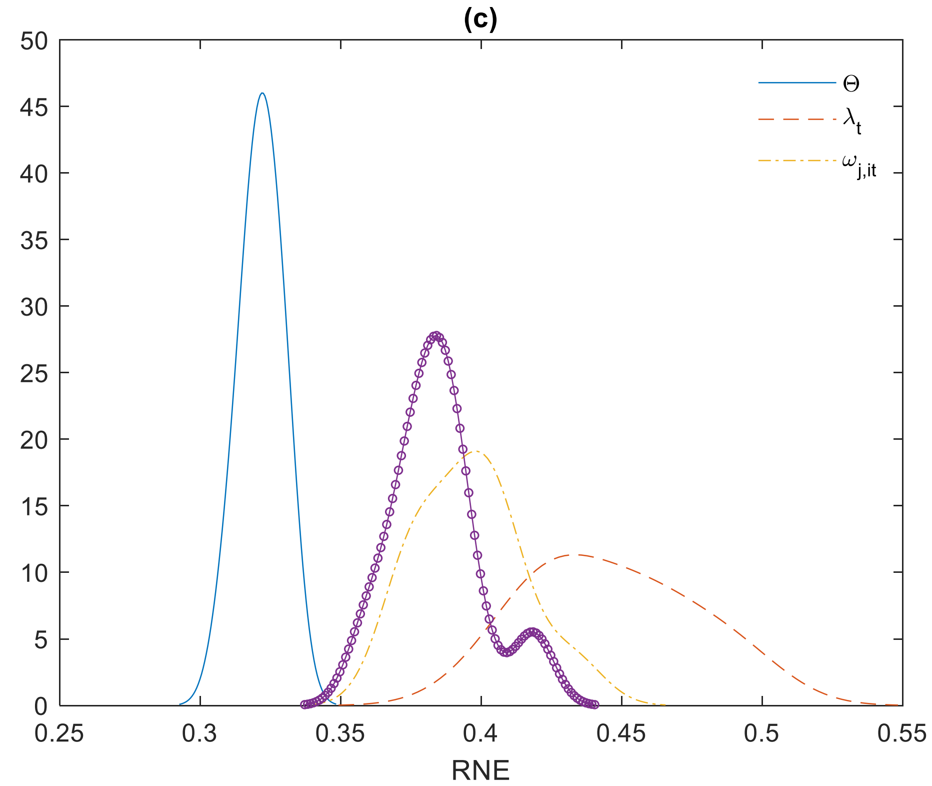
where $\boldsymbol{A}$ is a $P \times P$ lower triangular matrix (so that the product $\boldsymbol{A}'\boldsymbol{A}$ is positive definite). The different elements of $\boldsymbol{A}$ are drawn from independent normal distributions $\mathcal{N}(0, 10^2)$. Finally, although the scale parameter h is redundant is user here to increase prior uncertainty.
The model is re-estimated via SIR and percentage differences relative to posterior means corresponding to the benchmark prior are reported in Figure B1. These differences are reported in the form of kernel densities, separately for the parameters Θ, the latent variables $\boldsymbol{\lambda}_t$, and log relative prices $\boldsymbol{\omega}_{it}$ (recall that the dimensionality of this vector is $(J-1) \times 1$). In panel (a) we report percentage differences of posterior means, and in panel (b) reported are percentage differences of posterior standard deviations. From these results, it turns out that posterior moments of parameters and latent variables are fairly robust to prior assumptions.

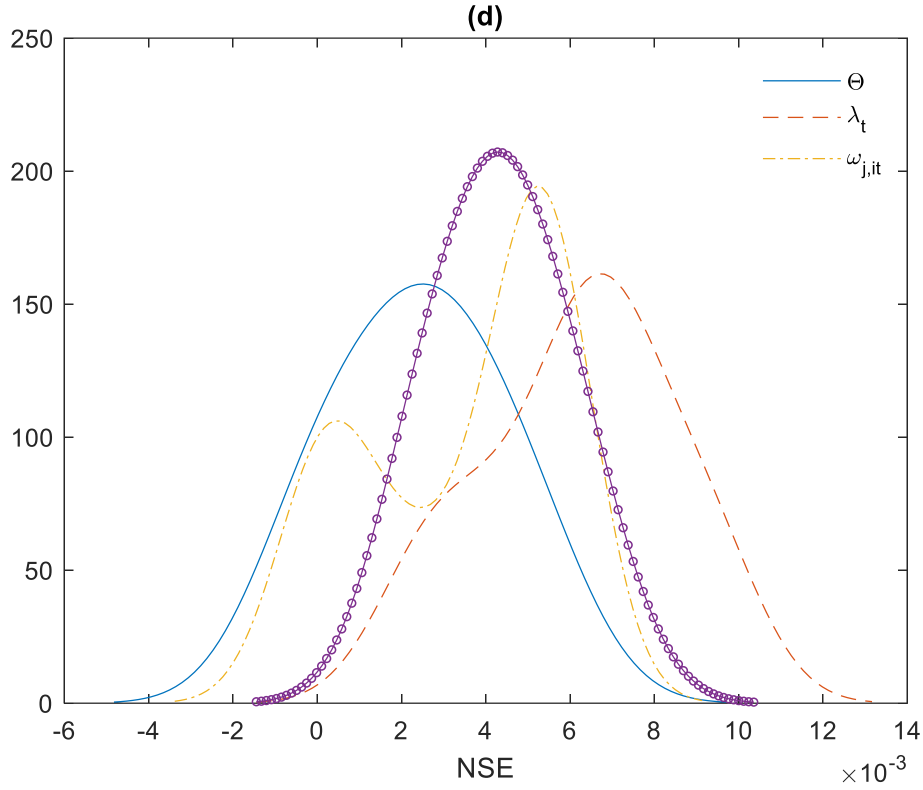
To assess numerical performance of MCMC we focus on relative numerical efficiency (RNE) and MCMC autocorrelation draws for Ω_{it} as results were roughly the same for other latent variables and parameters. RNE is a measure of closeness to i.i.d drawings from the posterior (Geweke, 1992) and, ideally, it should be equal to one, i.i.d sampling has been possible. We report median RNE for all MCMC draws of Ω_{it} $(\forall i = 1, \dots, n, t = 1, \dots, T)$ in panel (c) of Figure B.1. In panel (d) of the same Figure we report numerical standard errors (NSE, see Geweke, 1992). The conclusion is that all reported results are accurate to the decimal places reported.

FIGURE C.1. PRIOR SENSITIVITY ANALYSIS




(b)
600
500
400
300
200
100
0
-3
-2
-1
0
1
2
3
4
5
% difference relative to benchmark post. s.d.
×10-3
Θ
λt
ωj,it


(c)
50
45
40
35
30
25
20
15
10
5
0
0.25
0.3
0.35
0.4
0.45
0.5
0.55
RNE
Θ
λt
ωj,it


(d)
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200
150
100
50
0
-6
-4
-2
0
2
4
6
8
10
12
14
NSE
×10^-3
Θ
λt
ωj,it