Benford's law, small business financial reporting, and survival

Short title: **Benford's law and survival**

Keywords: ethics in reporting; Benford's law; small and medium-sized enterprises; failure; forensic accounting; financial statement manipulation

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Abstract

Benford's law states that in collections of numbers the leading digit is likely to be small. Drawing on this law, which is a widely used tool in forensic accounting, we ask whether small and medium-sized enterprises (SMEs) that violate Benford's law on the reporting of key accounting items are more likely to fail. Using a sample of 220,583 Portuguese SMEs (2010–2018), those companies violating Benford's law in reporting *cash* had the highest odds of failure, followed by *net income*, *current liabilities*, *assets*, and *sales*. The findings indicate that myopic strategic manipulation in financial reporting may lead to SME failure.

Keywords: ethics in reporting; Benford's law; small and medium-sized enterprises; failure; forensic accounting; financial statement manipulation

JEL Codes: L26; M49; G33

1. Introduction

Benford's law was proposed in the 19th century by Newcomb, an astronomer who noticed that, in library books of logarithms, earlier pages were thumbed more than later pages. Newcomb (1881) proved the law of the first and second digit, but Benford (1938) popularized it and showed that it applies to street addresses, number of deaths, length of rivers, several mathematical constants, and various other instances. Real-world distributions that span several *orders of magnitude* uniformly (*e.g.*, populations of cities, stock-market prices, etc.) are also likely to satisfy Benford's law to a very high degree of accuracy. According to the law, the first digit is proportional to the distance of successive numbers reading from left to right in log scale, which is $\log_{10}(d+1) - \log_{10}(d) = \log_{10}\left(1+\frac{1}{d}\right)$. Where P_d is the probability of the first non-zero digit, *D*. For example, numbers 146,321 or 0.0013 both have $D = 1$, and Benford's law states that numbers beginning with 1 will occur 30% of the time in nature, those with 2 as the first digit will occur approximately 17% of the time, and so on.[1] One can extend the analysis to

[1]

Digit	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

digits beyond the first, and every sequence of numbers can be tested.[2] Widely applied in forensic accounting, Benford's law is also used to compare reported numbers against naturally occurring numbers in economics, epidemiology, finance, and sociology (Miller 2015; Nigrini 2012, 2020).

Studies in ethics on forensic accounting in the context of small and medium enterprises (SMEs) are either exploratory or have focused on the motives of financial misreporting (Carpenter and Reimers 2005; Tschopp and Huefner 2015; Zhang et al. 2021; Zhang et al. 2020). However, SMEs bring together a set of factors that can make them particularly prone to mistakes, errors, or even intended manipulations of the financial statements. Weaker internal control systems, lower professionalization, and greater discretion exercised by SME owners may increase the likelihood of violating Benford's law in financial reporting. As such, such a violation may have unintended consequences for the firm, being the most serious one its ultimate failure. Whether ethical violations occur by chance, inadvertent errors, or otherwise, a greater focus on ethical checks in reporting could have long-term financial implications especially for SMEs that also have weaker control systems.

In this research note, our main objective is to assess whether SMEs violating Benford's law in their accounting reports are more likely to fail. Answering this research question is important for several reasons. First, the proposed research question allows us to complement the long-standing body of work in business ethics on financial reporting (Zhang et al. 2020; Patelli and Pedrini 2015; Staubus 2005). Although archival data is widely used to study SME and venture outcomes (for a review refer to Soto-Simeone et al. 2020), such archival data may be

Expected probability (in decimals)	.30103	.17609	.12494	.09691	.07918	.06695	.05799	.05115	.04576

[2] For instance, the probability that the first three digits are 3, 1, 4 (part of π) has probability $\log_{10}\left(1+\frac{1}{314}\right) = 0.00138$.

subject to errors of omission and errors of commission. Errors of omission, or false negatives, in reporting of archival performance data, may stem from less developed accounting and control systems, limited accounting and financial literacy of the owners, or more generally the liabilities of newness that drive the limited availability of materially strong systems that can ensure accurate reporting (Pešalj et al. 2018). Less developed accounting and control systems may also drive errors of commission or false positives in reporting. Facing challenges in sustaining relationships with suppliers, creditors, and customers to develop and sustain business relationships, SMEs may exercise greater discretion in accounting decisions and may not report less favorable accounting metrics. We do not ascribe accounting misconduct to such misreporting, but we consider the possibility that the available archival data, though certified by an external party in many countries, may not be as reliable for a variety of reasons that include but are not limited to the aforementioned reasons.

Second, a more important consideration for SME owners is whether consistently reporting accounting data that violates Benford's law is associated with firm failure. Much entrepreneurship research focuses on the strategic, tactical, or institutional and industry-related aspects of SME performance and survival (Kibler et al. 2017). However, the lack of accurate reporting of archival accounting data driving firm failure is an important contribution to the literature and may support the need to implement stronger reporting systems. Answering the proposed research question contributes to the much broader need to develop control systems in SMEs in particular and forensic accounting literature in general.

We draw on a census of SMEs in Portugal where all SMEs, irrespective of their size or age, are required to make their financial performance data public. The reported data is certified by a chartered accountant, adding potential veracity to the reported data. In Norway and Sweden

where auditing was required for SMEs, however, the laws have recently been either repealed (Norway) or relaxed (Sweden). Though data from Sweden (exemptions based on employee size and sales volume) or Norway (repeal) may seemingly indicate a stronger difference-in-differences design, repeal of the auditing legislation did not result in meaningful change in accounting reporting outcomes in Norway (Langli 2015). In Sweden, though the auditing laws were relaxed for smaller SMEs with sales or employees below a certain threshold, the exempt firms could still have their statements audited. This has resulted in unobserved selection bias in the pool of exempt firms – those who voluntarily audit versus those who chose not to audit (Bulatovic and Treis 2016). Nevertheless, future studies can replicate our findings using register data available in Norway and Sweden from the pre-repeal or pre-exemption periods, respectively.

Our sample includes 220,583 firms observed between 2010 and 2018, representing most of the available firms in Portugal. We compute the Z-statistic based on mean absolute deviation, a test less sensitive to sample size (Nigrini 2020). We compute the Z-statistic for the key line items in the income statement and balance sheet: assets, current assets, inventories, cash, net income, liabilities, current liabilities, and sales reported during all available years between 2010 and 2018. Thus, the range of observations is from a minimum of three years to a maximum of nine years. Controlling for employees, industry dummies, and region dummies along with the year of birth of the firm, we find that SMEs violating Benford's law in accounting reports on the aforementioned items were more likely to fail. The effect sizes are meaningful. Those violating Benford's law in reporting *cash* had the highest hazard of failure, followed by *net income*, *current liabilities*, *assets*, and *sales*. The higher effect sizes for liquidity (cash) followed by solvency (net income) suggest strategic manipulation driven by greater pressure to maintain

reliable signals with stakeholders, including creditors, who may be sensitive to liquidity and solvency issues (cf. Hoffman and Zimbelman 2009; Deleanu 2017). An alternate explanation is that firms may not fully record cash transactions and, to lower their tax bill, may report net income that is not consistent with the natural phenomenon explained by Benford's law.

In the following sections, we briefly discuss the theoretical background related to Benford's law, followed by the description of our sample. We then present our results and discuss the implications of our findings.

2. Theoretical Background

Accounting in SMEs is seldom explored in entrepreneurship research, with a few exceptions (e.g., Davila and Oyon 2009; Collier 2005). Yet, the broader theoretical narratives on liabilities of newness and smallness (Singh and Lumsden 1990), along with limited human capital and imperfect understanding of financial matters and literacy, resource constraints, and the generally less developed formal systems (Pennings et al. 1998), indirectly call into question the extent to which SMEs have reliable control and monitoring systems to ensure accurate reporting. Liabilities newness and smallness may limit the development of accounting and control systems. The need for accurate reporting goes beyond the legal requirements. Accurate reporting improves fiscal discipline, enhances financial health, and facilitates improved decision making (Petts 2015).

Though much of the forensic accounting literature has extensively used Benford's law for fraud detection, deviation from Benford's law may not necessarily indicate deliberate misrepresentation. Limited ability to collate and aggregate data, exercising proper discretion in making financial decisions, allowing adequate monitoring from creditors, limited coherence and

compatibility of internal systems, and the general informality in management are some reasons that Benford's law may be violated and may have long-term consequences for firm survival (cf. Beresford and Saunders 2005). Compared to the general application of Benford's law, our premise is not to construe the distortions as fraud. After all, as rational economic agents, consistent misreporting is neither possible nor sustainable in the long term (Nigrini 2012, 2020). Irrespective of the drivers of such distortions, the negative effects may accumulate over time, leading to negative consequences. Given that the survival of SMEs is generally at stake (Ropega 2011), accurate reporting may not only reinforce legitimacy with stakeholders but, more importantly, may also influence decision making in the precarious circumstances in which SMEs struggle to survive.

Irrespective of whether a violation of Benford's law by ventures or SMEs is driven by the characteristics of young and small firms or by deliberate misreporting, it nonetheless serves as an important tool for both researchers and practitioners. Table 1 outlines areas of entrepreneurship research and practice that could benefit from the application of Benford's law. For example, entrepreneurs could use this law as a tool to verify their financial reporting and spot inconsistencies that may signal potential errors. In other words, it has the potential to provide crude but informative inferences on the quality of internal controls. Other stakeholders such as banks or tax authorities can use Benford's law to assess reported numbers in funding applications.

----------Insert Table 1 about here----------

Compared to its use in forensic accounting to detect fraud, we propose using it as a potential litmus test of the veracity of performance reporting and certainly not as a definitive indication of malfeasance. We take this conservative stance because small and young firms have

weaker internal control systems, experience lower professionalization, and exercise greater accounting discretion—thereby increasing the likelihood, albeit perhaps inadvertently, of violating Benford's law in financial reporting. Lacking auditing requirements and poor record-keeping could also drive unintentional violations of Benford's law. Similarly, due to liabilities of newness and smallness in ventures, monitoring and control systems tend to be less mature and there may be inadequate investment in such systems, or even a lack of motivation to so invest. Finally, due to volatility and growth in key accounting metrics, the distribution of reported numbers is also more likely to deviate from the 'natural' order of numbers. Nevertheless, the widespread applicability of Benford's law suggests a potential use in empirical research into entrepreneurship while considering the aforementioned caveats.

It is not feasible to assess such distortions using traditional empirical tools because self-reports on distortions in reporting are either subject to bias or owners may simply be unaware of the idiosyncrasies that typically prevail in the control and monitoring systems of small private firms. Particularism, parsimony, and personalism are some of the governance characteristics (Carney 2005) that may inhibit the development of control and monitoring systems, increase opacity, and limit the application of the traditional empirical methods in assessing the drivers of such distortions. Parsimony refers to prudence in the use of the firm's resources. With much undiversified capital invested in the firm (Carney 2005) and overconfidence in their ability, SME owners rely on heuristics and firm-specific human capital and, therefore, may not fully invest in accounting and control systems. Given the tendency to exercise greater discretion, they may be inclined to make more exceptions in their reporting and accounting decisions. As claimants of residual returns, owners aim to lower costs and invest sparingly in less value-creating activities such as accounting systems. Personalism or concentration of ownership and control in the hands

of owners result in fewer internal constraints and greater reliance on informal governance mechanisms, resulting in lower professionalization (Carney 2005). The typically concentrated, personalized, and authoritative roles impose a rational-legal authority (d'Amboise and Muldowney 1988) that may reduce the perceived value of accounting systems. Finally, particularism or the perception of "our business" (Demsetz & Lehn, 1985) reduces the need for a rational-calculative approach to developing a business, and a greater focus on noneconomic goals (Hoque and Hopper 1994; Carney 2005) – goals that may not be amenable to rational-calculative accounting systems. These factors may affect the quality of information – in particular, the strategic and tactical decision making based on accounting performance information – and, over time, may significantly increase the odds of failure. Benford's law, which relies on observed numbers, is salient here – especially, given the nature of internal control systems and the general governance of SMEs.

The problem of distortion is further magnified when SMEs rely on external parties to produce financial statements because they possess neither the knowledge nor the resources to develop a formal system in-house. Often, SMEs deliver the "documents" (invoices, receipts, etc.) to the external accounting firm who, for the most part, does not go beyond the minimum procedures required by law to produce the balance sheet and other compulsory accounting statements. These accounting firms work for low fees and are limited in the time, personnel, and often the capacity to offer services such as fiscal or credit advice. Firms and accountants may also pursue aggressive strategic manipulation. For example, in Portugal, SMEs usually follow the Regulatory Decree nº25/2009 that establishes the depreciation and amortization scheme for assets. For simplicity, firms use the depreciation and amortization rates provided (in the regulatory decree) and do not explore the discretionary powers that managers may have to adapt

the useful life of the assets to the economic use that the firm has for such assets. An anecdotal example is the depreciation tax for vehicles of 25% a year. However, if the firm uses a vehicle for more than four years, the asset will be worth zero despite being still in use. This results in a mismatch between the accounting value and the economic value of such an asset. Furthermore, it influences the value of the assets and, ultimately, the net income of the firm.

Against this backdrop, we ask: *Are small and medium-sized enterprises who violate Benford's law in accounting reports over time more likely to fail?* Next, we provide a brief introduction to Benford's law.

3. Data and Methodology

3.1 Benford's Law

According to Benford's law (BL), the first digit of naturally occurring numbers is more likely to be equal to 1, and the probability of the first digit being identical to the subsequent numbers decreases progressively. With *d* representing the digits between 1 and 9, the probability is given as:

$$P(d) = \log_{10}(d+1) - \log_{10} d = \log_{10}\left(1 + \frac{1}{d}\right), \text{ for } (d = 1,\ldots,9)$$

Using this formula, the expected frequencies for digits in first position are illustrated below:

Digit	1	2	3	4	5	6	7	8	9
Expected Frequency	.30103	.17609	.12494	.09691	.07918	.06695	.05799	.05115	.04576

The simplest of Benford's tests is the Chi-square test that measures the difference between the observed and expected first digit of the Benford distribution:

$$\chi^2 = \sum \frac{(Oi - Ei)^2}{Ei}$$

Where Oi is the observed and Ei is the expected absolute frequencies for digit i. The Chi-square test is, however, sensitive to sample size.

The *Kuiper* test, or the modified Kolmogorov–Smirnov test) (Kuiper 1960) is less sensitive to sample size:

$$V = (D_n^+ + D_n^-)(\sqrt{n} + 0.155 + 0.24\sqrt{n})$$

where $D_n^+ = \sup [\mathrm{F(O_i)} - \mathrm{F(E_i)}]$ and $D_n^- = \sup [\mathrm{F(E_i)} - \mathrm{F(O_i)}]$, and $F(.)$ is the cumulative relative frequencies. The discrepancy between D^+ and D^- are the absolute sizes of the differences between the absolute and the observed distribution.

We use the modified *Mean Absolute Deviation (MAD)* test, which has fewer constraints and overcomes the limitation of the original MAD test, which is likely to give false positives. This test is based on Johnson and Weggenmann (2013) and addresses the false positive problem. Hence, an adjusted MAD can be used for the country's data set. The MAD statistic is calculated as follows:

$$\textit{Mean Absolute Deviation} = \frac{1}{N}\sum_1^N fi \left|Xi - \overline{X}\right|$$

Where Xi is the differences between the reported occurrence rate and the Benford rate, and $\overline{X}$ represents the mean of the difference between the actual occurrence rate and the Benford occurrence rate. We use the Z-statistic measure for the 1st, 2nd, and 3rd digits, which compared to another statistic (e.g., Chi-2, Kuiper test) is considered (Nigrini 2012) the correct test of whether the proportion of a digit differs significantly from the Benford distribution. The Z-statistic assesses whether the actual proportion of a specific digit deviates from the expected proportion. The traditional Chi-square and Kolmogorov-Smirnoff (and Kuiper test derived from Kolmogorov-

Smirnoff) tests account for the number of records in the conformity calculation. However, the mean absolute deviation test (Z-statistic) ignores sample size, thereby removing the constraints on requiring a large number of observations per venture.

3.2. Sample

Reliable financial data on small and medium-sized firms are rarely available as most governments do not require privately held firms to disclose their data. In several countries, though, these data are available through public registers. More recently, the auditing requirements for smaller firms have been lifted for firms below a certain size and employee count thresholds (e.g., Norway and Sweden). Among the countries where reliable financial data is available, Portugal represents a unique case where all firms irrespective of their size and age are required to publicly report their financial information, and the statements must be certified by a chartered accountant (*Informação Empresarial Simplificada* form). Our sample is a census of all firms in Portugal with financial information from 2010 to 2018. The sample for analysis includes 220,583 firms. Of these 220,583 firms, 24,029 firms failed (10.89%) during the period of observation.

3.3. Measures

Table 2 lists the measures used in our study. Based on the broader accounting literature, we use all the available accounting line-item variables in the data. We use the reported assets, current assets, inventories, cash, net income, liabilities, current liabilities, and sales. Consistent with (Nigrini 2012), we use Z-statistic for the 1^{st}, 2^{nd}, and 3^{rd} digits for each of the reported line items. To compute the deviation from Benford's law per venture, we have a maximum of nine years of

data and, therefore, the traditional Chi-square and Kuiper tests are not feasible at the level of each SME given a maximum of 9 observations (2010 to 2018).

----------Insert Tables 2-3 about here----------

In Appendix B, we provide Z-statistic distributions for the 1st, 2nd, and 3rd digits as well as the first two and last two digits for the variables. We find consistently that Benford's distribution is violated. As an additional test, we include Zipf's law (a law similar but not identical to Benford's law) which relates to a power-law probability distribution. In Appendix C, we again see that Zipf's law is violated for most reporting on the key variables.

Due to strict labor laws in Portugal, reporting on the employee count is less likely to be misreported. Therefore, we control for the log of employees. We further control for the year of firm establishment dummies to control for firm age. Finally, we control for industry and region dummies.

Tables A1 and A2 represent the distribution of firms by region and industry. The analysis of the variables divided by the 29 administrative regions that include the islands of the Azores and Madeira is presented in Table A1. Overall, Benford's law is violated most frequently for *net income* and least often for *current assets*. The small islands of the Azores archipelago and the Madeira archipelago present greater variation in the violation values, with the highest and smallest values registered. For example, when looking at the 1-digit analysis, Corvo island has the highest violations for *assets* and *current liabilities* and the smallest for *cash*, *net income*, and *sales*. The pattern is quite consistent when looking at 2 and 3 digits. In terms of the biggest cities, Lisbon (the capital) and Porto have values that are very close to the mean, irrespective of the digit considered.

The analysis of the variables divided by 2-digit industry codes is shown in Table A2. Overall, Benford's law is most often violated for *inventories* and less violated for *cash*. Some industries present interesting patterns. For example, the 1-digit analysis shows that the manufacture of tobacco products (industry code "CAE"=10) has the highest violation for *net income* and *sales* but the lowest violation for *assets*. The industry remediation activities and other waste management services (industry code "CAE"=36) has the highest values for *liabilities*, both *total* and *current*, but the lowest values for *net income*. Sewerage (industry code "CAE"=34) presents the highest values for *total assets*, *current assets*, and *cash* (here also for postal and courier activities, industry code "CAE"=47). Mining support service activities (industry code "CAE"=7) has the lowest violation values for *cash* and *liabilities*. Finally, real estate activities (industry code "CAE"=59) and scientific research and development (industry code "CAE"=63) have the lowest violation values for *sales* and *net income*, respectively.

3.4 Results

Given the censored nature of the outcome of failure, we use survival regressions, which is consistent with several studies in entrepreneurship. In Table 3, we present the results of survival regression based on Cox, exponential, Gompertz, and Weibull regressions. The hazard ratios are economically significant. Among the effects, SMEs deviating from Benford's law on *cash* reporting were among those with the highest odds of failure, followed by *net income*, *current liabilities*, and *assets*. For example, on average, firms reporting *cash* that deviated from Benford's law had about 1.3 to 1.26 times the higher hazard of failure. Interestingly, deviation from Benford's law in *sales* was also associated with higher odds of failure.

3.4.1. Controlling for size and volatility in performance

A concern with the current specification is that Benford's law may be an artifact of SMEs facing higher volatility in sales, assets, and profitability. Also, the SMEs who have overcome liabilities of smallness may be less subject to Benford's law. In Table 4, after adding the mean and standard deviation of sales, assets, and profitability to the specification in Table 3, our inferences are consistent with the main inferences.

The higher effect sizes for liquidity (cash) followed by solvency (net income) suggest strategic manipulation driven by greater pressure to maintain reliable signals for stakeholders—including creditors—who may be sensitive to liquidity and solvency issues (cf. Hoffman and Zimbelman 2009; Deleanu 2017). An alternative explanation is that firms may not fully record cash transactions and, to lower their tax bill, may report net income that is not consistent with the natural phenomenon explained by Benford's law. This is speculation on our part but, after controlling for mean and standard deviation of sales, assets, and profitability, the test statistic from Benford's law still explains firm failure with meaningful effect sizes.

4. Conclusion

Over the past two decades, research in business ethics has focused on the role of ethics in financial reporting (Zhang et al. 2020; Patelli and Pedrini 2015; Staubus 2005). Studies have focused on the role of enablers of accurate financial reporting, including CSR (Tschopp and Huefner 2015; Wang et al. 2018), the role of institutional forces (Chauvey et al. 2015), ethical values of a business (Choi and Pae 2011), organizational diversity (Labelle et al. 2010), tone of the top management team (Patelli and Pedrini 2015), among others. This study aimed to complement the multi contextual focus of business ethics research on financial reporting quality.

Consistent with the literature on the benefits of ethics, our findings make a financial case for ensuring ethical financial reporting. Though SMEs may not deliberately misreport and weaker control systems can be a primary culprit, the core message of our results is that focus on ethical frames to ensure accuracy in reporting is essential to the long-term economic health of an SME. Though privately-held SMEs are seldom subject to significant public scrutiny that public firms face, our results make a case for the importance of emphasizing ethics in financial reporting to SME owners on the financial value of such ethical values and systems.

Relatedly, much of entrepreneurship literature has focused on the strategic, institutional, or ecological factors explaining SME failure (Soto-Simeone et al. 2020). Our approach complements prior approaches by assessing whether distortions in financial reporting may spell firm failure. The purpose of this methodological research note was to apply a widely used tool in forensic accounting, Benford's law, to the context of SME survival. The inferences are two-fold. First, Portuguese SMEs, despite certification from chartered accountants, are likely to report performance that deviates from Benford's law. Second, such deviation has strong effects on firm failure. The hazard of failure is higher for deviations from Benford's law in the reporting of *cash* followed by *net income*, *current liabilities*, *inventories*, *assets*, and *current assets*. Our findings show that these distortions have practical and meaningful effects on failure.

We cannot infer misreporting on the part of SMEs. However, we posited that SMEs, due to their unique circumstances, may have systematic distortions in reporting, and that such deviations from Benford's law could drive firm failure in the long term. Accordingly, we ascribe our findings to the possibility that limited internal resources and routines along with the idiosyncrasies of private ownership governance are the driving forces behind failure. Greater deviation in reporting of *cash*, *net income*, *current liabilities*, and *assets* seems to indicate that

signaling is considered necessary to ensure confidence in a firm's operations. However, the results show that this potentially myopic and short-termist 'signaling' through strategic manipulation may be achieved at the expense of long-term firm failure.

Other institutional explanations may also be possible. Primarily, deviation from Benford's law in cash reporting may be due to potential incentives for tax or value-added tax (VAT) evasion (Braml and Felbermayr 2019; Marques et al. 2020). In several forensic accounting studies, there is indirect evidence of such evasion where businesses under-report cash holdings (Tavares and Iglesias 2010; Pappa et al. 2015; Casais et al. 2019). Furthermore, the violation of Benford's law in *net income* reporting perhaps implies tax evasion – documented to some degree in the Portuguese context (Tavares and Iglesias 2010; Poço et al. 2015). Relatively higher effect sizes may well indicate the possibility of strategic manipulation that may not pay off for small business owners. Though, in extreme cases, fraud may be possible, such consistent misreporting may not be sustainable.

The findings in this research suggest the possibility that the unique conditions of an SME may be the most critical driver of the observed phenomenon. The rationale for this interpretation is that SME owners who have concentrated ownership would not engage in misreporting on a sustained basis. Owners who have made significant personal and financial commitments to their companies would likely consider the long-term implications of such actions. Thus, it might behoove SME owners to consider Benford's law as a potential benchmark to compare their reporting numbers against. Lacking a systematic auditing and monitoring mechanism, forensic accounting tools may have an indirect benefit for SME owners.

Our findings are subject to the following limitations. First, it is difficult to prove manipulation or deliberate distortion, but the data at least demonstrates strategic manipulation of

the natural order of numbers. Benford's law is the most widely used tool in forensic accounting and, despite this, consistent with the challenges of forensic accounting, the reported numbers are the only indicators of possible distortion in reporting. Whether such distortions occur by accident or design is difficult to ascertain. Second, we drew on a reliable archival dataset and did not use any filters to avoid further bias from the selection effect. However, as discussed earlier, focusing on countries relaxing or repealing their auditing laws may provide additional evidence to confirm or invalidate the findings. Third, though qualitative interviews or surveys may not lead to the identification of deliberate misreporting, qualitative audits of accounting systems and decisions could be useful in understanding how some of the reporting distortions develop over time.

Next, we provide several caveats that we believe are important. For additional information, cautions, and applications, we refer interested readers to recent reviews (Barney and Schulzke 2016; Nigrini 2017; Morrow 2014). Morrow (2014) cautions against interpretations based on small samples. However, he also infers that differences in outcomes between small and large samples from the Kolmogorov-Smirnov test, the Kuiper test, the Chebyshev distance test, and the Euclidean Distance test are small. Though much research uses cross-sectional data, examples of panel data are limited (Druică et al. 2018; Schräpler 2011), and several studies sound a word of caution against the method (Diekmann and Jann 2010). False positives are also a key cause of concern (Barney and Schulzke 2016) as they may impose reputational and strategic costs on small firms. Barney and Schulzke (2016) provide a critical assessment of a variety of empirical tests of Benford's law, and Nigrini (2017) provides a review of the challenges and pitfalls, noting that inferences based on null hypothesis significance testing (NHST) must be taken with a grain of salt. Before applying the methodology, researchers must

carefully consider these challenges. Despite these cautions, Benford's law is validated in a variety of fields and can be considered an important tool for entrepreneurship researchers.

We must say again that not all violations are necessarily deliberate. Rather, the distortions in reporting could be driven by the internal systems and idiosyncrasies of the owners. Particularism, parsimony, and personalism are some of the governance characteristics (Carney 2005) that may inhibit the development of control and monitoring systems, increase opacity, and limit the application of the traditional empirical methods in assessing the drivers of such distortions. With much undiversified capital invested in their firms (Carney 2005) and overconfidence in their ability, SME owners rely on heuristics and firm-specific human capital. Therefore, they may not fully invest in accounting and control systems. The typically concentrated, personalized, and authoritative roles impose a rational–legal authority (d'Amboise and Muldowney 1988) that may reduce the perceived value of accounting systems. The particularism of the perception of "our business" (Demsetz & Lehn, 1985) reduces the need for a rational–calculative approach to developing a business, and a greater focus on noneconomic goals (Hoque and Hopper 1994; Carney 2005). The problem of distortion is further magnified when SMEs rely on external parties to produce financial statements because they possess neither the knowledge nor the resources to develop a formal system in-house. Often, SMEs deliver the "documents" (invoices, receipts, etc.) to the external accounting firm who, for the most part, does not go beyond the minimum procedures required by law to produce the balance sheet and other compulsory accounting statements. These accounting firms work for low fees and are limited in the time, personnel, and often the capacity to offer services such as fiscal or credit advice.[3]

[3] For example, in the empirical context of our third illustration on Portuguese ventures, firms and accountants may also pursue aggressive strategic manipulation. For instance, in Portugal, SMEs usually follow the Regulatory Decree

Finally, we would add the caveat that researchers should consider the volatility and size of small and young firms. In the Portuguese sample, we controlled for size and volatility and found consistent inferences, but this may not be the case in other instances. Therefore, we call on future researchers to consider this distinct context of ventures and SMEs in making inferences on the basis of Benford's law.

In conclusion, we asked if small and medium-sized enterprises who violate Benford's law on accounting reports over time are more likely to fail? We found that SMEs deviating from Benford's law on a variety of accounting line items are more likely to fail. The findings suggest that financial reporting, even when certified by an external party, may be subject to distortions. Financial reporting by SMEs has received scarce attention and, although SME owners may engage in strategic manipulation, such distortions could impact strategic and tactical decisions, weaken long-term relationships with stakeholders, and potentially lead on to failure. The current findings make a significant contribution to the literature in being among the first research efforts to highlight the role of forensic tools in helping SMEs address the evident shortcomings in their reporting. Benford's law can be an important forensic tool to assess the quality of financial reporting for a wide range of stakeholders in entrepreneurship and to help SMEs address the evident shortcomings in their financial reporting.

nº25/2009 that establishes the depreciation and amortization scheme for assets. For simplicity, firms use the depreciation and amortization rates provided (in the regulatory decree) and do not explore the discretionary powers that managers may have to adapt the useful life of the assets to the economic use that the firm has for such assets. An anecdotal example is the depreciation tax for vehicles of 25% a year. However, if the firm uses a vehicle for more than four years, the asset will be worth zero despite being still in use. This results in a mismatch between the accounting value and the economic value of such an asset. Furthermore, it influences the value of the assets and, ultimately, the net income of the firm.

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Table 1. Potential applications of Benford's law in select areas of entrepreneurship research

Research Area	**Possible Application**	**Theoretical applications**	**Practical applications**
Venture or SME survival	A venture or an SME with poorer control and reporting system may violate Benford's law over time, and these consistent violations could lead to firm failure driven by poorer internal control systems.	Complementing ongoing research on venture survival, Benford's law could be a non-strategic reason for firm failure.	Entrepreneurs and business owners can use this law as a tool to assess the extent of veracity in their financial reporting.
Venture investments	Reported data and projections provided by entrepreneurs could be assessed against Benford's law to assess the veracity of reports. Though financial projections are also increasingly questioned for 'hockey stick' patterns, entrepreneurs considering realistic numbers are less likely to violate Benford's law in their projections.	The extent to which violations in Benford's law can explain additional rounds of funding, IPO underpricing, and funding after due diligence.	Entrepreneurs can use Benford's law to assess how 'realistic' their reported numbers and projections are.
Government funding	Funding of early-stage loans and grants.	Research areas on R&D funding or loan defaults can use Benford's law as a mechanism to test hypotheses on R&D and loan default outcomes.	A range of government funding bodies providing third-party guarantee loans to R&D subsidies can assess whether reported financial numbers in applications violate Benford's law.
Alliances and acquisitions	In forming alliances, non-violation of Benford's law can signal veracity as an alliance partner. Similarly, due to less established business models, the target firm can improve signaling when not violating Benford's law.	Testing alliance and acquisition likelihood and outcomes based on Benford's law.	Ventures can use Benford's law to create signals of internal control.
Auditing in ventures	Though auditing is not required of private firms in most countries, in several countries (e.g., UK, Sweden) auditing rules have recently been relaxed due to the burden on ventures.	Whether firms not in violation of financial reporting in their statements are less likely to fail.	Governments can use Benford's law as a tool to assess reporting from non-audited firms.
The estimate of the goodness of fit with Benford distribution as a control in regressions	The computed Benford's law probabilities for key reporting metrics could be used as a control to lower confounds from reporting artifacts that are deliberate, due to limitations of internal controls, or errors.	Controls based on Benford's law probability could lower the effects of reporting artifacts.	

Table 2. Descriptives

variable	N	mean	sd	min	max
Outcome variable					
Firm failure	220,583	0.1089			
1st digit					
Assets (1st digit)	220,583	0.2440	0.5862	-0.7085	8.9765
Current assets (1st digit)	220,583	0.2439	0.5591	-0.7085	8.8960
Inventories (1st digit)	220,583	0.4971	1.1454	-0.7085	8.9990
Cash (1st digit)	220,583	0.2724	0.5478	-0.6943	8.9555
Net income (1st digit)	220,583	0.7281	1.4073	-0.6799	8.9992
Liabilities (1st digit)	220,583	0.2523	0.5865	-0.7085	8.9480
Current liabilities (1st digit)	220,583	0.2609	0.5688	-0.7085	8.9541
Sales (1st digit)	220,583	0.3085	0.7250	-0.7020	8.9952
2nd digit					
Assets (2nd digit)	220,583	0.2518	0.5842	-0.7085	8.9984
Current assets (2nd digit)	220,583	0.2499	0.5567	-0.7085	8.8805
Inventories (2nd digit)	220,583	0.4491	1.0751	-0.7085	9.0000
Cash (2nd digit)	220,583	0.2715	0.5414	-0.6940	8.9980
Net income (2nd digit)	220,583	0.7206	1.3970	-0.6799	8.9986
Liabilities (2nd digit)	220,583	0.2553	0.5773	-0.7085	8.9538
Current liabilities (2nd digit)	220,583	0.2612	0.5591	-0.7085	8.9978
Sales (2nd digit)	220,583	0.2836	0.6444	-0.6896	8.9995
3rd digit					
Assets (3rd digit)	220,583	0.2520	0.5852	-0.7085	8.9802
Current assets (3rd digit)	220,583	0.2495	0.5565	-0.7085	8.8971
Inventories (3rd digit)	220,583	0.4495	1.0776	-0.7085	8.9999
Cash (3rd digit)	220,583	0.2716	0.5429	-0.6964	8.9988
Net income (3rd digit)	220,583	0.7201	1.3959	-0.6798	8.9997
Liabilities (3rd digit)	220,583	0.2552	0.5772	-0.7085	8.9529
Current liabilities (3rd digit)	220,583	0.2611	0.5590	-0.7085	8.9611
Sales (3rd digit)	220,583	0.2835	0.6421	-0.6896	8.9998
Controls					
Year of birth	220,583	2001	14	1900	2017
Employees	220,583	9.5978	100.4488	0	23416.6700

Industry and region dummies included but not reported

Table 3. Survival regression estimates

	Cox regression			Exponential			Gompertz			Weibull		
	Digit 1	Digit 2	Digit 3	Digit 1			Digit 2			Digit 3		
VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Assets	1.200***	1.236***	1.259***	1.162***	1.201***	1.184***	1.168***	1.227***	1.219***	1.177***	1.243***	1.264***
	(0.0151)	(0.0144)	(0.0140)	(0.0216)	(0.0308)	(0.0287)	(0.0233)	(0.0347)	(0.0295)	(0.0244)	(0.0369)	(0.0336)
Current assets	1.130***	1.194***	1.174***	1.097***	1.120***	1.133***	1.133***	1.175***	1.206***	1.120***	1.165***	1.171***
	(0.0145)	(0.0143)	(0.0134)	(0.0188)	(0.0260)	(0.0268)	(0.0210)	(0.0310)	(0.0281)	(0.0212)	(0.0309)	(0.0309)
Inventories	1.167***	1.182***	1.175***	1.140***	1.169***	1.166***	1.148***	1.181***	1.179***	1.146***	1.175***	1.176***
	(0.00501)	(0.00499)	(0.00503)	(0.00576)	(0.00730)	(0.00733)	(0.00709)	(0.00991)	(0.00988)	(0.00678)	(0.0104)	(0.00949)
Cash	1.316***	1.342***	1.370***	1.235***	1.312***	1.301***	1.249***	1.337***	1.329***	1.262***	1.360***	1.356***
	(0.00879)	(0.00885)	(0.00876)	(0.0206)	(0.0353)	(0.0344)	(0.0158)	(0.0232)	(0.0242)	(0.0139)	(0.0214)	(0.0216)
Net income	1.276***	1.272***	1.275***	1.225***	1.274***	1.266***	1.222***	1.272***	1.263***	1.223***	1.274***	1.263***
	(0.00380)	(0.00379)	(0.00377)	(0.00388)	(0.00480)	(0.00478)	(0.00415)	(0.00561)	(0.00542)	(0.00411)	(0.00577)	(0.00515)
Liabilities	1.066***	1.161***	1.134***	1.068***	1.075***	1.071***	1.118***	1.164***	1.141***	1.116***	1.146***	1.140***
	(0.0117)	(0.0109)	(0.0117)	(0.0173)	(0.0210)	(0.0220)	(0.0280)	(0.0378)	(0.0365)	(0.0202)	(0.0285)	(0.0284)
Current liabilities	1.278***	1.262***	1.294***	1.201***	1.276***	1.268***	1.190***	1.263***	1.278***	1.204***	1.292***	1.287***
	(0.0130)	(0.0109)	(0.0122)	(0.0157)	(0.0219)	(0.0232)	(0.0280)	(0.0388)	(0.0362)	(0.0157)	(0.0207)	(0.0252)
Sales	1.065***	1.230***	1.254***	1.103***	1.069**	1.082***	1.183***	1.230***	1.241***	1.206***	1.252***	1.265***
	(0.00719)	(0.00877)	(0.00907)	(0.0213)	(0.0316)	(0.0321)	(0.0198)	(0.0244)	(0.0229)	(0.0136)	(0.0181)	(0.0177)
Log of employees	0.463***	0.466***	0.468***	0.478***	0.459***	0.462***	0.473***	0.461***	0.463***	0.475***	0.463***	0.466***
	(0.00515)	(0.00515)	(0.00517)	(0.00627)	(0.00752)	(0.00748)	(0.00525)	(0.00585)	(0.00607)	(0.00516)	(0.00569)	(0.00584)
Year of birth	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included
Industry (2-digit) dummies	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included
Region dummies	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included
Constant				9.22e-05***	0***	5.13e-11***	0.000101***	0***	0***	8.84e-05***	0***	0***
				(4.27e-05)	(0)	(0)	(4.79e-05)	(0)	(0)	(4.28e-05)	(0)	(0)

Notes.
N=220,583
Hazard ratios reported
*** p<0.01, ** p<0.05, * p<0.1

Table 4. Survival regression estimates (size and volatility)

	Cox regression			Exponential			Gompertz			Weibull		
	Digit 1	Digit 2	Digit 3	Digit 1	Digit 2	Digit 3	Digit 1	Digit 2	Digit 3	Digit 1	Digit 2	Digit 3
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10	(11)	(12)
Digit # Assets Z-statistic	1.179***	1.305***	1.303***	1.127***	1.156***	1.162***	1.177***	1.293***	1.290***	1.180***	1.278***	1.280***
	(0.0213)	(0.0239)	(0.0239)	(0.0234)	(0.0221)	(0.0207)	(0.0394)	(0.0368)	(0.0307)	(0.0390)	(0.0354)	(0.0306)
Digit # Current assets Z-statistic	1.108***	1.222***	1.226***	1.074***	1.113***	1.103***	1.101***	1.217***	1.218***	1.089**	1.198***	1.187***
	(0.0199)	(0.0225)	(0.0226)	(0.0226)	(0.0217)	(0.0200)	(0.0378)	(0.0358)	(0.0291)	(0.0371)	(0.0347)	(0.0295)
Digit # Inventories Z-statistic	1.006	1.030***	1.026***	1.003	1.019***	1.017***	1.008	1.028***	1.026**	1.008	1.034***	1.032***
	(0.00537)	(0.00549)	(0.00546)	(0.00572)	(0.00605)	(0.00613)	(0.00843)	(0.0107)	(0.0109)	(0.00832)	(0.00993)	(0.0101)
Digit # Cash Z-statistic	1.075***	1.104***	1.110***	1.069***	1.083***	1.086***	1.084**	1.112***	1.119***	1.085**	1.113***	1.120***
	(0.00949)	(0.00989)	(0.00980)	(0.0225)	(0.0202)	(0.0184)	(0.0399)	(0.0382)	(0.0343)	(0.0372)	(0.0357)	(0.0329)
Digit # Net income Z-statistic	1.102***	1.116***	1.119***	1.080***	1.086***	1.086***	1.102***	1.117***	1.118***	1.099***	1.113***	1.114***
	(0.00404)	(0.00406)	(0.00405)	(0.00419)	(0.00418)	(0.00412)	(0.00632)	(0.00633)	(0.00608)	(0.00621)	(0.00620)	(0.00604)
Digit # Liabilities Z-statistic	1.050***	1.092***	1.099***	1.059***	1.075***	1.073***	1.052*	1.094***	1.099***	1.040	1.078**	1.085***
	(0.0155)	(0.0150)	(0.0157)	(0.0189)	(0.0190)	(0.0189)	(0.0297)	(0.0337)	(0.0327)	(0.0292)	(0.0328)	(0.0320)
Digit # Current liabilities Z-statistic	1.095***	1.159***	1.162***	1.086***	1.111***	1.115***	1.108***	1.170***	1.175***	1.115***	1.175***	1.177***
	(0.0150)	(0.0144)	(0.0151)	(0.0181)	(0.0186)	(0.0181)	(0.0280)	(0.0310)	(0.0284)	(0.0281)	(0.0311)	(0.0292)
Digit # Sales Z-statistic	0.978***	1.169***	1.173***	0.999	1.092***	1.087***	0.977	1.163***	1.166***	0.978	1.143***	1.150***
	(0.00827)	(0.0119)	(0.0115)	(0.0139)	(0.0148)	(0.0145)	(0.0241)	(0.0201)	(0.0195)	(0.0227)	(0.0241)	(0.0232)
Controls												
Mean sales	0.467***	0.492***	0.493***	0.550***	0.565***	0.564***	0.478***	0.503***	0.503***	0.475***	0.498***	0.499***
	(0.00541)	(0.00564)	(0.00564)	(0.00650)	(0.00631)	(0.00630)	(0.00757)	(0.00735)	(0.00742)	(0.00752)	(0.00729)	(0.00735)
Mean assets	0.343***	0.341***	0.339***	0.432***	0.433***	0.432***	0.347***	0.346***	0.344***	0.360***	0.359***	0.358***
	(0.00360)	(0.00353)	(0.00351)	(0.00446)	(0.00438)	(0.00438)	(0.00493)	(0.00481)	(0.00481)	(0.00500)	(0.00494)	(0.00496)
Mean EBITDA	0.984***	0.984***	0.985***	0.985***	0.985***	0.985***	0.984***	0.984***	0.984***	0.984***	0.984***	0.984***
	(0.00234)	(0.00225)	(0.00226)	(0.00244)	(0.00240)	(0.00240)	(0.00267)	(0.00254)	(0.00246)	(0.00339)	(0.00319)	(0.00313)
S.D. in sales	1.994***	1.963***	1.956***	1.711***	1.695***	1.695***	1.943***	1.915***	1.910***	1.986***	1.959***	1.955***
	(0.0222)	(0.0215)	(0.0214)	(0.0196)	(0.0189)	(0.0188)	(0.0298)	(0.0287)	(0.0284)	(0.0304)	(0.0292)	(0.0289)
S.D. in EBITDA	1.631***	1.599***	1.605***	1.391***	1.377***	1.378***	1.633***	1.602***	1.607***	1.566***	1.534***	1.538***
	(0.0145)	(0.0140)	(0.0141)	(0.0133)	(0.0124)	(0.0124)	(0.0213)	(0.0206)	(0.0206)	(0.0201)	(0.0193)	(0.0193)
S.D. in assets	1.644***	1.647***	1.655***	1.472***	1.472***	1.474***	1.640***	1.644***	1.651***	1.587***	1.587***	1.591***
	(0.0175)	(0.0173)	(0.0173)	(0.0163)	(0.0160)	(0.0160)	(0.0248)	(0.0244)	(0.0248)	(0.0235)	(0.0231)	(0.0233)
Log of employees	0.854***	0.846***	0.846***	0.814***	0.806***	0.807***	0.831***	0.822***	0.823***	0.862***	0.853***	0.854***
	(0.0135)	(0.0134)	(0.0134)	(0.0124)	(0.0120)	(0.0120)	(0.0159)	(0.0152)	(0.0153)	(0.0165)	(0.0158)	(0.0159)
Year of birth	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included
Industry (2-digit) dummies	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included
Region dummies	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included
Constant				0.0236***	0.0197***	0.0180***	0***	0***	0***	0***	0***	0***
				(0.0139)	(0.0117)	(0.0110)	(0)	(0)	(0)	(0)	(0)	(0)
Observations	194,020	194,020	194,020	208,517	208,517	208,517	208,517	208,517	208,517	208,517	208,517	208,517

Hazard ratios reported
*** p<0.01, ** p<0.05, * p<0.1