# Eco-efficiency estimation with quantile stochastic frontiers

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**Abstract**

This paper adopts the quantile stochastic frontier framework in order to construct eco-efficiency measures. Using the estimates from the quantile stochastic frontier, the eco-performance of the U.S. states for nitrogen oxides ($NO_X$), carbon dioxide ($CO_2$), and sulfur dioxide ($SO_2$) emissions is evaluated. Numerical Bayesian inference is applied by utilizing Markov Chain Monte Carlo techniques. A decoupling analysis involving the evaluation of the nonsynchronous change among states' economic output and environmental degradation levels is also performed. The findings suggest that the U.S. states' have followed a decoupling process among their GDP and emission levels over the period 1990-2017. The eco-efficiency estimates derived from the quantile stochastic frontier suggest that the eco-productivity levels of the U.S. states have improved over time. This finding is verified across all quantiles and is reflected on the findings obtained from the decoupling analysis

**Keywords:** OR in environment and climate change; Bayesian Inference**;** Decoupling; Eco-efficiency; Quantile Stochastic Frontier;

## 1. Introduction

The cornerstone of sustainability analysis is the investigation of environmental and economic (E-E) performance. According to Koskela and Vehmas (2012) this E-E relationship can be operationalized through the construction of eco-performance or eco-efficiency indicators. Huppes and Ishikawa (2005) suggests that eco-efficiency is an instrument for evaluating sustainability measuring the exchange between environmental quality and economic welfare. Koskela and Vehmas (2012, p.548) describe different types of eco-efficiency. The first one is simply indicates how more output can be obtained from less natural resources. The second definition is based on maximizing value added with less environmental damage, whereas, the third is focused on minimizing the environmental damage from the obtained economic output. Finally a fourth type of eco-efficiency relates to management strategy. Aparicio et al. (2020) assert that eco-efficiency models (Korhonen and Luptacik, 2004; Kuosmanen and Kortelainen, 2005), are not based on the axiomatic production efficiency theoretical framework but rather aggregate environmental damage with economic outputs. They can be constructed either by assuming the minimization of environmental damage given the economic outputs, or by maximizing the economic output given the environmental damages. There are two types of approaches under which the eco-efficiency estimators can be constructed. The first one is based on parametric methodological framework known as Stochastic frontier approach (SFA; Aigner et al., 1977) and the other which is based on nonparametric methodological framework known as data envelopment analysis (DEA; Charnes et al., 1978; Banker et al., 1984). The most popular eco-efficiency indicator is the DEA based indicator introduced by Kuosmanen and Kortelainen (2005, hereafter KK). As has been stressed by several studies (Gómez-Limón et al., 2012; Zhu et al., 2018; Vásquez-Ibarra et al., 2020), the DEA based eco-efficiency estimator are so popular because it provides benchmark targets, can handle multiple inputs and outputs and from modelling point of view does not assume any functional forms among the environmental pressures and economic outcomes. However, as Tsionas (2002, 2003, 2012) and Kumbhakar and Tsionas (2006, 2008, 2020) assert that SFA approaches account for measurement errors and can separate inefficiency from noise. In addition when assuming a specific distribution on the errors, the SFA approach allows the researcher to obtain observation specific inefficiency measures. Given those advances and to our knowledge there are only two studies utilizing the SFA framework on the measurement of eco-efficiency. The first study was conducted by Orea and Wall (2017) applying the SFA framework in order to evaluate the eco-efficiency levels of 50 Spanish dairy farms. The second study is conducted by Song and Chen (2019) and evaluates the eco-efficiency levels of grain production in Chinese regions.

Our research builds and extents these two studies by utilizing quantile SFA models[1] introduced recently by Tsionas (2020) and in order to measure U.S. states' eco-efficiency levels over the period 1990-2017. According to Assaf et al. (2020) and Tsionas et al. (2020) the proposed methodological approach for eco-efficiency measurement poses several methodological advances compared to the standard SFA approaches. Specifically, every quantile of the conditional distribution of the economic output (or environmental damage) is allowed to have a quantile specific inefficiency. Therefore the adopted methodological approach relates a specific quantile one side error term with the economic output to environmental degradation specification. Therefore based both on the asymmetric Laplace and on the half-normal errors, the model enable us to estimate US states' specific quantile technical eco-inefficiency levels. In addition every quantile has a different eco-frontier technology provide us with greater flexibility in our analysis accounting for potential heterogeneity. Moreover, we adopt a symmetric generalized McFadden functional form in our estimation, which according to Kumbhakar (1994) is more flexible compared to Cobb-Douglas and Translog specifications usually applied in the SFA approaches. Finally, among other components, quantiles as treated as parameters in our approach using Markov chain Monte Carlo (MCMC) methods in order to obtain posterior distributions. Therefore, even though we analyze all quantiles the most likely (optimal) quantile is also estimated. Finally, in accordance with the quantile SFA eco-efficiency estimates, we complement US states' eco-efficiency estimation by applying a decoupling analysis. The notion of decoupling is strongly relates to eco-efficiency analysis, investigating states' nonsynchronous change among economic output and environmental degradation (OCED, 2002). Specifically, as in similar studies (among others, De Freitas and Kaneko, 2011; Andreoni and Galmarini, 2012; Conrad and Cassar, 2014; Moutinho et al., 2018; Yang et al., 2018), we follow the methodological approach introduced by Tapio (2005) in order to estimate the decomposition of decoupling among US states' economic growth from nitrogen oxides ($NO_x$), carbon dioxide ($CO_2$), and sulfur dioxide ($SO_2$) emissions. The structure of the paper is as follows: Section 2 reviews the studies utilizing the KK eco-efficiency indicator, whereas Section 3 describes our data and presents the quantile SFA based eco-efficiency measure. Section 4 analyses our empirical findings, whereas the last Section concludes our paper.

## 2. Literature review

The relative literature of the measurement of eco-efficiency is heavily based on the DEA methodology (Zhou et al., 2018). Following the eco-efficiency methodological approach of Kortelainen and Kuosmanen (2007, KK) they introduce an eco-efficiency indicator

[1]The quantile estimation of efficiency measurement has drawn attention both under the nonparametric (Aragon et al., 2005; Daouia and Simar, 2007; Jradi and Ruggiero, 2019) and under the parametric (Bernini et al., 2004; Knox et al., 2007; Liu et al., 2008; Jradi et al., 2019) methodological framework.

analyzing consumer durables. The contribution of the proposed model is based on the evaluation of eco-efficiency performance in terms of absolute scale shadow prices. Similarly, Kuosmanen and Kortelainen (2007) present a DEA based environmental cost–benefit analysis derived by the KK eco-efficiency measure. The advantage of the proposed measure lies on the fact that prices are endogenous determined by the model, whereas, the overall measurement is based on absolute shadow prices. Moreover, Kortelainen (2008) based on the DEA eco-efficiency indicator presents a Malmquist productivity index presenting an eco-productivity measure alongside with its decomposition. The eco-productivity index is then applied on a sample of 20 EU countries during 1990-2003. Zhang et al. (2008) provides a slack-based version of the KK eco-efficiency DEA estimator. By utilizing several pollutants, they measure the eco-efficiency levels of 30 provinces in China. They provide evidence of a positive correlation among provinces' GDP per capita levels and their estimated eco-efficiency levels. Moreover, Cha et al., (2008) provide a global worming eco-efficiency indicator based on Kyoto Mechanism projects, utilizing in a DEA framework the value added of a system as an output and its global warming influence as an input. Picazo-Tadeo et al. (2011) using the KK indicator in order to measure farming eco-efficiency for a sample of 171 Spanish farms. Their results suggest that the agri-environmental programs applied to the Spanish agricultural sector enhanced farming eco-efficiency levels. Camarero et al., (2013) in a first stage analysis, they utilize the KK indicator in order to estimate 22 OECD countries' eco-efficiency levels over the period 1980-2008. Then in a second stage analysis they perform the Phillips and Sul (2007) test in order to identify eco-efficiency convergence clubs. Their findings signify an overall identification of five convergence clubs. Similarly, Camarero et al., (2014) utilizing a directional distance function approach evaluate 27 EU countries' eco-efficiencies levels during 1990-2009. Then by applying the convergence approach by Phillips and Sul (2007), the existence of eco-efficiency convergence clubs among the EU countries is evaluated. Their findings suggest the existence of 6 convergence eco-efficiency clubs. In the same manner, Gómez-Calvet et al. (2016) utilize a slack-free directional distance function in order to construct an eco-efficiency indicator for the 27 EU countries over the period 1993-2010. Then in a second stage analysis their findings suggest that the eco-efficiency convergence is not a continuous process. In their study Godoy-Durán et al. (2017) suggest that the eco-efficiency indicator has been accepted by the environmental economic literature as a sustainability indicator. They apply an eco-efficiency indicator on a sample of small-scale family farms in southeast Spain. They provide evidence that environmental innovation improves farms' eco-efficiency levels. Analyzing Polish regional eco-efficiency levels Rybaczewska-Błażejowska and Masternak-Janus (2018), reveal that heavy industry remains as the main source of the generation of Polish environmental pressures. Moreover, for the case of Latin America Moutinho et al., (2018) measure countries' eco-efficiency levels over the period 1994-2013. They utilize an eco-

Malmquist productivity index decomposing countries' eco-productivity levels into eco-technical efficiency and eco-technological change levels. Grovermann et al. (2019) in a first stage analysis, measure the eco-efficiency levels of 79 countries. Then in seconds stage analysis they apply a truncated regression suggesting that education and legal rights affect the estimated eco-efficiency levels. Broadstock et al., (2019) using conditional eco-efficiency estimators provide evidence that corporate choices on environmental, social, and governance (ESG) strategic investment compliance affect firms' eco-efficiency levels. Ina different context, Tang et al. (2020) evaluated the eco-efficiency levels of 30 Chinese provinces over the period 1986 to 2016 utilizing and eco-Malmquist-Luenberger productivity indicator. Kounetas et al., (2020) provide an order-m eco-efficiency frontier which is robust to atypical values and sample characteristics. In a second stage analysis they perform different convergence approaches investigating whether the order-m eco-efficiency indicators converge overtime. Using a sample of 1572 firms over the years 2009–2017, Trinks et al., (2020) estimate a carbon efficiency model which is based on KK eco-efficiency indicator. They define carbon efficiency indicator as the ratio of target-to-actual carbon emissions and provide an association between firms' financial performance and their carbon efficiency levels. Finally, Gamboa et al. (2020) utilize an eco-efficiency indicator in a sample of 367 smallholder farmers, providing evidence of low eco-efficiency levels among producers due to extensive use of mineral fertilizers.

## 3. Description of variables and methodological framework

### *3.1. Description of variables*

Based on the notion of eco-efficiency as described by Kuosmanen and Kortelainen (2005) we use regional GDP (measured in millions of 2009 USD) as the states' value-added levels. Regional GDP has been extracted from Bureau of Economic Analysis (BEA) and is adjusted from inflation. In addition, we use: nitrogen oxides ($NO_x$), carbon dioxide ($CO_2$), and sulfur dioxide ($SO_2$) as the main pollutants of our eco-efficiency model. All pollutants are measured in metric tons and have been extracted from U.S. Energy Information Administration (EIA). Table 1 presents the descriptive statistics of the variables used for all the states over the entire period (1990-2017).

### *3.2. The eco-inefficiency model*

The eco-efficiency estimator has been introduced by Kuosmanen and Kortelainen (2005) and it is based on the ability of an economy to grow by controlling the environmental pressures. Formally, the eco-efficiency indicator can be defined as:

$$\frac{\phi}{\sum_i^B u_i z_i} \leq 1 \ . \tag{1}$$

In equation (1) $u_i$ represent the weights which are nonnegative and assigned to environmental pressures $z_i$ (i.e. the pollutants), whereas, $\phi$ represents the value added (i.e. GDP). Moreover, we can express (1) as:

$$\phi \le \sum_{i=1}^{B} u_i z_i. \quad (2)$$

Then in the SFA context (2) can be expressed as:

$$\phi = \alpha_0 + \sum_{i=1}^{B} u_i z_i + \mu_i - \psi_i, i = 1, \dots, n. \quad (3)$$

In its general form eco-efficiency estimation can be obtained from:

$$\phi_i = z'_i \gamma + \mu_i - \psi_i, i = 1, \dots, n. \quad (4)$$

Note that the error component is decomposed as: $\varepsilon_i = \mu_i - \psi_i$, where, $\psi_i$ represents the eco-inefficiency and $\mu_i$ the noise. Based on the quantile SFA model introduced by Tsionas (2020) (4) will take the following form:

$$\phi_i = z'_i \gamma(k) + \mu_i(k) - \psi_i(k), i = 1, \dots, n, \quad (5)$$

where $k$ represents the different quantiles $(0 < k < 1)$, and the additional parameters $\gamma, \mu_i$ and $\psi_i$ can vary by the quantiles. Equation (5) is based on the quantile regression considering the following Laplace density:

$$p(\mu) \propto \tau^{-1} \exp\left\{-\tau^{-1}|\mu|\left[kI_{[0,\infty)}(\mu) + (1-k)I_{(-\infty,0]}(\mu)\right]\right\}, \quad (6)$$

with the following likelihood function:

**Table 1**: Descriptive statistics

| *YEAR* | *STAT* | *GDP (Millions USD)* | *$CO_2$ emissions (Metric tonnes)* | *$SO_2$ emissions (Metric tonnes)* | *$NO_X$ emissions (Metric tonnes)* | *YEAR* | *STAT* | *GDP (Millions USD)* | *$CO_2$ emissions (Metric tonnes)* | *$SO_2$ emissions (Metric tonnes)* | *$NO_X$ emissions (Metric tonnes)* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1990 | *mean* | 183554.316 | 76615644.784 | 606334.353 | 312206.922 | 2004 | *mean* | 283878.225 | 97528688.549 | 404266.863 | 162472.000 |
| | *std* | 223339.782 | 76311885.414 | 821732.036 | 293315.321 | | *std* | 339933.603 | 92977598.900 | 493543.240 | 133964.084 |
| 1991 | *mean* | 183233.086 | 76488851.255 | 601663.941 | 310318.373 | 2005 | *mean* | 293747.598 | 99758359.333 | 405472.275 | 155337.137 |
| | *std* | 220204.903 | 76257873.724 | 831492.625 | 296223.149 | | *std* | 353372.197 | 95033421.496 | 508952.257 | 125506.379 |
| 1992 | *mean* | 188701.291 | 77344121.922 | 589423.686 | 303072.627 | 2006 | *mean* | 302264.622 | 97604617.686 | 373472.980 | 148997.922 |
| | *std* | 222900.911 | 76014479.733 | 804918.775 | 284042.336 | | *std* | 367176.181 | 94975405.211 | 469715.091 | 123700.303 |
| 1993 | *mean* | 192466.274 | 80668729.412 | 586897.745 | 313594.157 | 2007 | *mean* | 307011.041 | 99883626.902 | 354576.353 | 143137.647 |
| | *std* | 224465.120 | 80598235.892 | 796434.077 | 295185.927 | | *std* | 373847.320 | 95127405.115 | 452300.248 | 122084.537 |
| 1994 | *mean* | 201397.784 | 81887780.431 | 567527.216 | 305924.490 | 2008 | *mean* | 306087.724 | 97412239.647 | 307050.902 | 130596.980 |
| | *std* | 230762.423 | 79470087.241 | 769892.676 | 281554.623 | | *std* | 372115.842 | 92788572.046 | 376383.304 | 112403.249 |
| 1995 | *mean* | 208368.860 | 82904804.078 | 484865.961 | 249885.176 | 2009 | *mean* | 298009.396 | 89000299.137 | 234130.353 | 93939.216 |
| | *std* | 238793.904 | 80998186.739 | 569275.972 | 246198.998 | | *std* | 363801.696 | 86579809.990 | 288616.303 | 77322.966 |
| 1996 | *mean* | 217362.687 | 85763907.490 | 509462.627 | 253889.137 | 2010 | *mean* | 304830.131 | 93670428.039 | 211782.353 | 97687.216 |
| | *std* | 249105.891 | 84608198.278 | 618605.997 | 254287.603 | | *std* | 371623.845 | 91012892.868 | 259171.008 | 79918.447 |
| 1997 | *mean* | 228706.986 | 88600843.922 | 528614.941 | 254917.294 | 2011 | *mean* | 309049.943 | 89689048.235 | 189999.608 | 94369.059 |
| | *std* | 263613.878 | 86757827.199 | 634763.644 | 250868.997 | | *std* | 377348.732 | 92098631.578 | 242638.515 | 82435.819 |
| 1998 | *mean* | 238565.096 | 92219588.902 | 528018.863 | 253282.275 | 2012 | *mean* | 315368.159 | 84583324.588 | 145253.725 | 84224.549 |
| | *std* | 277171.059 | 89741315.218 | 624967.518 | 246407.024 | | *std* | 389284.916 | 86248553.596 | 173129.822 | 73671.662 |
| 1999 | *mean* | 249675.420 | 92796168.471 | 503661.529 | 233548.980 | 2013 | *mean* | 320278.200 | 85247311.294 | 141520.784 | 84841.255 |
| | *std* | 293744.905 | 90372860.428 | 590516.625 | 225580.748 | | *std* | 400516.304 | 87121085.701 | 167546.050 | 75733.798 |
| 2000 | *mean* | 259921.463 | 96895468.431 | 466815.961 | 221102.941 | 2014 | *mean* | 327638.037 | 85030755.882 | 135453.216 | 82367.176 |
| | *std* | 310973.564 | 93937364.559 | 544052.199 | 210191.770 | | *std* | 413583.018 | 86486308.780 | 165175.053 | 72236.024 |
| 2001 | *mean* | 261955.755 | 94847322.471 | 438210.471 | 207446.235 | 2015 | *mean* | 336836.659 | 79664794.627 | 99912.667 | 71531.294 |
| | *std* | 313007.949 | 89933060.792 | 512479.308 | 187912.916 | | *std* | 431311.418 | 80729764.473 | 119114.100 | 63596.576 |
| 2002 | *mean* | 266708.902 | 95057376.078 | 426722.824 | 203671.843 | 2016 | *mean* | 341995.459 | 75623565.176 | 70852.275 | 63929.294 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *std* | 318050.430 | 92032756.463 | 504604.333 | 185345.268 | | *std* | 440728.001 | 77629264.747 | 85531.933 | 59025.941 |
| 2003 | *mean* | 273756.341 | 95886050.980 | 417482.706 | 177714.706 | 2017 | *mean* | 348761.867 | 72539214.588 | 64974.471 | 59049.490 |
| | *std* | 327268.890 | 92191219.235 | 507216.130 | 155995.251 | | *std* | 451597.177 | 77510903.135 | 87569.504 | 55551.869 |

**Table 2**: Decoupling criteria

| ***Environmental pressures*** | | | ***GDP*** | ***Elasticities*** | ***Characterization*** |
|---|---|---|---|---|---|
| $\Delta CO_2>0$ | $\Delta NO_X>0$ | $\Delta SO_2>0$ | $\Delta GDP>0$ | $\theta > 1.2$ | *Negative Decoupling (Expansionary Negative Decoupling)* |
| $\Delta CO_2>0$ | $\Delta NO_X>0$ | $\Delta SO_2>0$ | $\Delta GDP<0$ | $\theta < 0$ | *Negative Decoupling (Strong Negative Decoupling)* |
| $\Delta CO_2<0$ | $\Delta NO_X<0$ | $\Delta SO_2<0$ | $\Delta GDP<0$ | $0 \leq \theta \leq 0.8$ | *Negative Decoupling (Weak Negative Decoupling)* |
| $\Delta CO_2>0$ | $\Delta NO_X>0$ | $\Delta SO_2>0$ | $\Delta GDP>0$ | $0 \leq \theta \leq 0.8$ | *Decoupling (Weak Decoupling)* |
| $\Delta CO_2<0$ | $\Delta NO_X<0$ | $\Delta SO_2<0$ | $\Delta GDP>0$ | $\theta < 0$ | *Decoupling (Strong Decoupling)* |
| $\Delta CO_2<0$ | $\Delta NO_X<0$ | $\Delta SO_2<0$ | $\Delta GDP<0$ | $\theta > 1.2$ | *Decoupling (Recession Decoupling)* |
| $\Delta CO_2>0$ | $\Delta NO_X>0$ | $\Delta SO_2>0$ | $\Delta GDP>0$ | $0.8 \leq \theta \leq 1.2$ | *Link (Growing Link)* |
| $\Delta CO_2<0$ | $\Delta NO_X<0$ | $\Delta SO_2<0$ | $\Delta GDP<0$ | $0.8 \leq \theta \leq 1.2$ | *Link (Recession Link)* |

$L_k(\gamma,\tau;\phi,Z) \propto \tau^{-n} \exp\left\{-\tau^{-1}\sum_{i=1}^{n} | \phi_i - z'_i\gamma| \left[kI_{[0,\infty)}(\phi_i - z'_i\gamma) + (1-k)I_{(-\infty,0]}(\phi_i - z'_i\gamma)\right]\right\}$. (7)

Given that $p(w_i) = \exp(-w_i)$, the Laplace distribution having set $\sigma_\mu = (2\tau)^{1/2}$ can be defined as:

$$p(\mu_i|w_i) \propto \left(\sigma_\mu^2 w_i\right)^{-\frac{1}{2}} \exp\left\{-\frac{\mu_i^2}{2\sigma_\mu^2 w_i}\left[kI_{[0,\infty)}(\mu_i) + (1-k)I_{(-\infty,0]}(\mu_i)\right]\right\}. \quad (8)$$

Also, from equation (5) is assumed that $\psi_i \sim \mathcal{N}_+(0,\sigma_\psi^2)$ with a density of $\varepsilon_i$ defined as:

$$p(\varepsilon_i) \propto \tau^{-1}\sigma_\psi^{-1}\int_0^\infty \exp\left\{-\tau^{-1}|\varepsilon_i|\left[kI_{[0,\infty)}(\varepsilon_i) + (1-k)I_{(-\infty,0]}(\varepsilon_i)\right] - \frac{1}{2\sigma_\psi^2}\psi_i^2\right\} d\psi_i. \quad (9)$$

In addition, the posterior can be defined as[2]:

$$p_k(\gamma,\tau,\sigma_\psi,\{\psi_i\}_{i=1}^n|\phi,z) \propto \tau^{-n}\sigma_\psi^{-n} \cdot p(\gamma,\tau,\sigma_\psi) \cdot \exp\left\{-\tau^{-1}\sum_{i=1}^{n} | \phi_i - z'_i\gamma + \psi_i|\left[kI_{[0,\infty)}(\phi_i - z'_i\gamma + \psi_i) + (1-k)I_{(-\infty,0]}(\phi_i - z'_i\gamma + \psi_i)\right] - \frac{1}{2\sigma_\psi^2}\sum_{i=1}^{n}\psi_i^2\right\} \quad (10)$$

Moreover, we can treat $\{w_i\}_{i=1}^n$ as parameters and, in turn, (10) can be expressed as:

$$p_k(\gamma,\sigma_\mu,\sigma_\psi,\{\psi_i\}_{i=1}^n,\{w_i\}_{i=1}^n|\phi,z) \propto \sigma_\mu^{-n}\sigma_\psi^{-n} \cdot \exp\left\{-\frac{1}{2\sigma_\mu^2}\sum_{i=1}^{n}\frac{\left(\phi_i - z'_i\gamma + \psi_i\right)^2}{w_i}\left[kI_{[0,\infty)}(\phi_i - z'_i\gamma + \psi_i) + (1-k)I_{(-\infty,0]}(\phi_i - z'_i\gamma + \psi_i)\right] - \frac{1}{2\sigma_\psi^2}\sum_{i=1}^{n}\psi_i^2 - \sum_{i=1}^{n} w_i\right\}. \quad (11)$$

Then the priors can be defined as:

$$\begin{gathered} p(\gamma|\sigma_\mu,\sigma_\psi) \propto 1, \\ p(\sigma_\mu) \propto \sigma_\mu^{(\underline{n}_\mu+1)} \exp\left(-\frac{\underline{k}_\mu}{2\sigma_\mu^2}\right), \\ p(\sigma_\psi) \propto \sigma_\psi^{(\underline{n}_\psi+1)} \exp\left(-\frac{\underline{k}_\psi}{2\sigma_\psi^2}\right). \end{gathered} \quad (12)$$

From (12) the prior parameters are $\underline{n}_\mu, \underline{n}_\psi, \underline{k}_\mu, \underline{k}_\psi \geq 0$.[3] In addition we set:

$$p(k) \propto 1, 0 \leq k \leq 1, \quad (13)$$

whereas the conditional posterior can be defined as:

$$p(k|\gamma,\tau,\sigma_\psi,\{\psi_i,w_i\}_{i=1}^n,\phi,z) \propto \exp(Dk), 0 \leq k \leq 1, \quad (14)$$

where $D = -\tau^{-1}\sum_{i=1}^{n}\left\{I_{[0,\infty)}(\phi_i - z'_i\gamma + \psi_i) - I_{(-\infty,0]}(\phi_i - z'_i\gamma + \psi_i)\right\}$.

## 4. Empirical findings

As a first step, we perform a decoupling analysis based on the work by Tapio (2005) and then in a second stage analysis we present our findings derived from the proposed eco-

[2]See Appendix for technical details regarding the estimation of posterior conditional distributions.

[3]We set $\underline{n}_\mu, \underline{n}_\psi = 1, \underline{k}_\mu, \underline{k}_\psi = 10^{-4}$, which corresponds to a flat prior.

efficiency indicator. According to De Freitas and Kaneko (2011) and Moutinho et al. (2018) the decoupling analysis can be used in accordance with other eco-performance indicators presenting useful insights among environmental damages and economic output. In fact, both the proposed eco-efficiency and the decoupling indicator is based on the notion of "impact decoupling" which is based on the growth of economic output alongside with the deterioration of environmental damages (UNEP, 2011). Tapio (2005), based on elasticities of decoupling indicators introduced a decoupling index. Specifically, the decoupling index of the three pollutants can be expressed as:

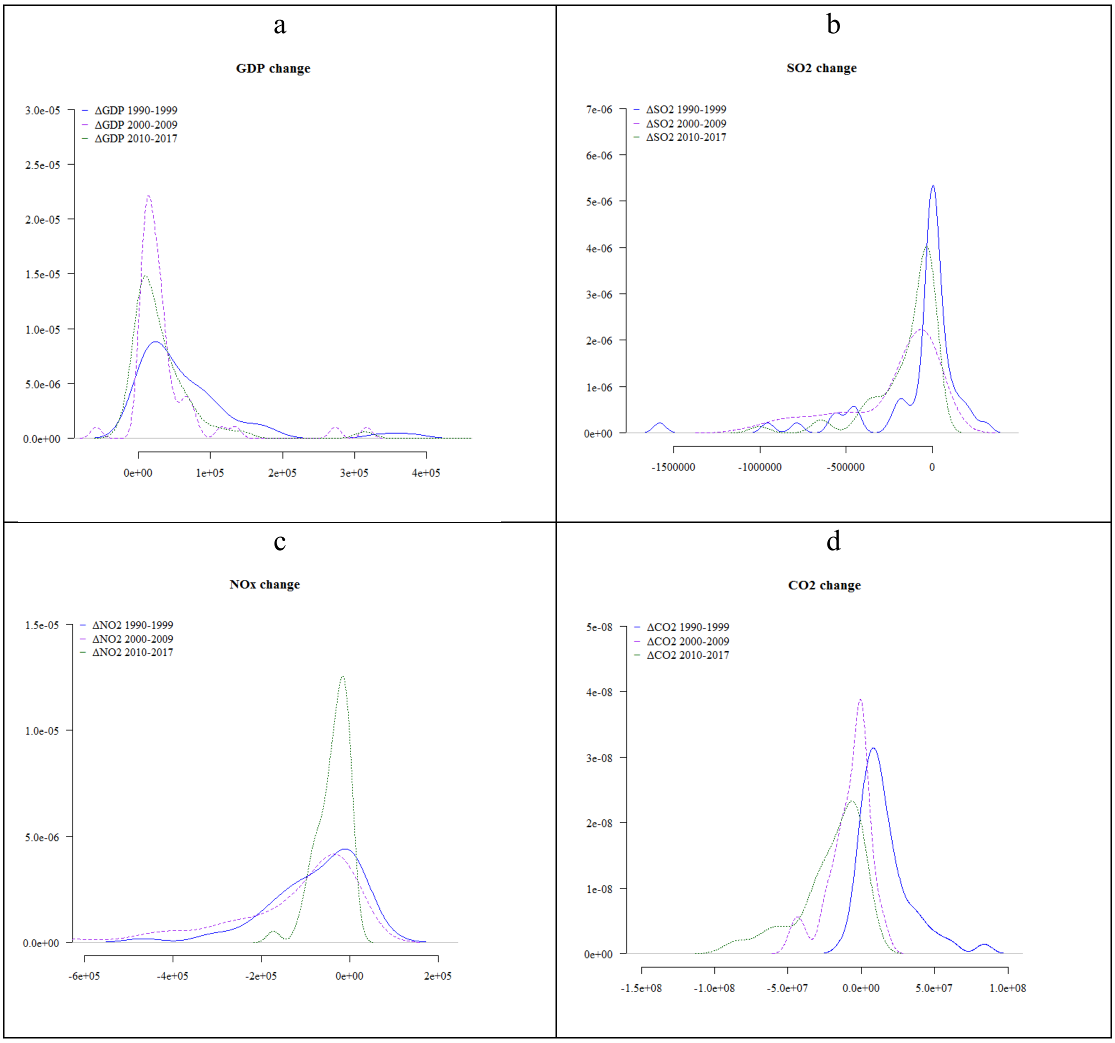
$$\theta(CO_2, GDP) = \frac{\frac{\Delta CO_2}{CO_2}}{\frac{\Delta GDP}{GDP}}, \theta(SO_2, GDP) = \frac{\frac{\Delta SO_2}{SO_2}}{\frac{\Delta GDP}{GDP}}, \theta(NO_X, GDP) = \frac{\frac{\Delta NO_X}{NO_X}}{\frac{\Delta GDP}{GDP}}. \quad (15)$$

Table 2 presents the characterizations of the obtained decoupling measure based on the estimated values of the three decoupling indexes. Specifically, according to Tapio (2005) the estimated elasticities and the growth among the states' GDP levels and the environmental pollutants can characterize the obtained decoupling states as: "Expansionary Negative Decoupling"; "Strong Negative Decoupling"; "Weak Negative Decoupling"; "Weak Decoupling"; "Strong Decoupling"; "Recession Decoupling"; "Growing Link" and "Recession Link". These detailed characterizations can be grouped in three general states: "Decoupling"; "Negative Decoupling" and "Link".

We construct the decoupling indicators for the U.S. states over three non-overlapping periods (1990-1999; 2010-2009; 2010-2017)[4]. Based on those periods we first analyze the differences among the variables. Figure 1a clearly presents an overall GDP growth over the examined periods being more pronounced during the period 1990-1999. According to Krueger and Kumar (2004) the U.S. economy during the 90s outperformed. The higher economic growth rates during that period was mainly driven by the extensive consumption of new goods and services (Greenwood and Uysal, 2005). Also, it is noticeable from Fig.1 that during that period all pollutants had a positive change with the greatest positive change to be reported for $CO_2$ emissions (Fig. 1d). It is obvious that the increased demand and the consumption of goods and services (i.e. energy, extensive trade, etc.) has caused emissions to increase among that period. However, for the other two examined periods (i.e. 2000-2009; 210-2017) the positive change for the pollutants is reported but they are less pronounced compared to the 1990-1999 period. Similarly, Fig. 2 presents the densities of the constructed decoupling indexes (DI). The plots suggest that for the periods 2010-2009 and 2010-2017 the majority of the states' DI values is negative, being more emphatic for the cases of $SO_2$ and $NO_X$ and less pronounced for $CO_2$ emissions. However, for the period 1990-1999 the majority of states report positive DI values

[4]The analytical estimations of the obtained elasticities "$\theta$" are available upon request. The overlapping time periods was chosen based on the remarks raised by Jorgenson and Wilcoxen (1990) regarding the adjusting period from short-run to steady-state.

**Figure 1**: Density plots of decoupling and eco-efficiency components




with the exception of $NO_X$ emissions. These findings are not surprising since during the 1990s, U.S. economic growth was mainly driven by "unrestricted" energy use. Leflaive (2008) reports that it has been until 2007 which the "Energy Independence and Security Act" was signed concentrated to regulations regarding energy efficiency and the usage of renewable energy among U.S. states. Song et al. (2019) provide similar findings suggesting that U.S. development is experiencing a strong decoupling. Interesting enough from both Fig. 1 and Fig. 2 we can see that some of distributions appear to be similar. Therefore, it is interesting to see if both the estimated changes and the DI indexes have been differentiated during the examined periods. For that reason we perform on the estimated measures a bootstrapped based test introduced by Li et al. (2006), for testing the mean equality among two densities. Table 3 presents our findings comparing in a pair manner the estimated density of every measure among the examined periods. Under the null hypothesis the distribution of means are equal. The results presented suggest that for the case of ΔGDP the null hypothesis is rejected only between the period 1990-1999 and 2000-2009. Similarly, in all cases the null hypothesis is rejected for $\Delta CO_2$ and for DI

$SO_2$, whereas, for some other cases like $\Delta SO_2$ and DI NOx the null hypothesis could not be rejected. According to Environmental Protection Agency U.S. $SO_2$ emissions have decreased by 75% over the last decade, whereas, gas emissions generated from power plants have also decreased by 20%.[5]

**Figure 2**: Density plots of decoupling indexes

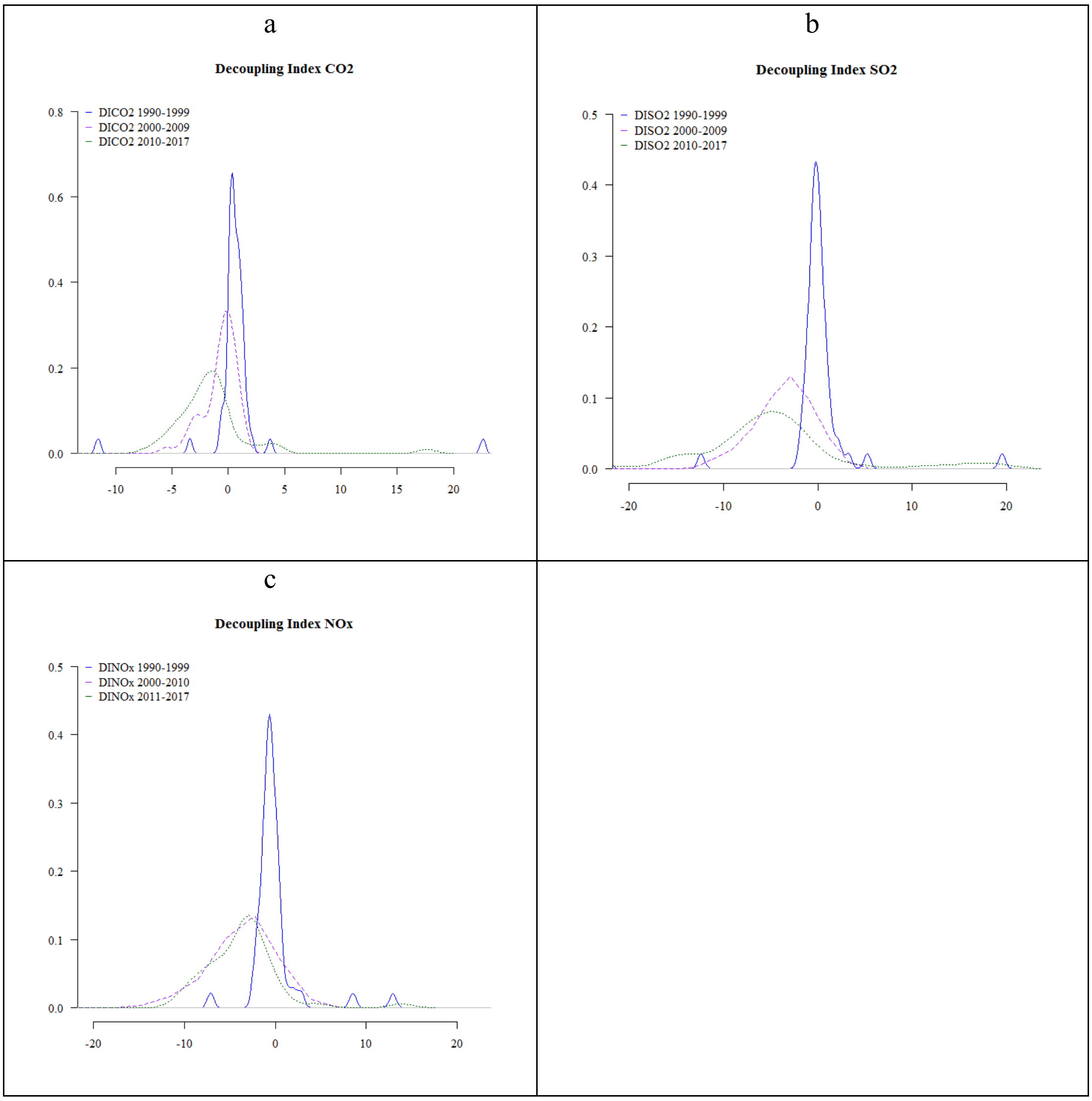



**Table 3**: Results of Li et al. (2009) test for mean equality

[5]https://www.epa.gov/newsreleases/epas-2019-power-plant-emissions-data-demonstrate-significant-progress

| GDP Change | | | DI $CO_2$ | | |
|---|---|---|---|---|---|
| | *2000-2009* | *2010-2017* | | *2000-2009* | *2010-2017* |
| *1990-1999* | 2.127895*** | 4.351936 | 1990-1999 | 11.95207 | 0.4327193*** |
| *2000-2009* | | -4.661233 | 2000-2009 | | 1.407984*** |
| $CO_2$ Change | | | DI $SO_2$ | | |
| | *2000-2009* | *2010-2017* | | *2000-2009* | *2010-2017* |
| *1990-1999* | 9.695959*** | 14.93176*** | 1990-1999 | 10.80082*** | 17.31155*** |
| *2000-2009* | | 1.714191* | 2000-2009 | | 1.247155*** |
| $SO_2$ Change | | | DI NOx | | |
| | *2000-2009* | *2010-2017* | | *2000-2009* | *2010-2017* |
| *1990-1999* | 6.092738 | 6.448754* | 1990-1999 | 14.77057 | 13.95677 |
| *2000-2009* | | -1.110785 | 2000-2009 | | -1.739764 |
| NOx Change | | | | | |
| | *2000-2009* | *2010-2017* | | | |
| *1990-1999* | 0.7852268 | 9.95035*** | | | |
| *2000-2009* | | 8.012885*** | | | |

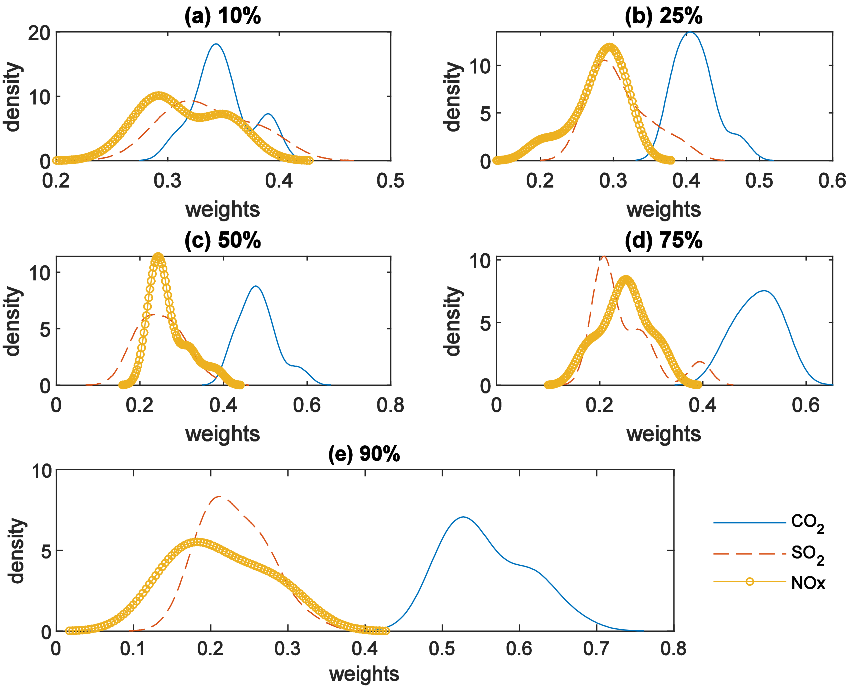
*** Null of equality is rejected at the 0.1% level
*Null of equality is rejected at the 5% level

Moreover, Table 4, presents the different decoupling classifications of the U.S. states. The results for the case of states' $CO_2$ emissions in relation to their GDP levels, suggest that 13 states during the period 1990-1999 are reported as "Negative Decoupling" with the majority of cases characterized as "Expansionary Negative Decoupling" (i.e. $\Delta CO_2>0$; $\Delta GDP>0$; $\theta > 1.2$). However, for the next two examined periods (2010-2019 and 2010-2017) the results suggest that the majority of states are in a "Decoupling" juncture and in most cases characterized as "Strong Decoupling" (i.e. $\Delta CO_2<0$; $\Delta GDP>0$; $\theta < 0$). For the case of $SO_2$ emissions we have a similar picture. Specifically, during the period 1990-1999, seven states report a "Negative Decoupling" characterized as "Expansionary Negative Decoupling" (i.e. $\Delta SO_2>0$; $\Delta GDP>0$; $\theta > 1.2$). Similarly, as in the case of $CO_2$ emissions, in the majority of cases the next two periods (2010-2009; 2010-2017) suggest a "Strong Decoupling" juncture among the US sates (i.e. $\Delta SO_2<0$; $\Delta GDP>0$; $\theta < 0$). In contrast for the case of $NO_X$ emissions our findings suggest that for all examined periods a "Decoupling" juncture is reported, with the majority of cases to be characterized as "Strong Decoupling" (i.e. $NO_X<0$; $\Delta GDP>0$; $\theta < 0$). Strong decoupling for the U.S. economy is also reported by Song et al. (2019) especially during the period 2014-2016, characterized by high economic development levels.

Furthermore, Fig. 3 reports the weights of pollutants across various quantiles. As we move across to the 90% quantile the weights of the so-called acidification pollutants ($NO_X$ and

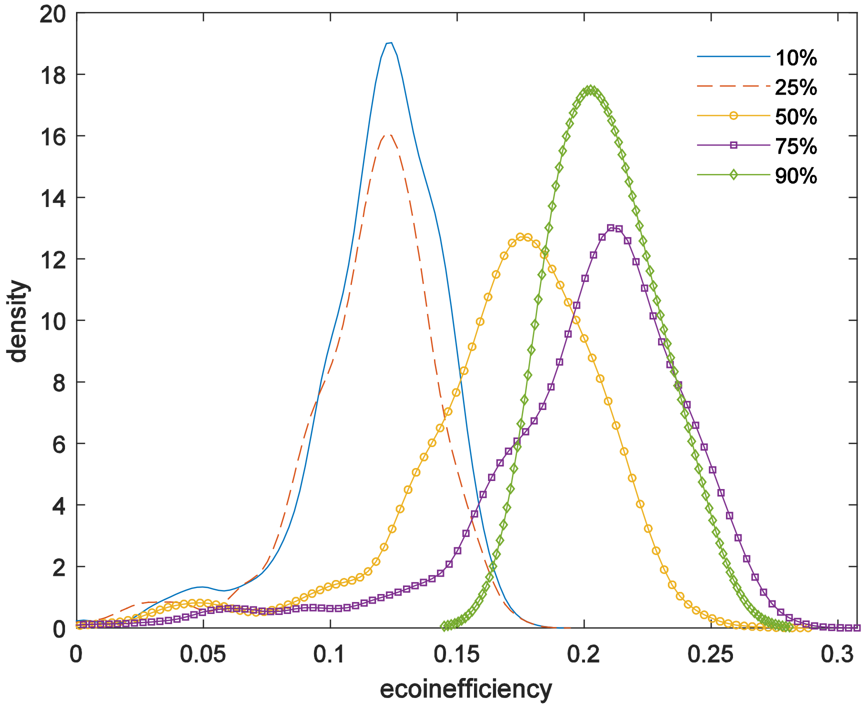
$SO_2$) remained quite similar across the different quantiles. However, the weight for the $CO_2$ emissions (global warming pollutant) has been increased from the 25% quantile onwards.

**Figure 3**: Densities of the weights of pollutants by quantile




Similarly, Fig. 4 presents the estimated eco-inefficiencies levels over the various quantiles. We notice that the 10% and 25% quantiles signify similar eco-inefficiency levels. In addition, the 75% and the 90% quantile suggest a similar increase of states' eco-inefficiency levels, whereas, the median quantile indicates large concentration of eco-inefficiency levels around 0.18. Our findings support the study by Kounetas et al., (2020) reporting states' relatively high eco-efficiency levels. In addition, it is noticeable that both the 50% and the 90% quantile appear to have long left tails (negatively skewed) suggesting that we have dispersed eco-inefficiencies among the states. As a result, in both quantiles, states' mean eco-inefficiency levels are located on the left from the peak of the eco-inefficiency distribution.

**Figure 4**: Densities of the US states' technical eco-inefficiency levels by quantile




In addition, Fig. 5 presents sample distributions of posterior means of the eco-productivity change (ECOPG) and its components, eco-technical change (ECOTC) and eco-efficiency change (ECOEC). The ECOPG and their components have been calculated per two years for the entire period (i.e. 1990-1991;1991-1992;1992-1993,…,2016-2017). The eco-productivity contains state's ability to catch-up to the eco-frontier and its ability to move the eco-frontier ($ECOPG = ECOEC + ECOTC$). Specifically, Fig. 5d suggests that states' eco-productivity levels is positive mainly driven by their eco-technical change levels. However, it is also reported that states' eco-efficiency levels are low and, in some cases, negative suggesting the inability of some states to catch-up the eco-frontier over the examined period. Moreover, Fig.5a presents the $ECOPG$ levels by quantile. All measures are slightly different having a unimodal distribution, with the posterior of 75% quantile to be bimodal. Also, in the 90% quantile we can find evidence that some states' eco-productivity levels can be also negative. For the case of eco-efficiency change (Fig.5b) the 10%, 25% and 90% quantiles have little dissimilarities, having their peak just above zero (positive *ECOEC*). In contrast the posterior of 75% quantile is bimodal, having the lower peak in the area of negative eco-efficiency change values. Clearly our findings suggest that many states report negative eco-efficiency change values over the examined period. In contrast when looking states eco-technical change (Fig.5c) we can see a clear difference among the quantiles. Specifically, *ECOTC* is positive for all quantiles with a

**Table 4**: States' decoupling classifications

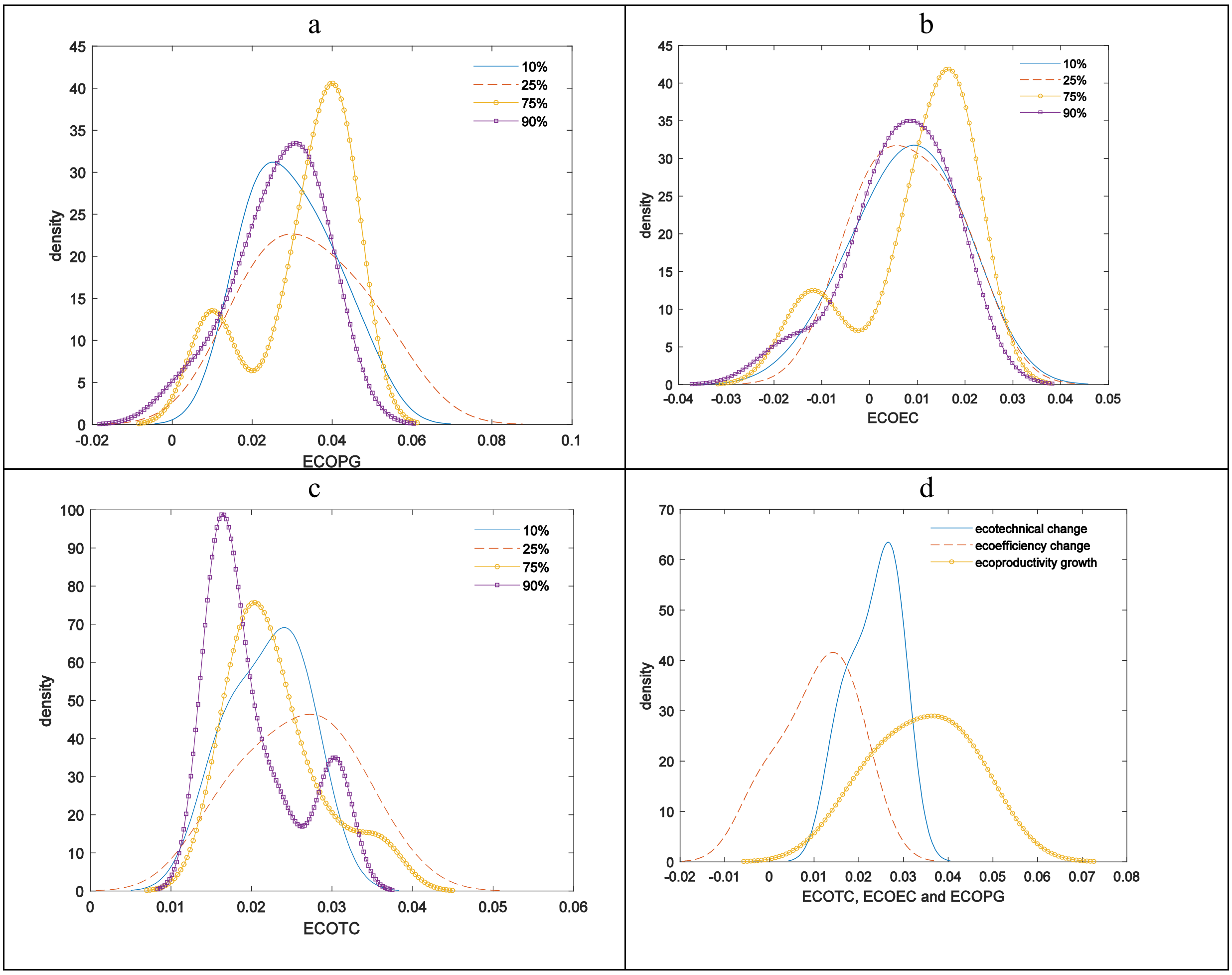
| | $CO_2$ | | | $SO_2$ | | | $NO_X$ | | |
|---|---|---|---|---|---|---|---|---|---|
| ***State Name*** | ***1990-1999*** | ***2000-2009*** | ***2010-2017*** | ***1990-1999*** | ***2000-2009*** | ***2010-2017*** | ***1990-1999*** | ***2000-2009*** | ***2010-2017*** |
| ***Alaska*** | Strong Negative Decoupling | Strong Decoupling | Recession Decoupling | Recession Decoupling | Strong Decoupling | Recession Decoupling | Strong Negative Decoupling | Strong Decoupling | Strong Negative Decoupling |
| ***Alabama*** | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Arkansas*** | Weak Decoupling | Weak Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Arizona*** | Weak Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***California*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling |
| ***Colorado*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Connecticut*** | Weak Decoupling | Strong Decoupling | Recession Decoupling | Strong Decoupling | Strong Decoupling | Recession Decoupling | Strong Decoupling | Strong Decoupling | Recession Decoupling |
| ***District of Columbia*** | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling |
| ***Delaware*** | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Florida*** | Growing Link | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling |
| ***Georgia*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Hawaii*** | Strong Negative Decoupling | Strong Decoupling | Strong Decoupling | Strong Negative Decoupling | Strong Decoupling | Weak Decoupling | Strong Negative Decoupling | Strong Decoupling | Strong Decoupling |
| ***Iowa*** | Growing Link | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Idaho*** | Expansionary Negative Decoupling | Growing Link | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Growing Link |
| ***Illinois*** | Expansionary Negative Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Indiana*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ***Kansas*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Kentucky*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Louisiana*** | Expansionary Negative Decoupling | Strong Decoupling | Recession Decoupling | Expansionary Negative Decoupling | Strong Decoupling | Recession Decoupling | Weak Decoupling | Strong Decoupling | Weak Negative Decoupling |
| ***Massachusetts*** | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Maryland*** | Growing Link | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling |
| ***Maine*** | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling |
| ***Michigan*** | Weak Decoupling | Weak Negative Decoupling | Strong Decoupling | Weak Decoupling | Recession Decoupling | Strong Decoupling | Strong Decoupling | Recession Decoupling | Strong Decoupling |
| ***Minnesota*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Missouri*** | Growing Link | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Mississippi*** | Expansionary Negative Decoupling | Weak Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling |
| ***Montana*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***North Carolina*** | Growing Link | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling |
| ***North Dakota*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Growing Link | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling |
| ***Nebraska*** | Growing Link | Growing Link | Strong Decoupling | Weak Decoupling | Growing Link | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling |
| ***New Hampshire*** | Strong Decoupling | Growing Link | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***New Jersey*** | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***New Mexico*** | Weak Decoupling | Weak Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Nevada*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ***New York*** | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Ohio*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Oklahoma*** | Growing Link | Weak Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Oregon*** | Expansionary Negative Decoupling | Weak Decoupling | Strong Decoupling | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling |
| ***Pennsylvania*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Rhode Island*** | Expansionary Negative Decoupling | Growing Link | Strong Decoupling | Strong Decoupling | Strong Decoupling | Expansionary Negative Decoupling | Expansionary Negative Decoupling | Weak Decoupling | Strong Decoupling |
| ***South Carolina*** | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***South Dakota*** | Growing Link | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling |
| ***Tennessee*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Texas*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Utah*** | Weak Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Virginia*** | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling | Growing Link | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling |
| ***Vermont*** | Weak Decoupling | Strong Decoupling | Expansionary Negative Decoupling | Expansionary Negative Decoupling | Strong Decoupling | Expansionary Negative Decoupling | Expansionary Negative Decoupling | Strong Decoupling | Strong Decoupling |
| ***Washington*** | Growing Link | Strong Decoupling | Strong Decoupling | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Wisconsin*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***West Virginia*** | Weak Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling | Strong Decoupling |
| ***Wyoming*** | Weak Decoupling | Strong Decoupling | Recession Link | Strong Decoupling | Strong Decoupling | Recession Decoupling | Strong Decoupling | Strong Decoupling | Recession Decoupling |

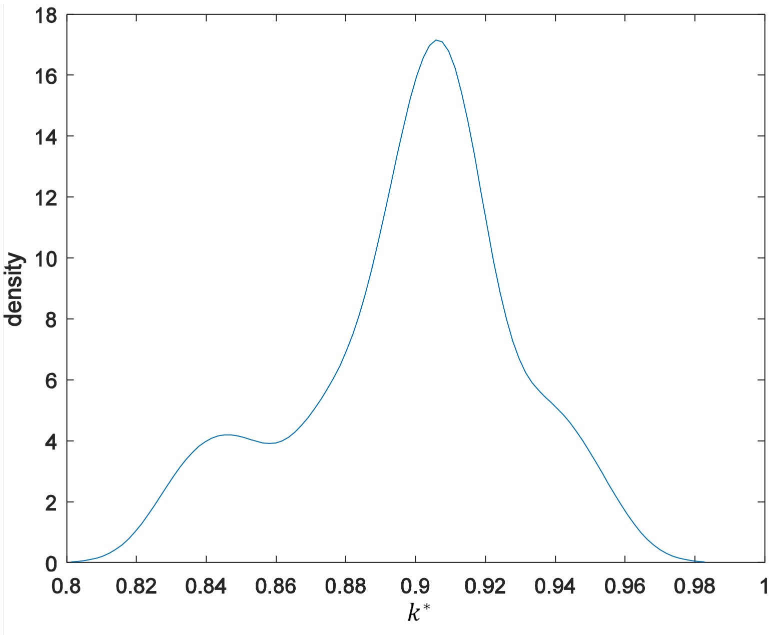
unimodal distribution for the 10%, 25% and 75% quantile, but with a bimodal distribution of the 90% quantile. The reported positive eco-technical change complements our previous decoupling analysis suggesting that states' growth rates in the majority of the cases are obtained by reducing the negative environmental impact. Our findings complement those presented by Leflaive (2008) suggesting that from early 1990s US government have invested on a flexible innovation system policies, which are based on decentralization and on the enhancement of states' environmental technology innovation.

**Figure 5**: Densities of the US states' productivity levels and components by quantile




Finally, the marginal posterior of $k$ is reported in Fig. 6 (see Appendix for details). In fact, as stressed by Assaf et al. (2020) and Tsionas et al. (2020) the main advantage of the methodology adopted is the utilization of Bayesian estimation treating the quantile $k$ as a parameter. Specifically, Fig. 6 presents the optimal quantile $k^*$ as has been derived from its marginal posterior distribution. The results suggest that the dominant model is near the 90th percentile. Given that the 90% quantile is the most likely (or optimal) gives an idea about a representation regarding our sample. However, as clearly stated by Assaf et al. (2020) other functions at different values of quantiles must also be examined in order to provide us with different valuable insights of the states' eco-inefficiencies fluctuations.

**Figure 6**: Density of the marginal posterior of the optimal quantile




**5. Conclusions**

The paper adopts the newly introduced quantile stochastic frontier framework (Tsionas, 2020) in order to construct eco–efficiency measures for the U.S. states over the period 1990-2017. Based on the quantile regression in the stochastic frontier framework we present how eco-efficiency indicators can be constructed. The eco-efficiency indicators are quantile specific and are based both on asymmetric Laplace and on half-normal errors. Moreover, we treat quantiles as parameters and with the adoption of Markov chain Monte Carlo (MCMC) methods we estimate posterior distributions. In principle, the examination of different eco-efficiency quantiles allows us to have a better view of the estimated eco-efficiencies. However, with the obtained posterior distribution of the quantiles we are able to indicate the optimal (most likely) quantile. In addition when we estimate the eco-efficiencies we utilize a globally concave symmetric generalized McFadden form, which provide us with greater flexibility compared to other functional forms used in SFA studies (i.e. Cobb-Douglas, translog, etc.).

Our empirical findings suggest that even though in absolute terms the U.S. economy is regarded as one of the largest emitter of pollutants (Rodríguez et al., 2018), our decoupling analysis indicates that over the examined period U.S. states are in a decoupling juncture for all the evaluated pollutants. This finding is aligned with our eco-productivity analysis suggesting positive eco-productivity growth among the states. In fact this outcome is also a result from the environmental regulations of US government, which are based on the enhancement of flexible innovation system policies among the U.S. regions (Leflaive, 2008). As the United States Environmental Protection Agency (EPA) reports[6] from 1990 to 2017, U.S. has managed to reduce their emissions by 74% through the implementation of different environmental policies.

**Appendix:** *Posterior conditional distributions for Gibbs sampling.*

As explained by Tsionas (2020), from equation (11) we can obtain:

$$p_k(\sigma_\mu|\gamma,\sigma_\psi,\{\psi_i\}_{i=1}^n,\{w_i\}_{i=1}^n,\phi,Z) \propto \sigma_\mu^{-(n+\underline{n}_\mu+1)} \exp\left\{-\frac{K_\mu}{2\sigma_\mu^2}\right\}, \quad \text{(A1)}$$

where $K_\mu = \underline{k}_v + \sum_{i=1}^n \frac{(\phi_i - z'_i\gamma+\psi_i)^2}{w_i}\left[kI_{[0,\infty)}(\phi_i - z'_i\gamma+\psi_i) + (1-k)I_{(-\infty,0]}(\phi_i - z'_i\gamma+\psi_i)\right]$,

$$p_k(\sigma_\psi|\gamma,\sigma_\mu,\{\psi_i\}_{i=1}^n,\{w_i\}_{i=1}^n,\phi,Z) \propto \sigma_\psi^{-(n+\underline{n}_\psi+1)} \exp\left\{-\frac{K_\psi}{2\sigma_\psi^2}\right\}, \quad \text{(A2)}$$

where $K_\psi = \underline{k}_\psi + \sum_{i=1}^n \psi_i^2$,

$$p_k(w_i|\gamma,\sigma_\mu,\sigma_\psi,\{\psi_i\}_{i=1}^n,\phi,Z) \propto \exp\left\{-w_i - \Omega_i w_i^{-1}\right\}, w_i > 0, \quad \text{(A3)}$$

where $\Omega_i = \frac{1}{2\sigma_\mu^2}(\phi_i - z'_i\gamma+\psi_i)^2\left[kI_{[0,\infty)}(\phi_i - z'_i\gamma+\psi_i) + (1-k)I_{(-\infty,0]}(\phi_i - z'_i\gamma+\psi_i)\right]$,

$$p_k(\psi_i|\gamma,\sigma_\mu,\sigma_\psi,\{w_i\}_{i=1}^n,;\phi,Z) \propto \exp\left\{-\frac{(\lambda_i+\psi_i)^2}{2\sigma_\mu^2 w_i}\left[kI_{[0,\infty)}(\lambda_i+\psi_i) + (1-k)I_{(-\infty,0]}(\lambda_i+\psi_i)\right] - \frac{1}{2\sigma_\psi^2}\psi_i^2\right\}, \psi_i \geq 0, \quad \text{(A4)}$$

where $\lambda_i = \phi_i - z'_i\gamma$.

The conditional posterior distribution of $\gamma$ is given by:

$$p_k(\gamma|\sigma_\mu,\sigma_\psi,\{\psi_i\}_{i=1}^n,\{w_i\}_{i=1}^n,\phi,Z) \propto \exp\left\{-\frac{1}{2\sigma_\mu^2}\sum_{i=1}^n \frac{(\phi_i - z'_i\gamma+\psi_i)^2}{w_i}\left[kI_{[0,\infty)}(\phi_i - z'_i\gamma+\psi_i) + (1-k)I_{(-\infty,]}(\phi_i - z'_i\gamma+\psi_i)\right]\right\}.$$

(A5)

Therefore,

$$p_k(\gamma|\sigma_\mu,\sigma_\psi,\{\psi_i\}_{i=1}^n,\{w_i\}_{i=1}^n,\phi,Z) \propto \exp\left\{-\frac{1}{2\sigma_\mu^2}(\phi+\psi-Z\gamma)'W^{-1}(\phi+\psi-Z\gamma)\left[k^{n_o(\gamma,\psi)} + (1-k)^{n-n_o(\gamma,\psi)}\right]\right\},$$

where $n_o(\gamma,\psi) = \sum_{i=1}^n I_{[0,\infty)}\,(\phi_i - z'_i\gamma+\psi_i)$, $W = diag[w_1,\ldots,w_n]$. (A6)

[6] https://www.epa.gov/air-trends/air-quality-national-summary

From (A6) when $k = 1$ (or zero) we can obtain:

$$\gamma | \sigma_\mu, \sigma_\psi, \{\psi_i\}_{i=1}^n, \{w_i\}_{i=1}^n, \phi, Z \sim \mathcal{N}_q(\hat{\gamma}, M), \tag{A7}$$

where $\hat{\gamma} = (Z'W^{-1}Z)^{-1}Z'W^{-1}(\phi + \psi)$, $M = \sigma_\mu^2(Z'W^{-1}Z)^{-1}$.

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