Dykman *et al.* Reply: The phase shift between the periodic response of a system (the signal) and the periodic driving force that gives rise to it is defined *uniquely* in statistical physics. The ensemble-averaged signal $\langle q(t)\rangle = \sum_n a(n)\cos[n\Omega t + \phi(n)]$, where $2\pi/\Omega$ is the period of the force. All of the phases $\phi(n)$ can be measured experimentally. In the commonly considered case of a cosine force $A\cos(\Omega t)$ and a nearly cosine signal, the term "phase shift" refers to $\phi \equiv \phi(1)$. There is no ambiguity about this; neither are there two different phase shifts [1]. It was ϕ that was investigated both in [2] and in earlier theoretical papers [3,4]. Provided the periodic force is weak, ϕ can be expressed in terms of a linear susceptibility [5]. It was the finiteness of the phase ϕ that Gammaitoni *et al.* claimed [6], wrongly [2], to have been "ruled out as apparently spurious" in stochastic resonance.

The topic of our Letter [2] was phase shifts *in stochastic resonance* (SR), a noise-induced enhancement of the signal-to-noise ratio R that is significant when [4] $\Omega \ll \tau_r^{-1}$, where τ_r^{-1} ($=1$ for the overdamped bistable system of [2]) is the reciprocal intrawell relaxation time (not "librational frequency"). In the range $\Omega\tau_r \gg 1$ [1], on the other hand (actually, $\Omega\tau_r > \sim 0.5$ for the system of [2]), SR does not occur; see Fig. 1, inset. In contrast to the exponentially fast rise of $-\phi(D)$ with increasing D (followed by a slower decrease) observed [2] for small A and Ω, $-\phi(D)$ for *large* Ω (Fig. 1) displays a much shallower maximum (but nonetheless increases, rather than decreases [1], for small D; we have noted that the signal, too, initially increases with D). The steep initial rise of $-\phi(D)$ for *small* Ω [2] is associated with the onset of the noise-induced interwell transitions that are responsible for SR; moreover, ϕ is evidently a more sensitive indicator of these transitions than R. The monotonic decrease of $-\phi(D)$ in [7] does not contradict this result, because the signal from the experiment had apparently [8] been passed through a two-state filter prior to analysis, thus removing the effect of the intrawell vibrations and mimicking the two-state approximation of earlier theories [3,4].

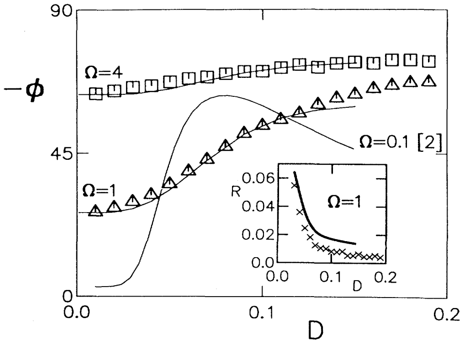

FIG. 1. Phase shift $-\phi$ of the signal induced by a weak periodic force of frequency Ω in the overdamped bistable system (1) of [2], as a function of noise intensity D. Inset: Signal-to-noise ratio R as a function of D at large Ω. The data points are by digital simulation; the curves are LRT based on Eqs. (6)-(10) of [2], to first order in $L_n(\omega)$.

Applied consistently to the SR problem—which involves more than merely a linearization of the transition probabilities as suggested in [9]—our linear response theory (LRT) approach has been shown [2,10] to yield good agreement with experimental measurements of the amplitude and phase of the signal over a wide range of parameters for small amplitudes of the force.

We acknowledge valuable discussions with N. D. Stein.

M. I. Dykman,(1) R. Mannella,(2)
P. V. E. McClintock,(3) and N. G. Stocks(3)
(1)Department of Physics
Stanford University
Stanford, California 94305
(2)Dipartimento di Fisica
Università di Pisa
Piazza Torricelli 2, 56100 Pisa, Italy
(3)School of Physics and Materials
Lancaster University
Lancaster, LA1 4YB, United Kingdom

Received 2 November 1992
PACS numbers: 05.40.+j, 02.50.−r

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