Dividend Taxation and Financial Business Cycles^{*}

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Abstract

We examine the interactions between different dividend tax systems and financial shocks in a dynamic stochastic general equilibrium (DSGE) model with an occasionally-binding investment credit limit. We show that dividend taxes largely determine the collateral value of assets, thereby occasionally distorting investment decisions and altering the propagation of financial shocks. Permanently lower dividend taxes dampen financially-driven business cycles in a state-contingent fashion. They also help explain substantial macroeconomic asymmetries following equally-sized expansionary and contractionary financial shocks.

Keywords: Occasionally-Binding Borrowing Constraints; Investment; Asset Prices; Financial Shocks.

JEL Classification: E22; E32; E44; H25; H30.

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1 Introduction

Motivated by the observation that both the 1980s financial liberalization period and the 2007-2008 global financial crisis coincided or were preceded by major U.S. tax reforms, this note studies the impact of different dividend tax systems on the propagation mechanisms of financial shocks.

We evaluate the role of permanently lower dividend taxes in explaining investment dynamics and financial business cycles in a production-based DSGE model augmented for an occasionallybinding investment credit limit – à la Ghilardi and Zilberman (2024). Tying investment loans to collateralized capital through Tobin's q establishes an occasionally-tight relationship between τ^D , q, the credit constraint tightness ϕ , and investment I. Importantly, we argue that a permanent reduction in τ^D considerably attenuates the response of key variables to favorable loan-to-value (LTV) shocks associated with credit regime switching. However, such lower tax system only marginally mitigates real fluctuations following adverse shocks that confine corporate firms within constrained credit regions. Thus, the $\tau^D - q - \phi - I$ conduit also explains empirically-observed asymmetrical business cycle traits following equally-sized expansionary and contractionary credit disturbances (Jensen et al. 2020).

Our model offers an alternative interpretation for the moderated real fluctuations observed in the 1980s, which were linked to permanent dividend tax reforms and extensive financial innovations (McGrattan and Prescott 2005; Atesagaoglu 2012). Additionally, we shed light on the probable limited influence of the substantial 2003 dividend tax cut on investment dynamics during the 2007-2008 financial crisis.

We contribute to the literature studying the impact of corporate tax changes on the macroeconomy by considering how payout taxes influence financial business cycles. Santoro and Wei (2011) inspect the transmission channels of productivity shocks under various corporate profit and distribution tax systems. In their model, payout taxes follow the 'new' view wherein permanent τ^D alterations have no impact on investment (e.g., McGrattan and Prescott 2005). Here, the 'old' view, where τ^D distorts I (e.g., Poterba and Summers 1983), holds so long as firms seek external investment finance. Our methodology nevertheless largely follows Santoro and Wei (2011) as we investigate the transmission mechanisms of financial shocks, as opposed to technology shocks, under a permanently high and low τ^D . We specifically extend the Ghilardi and Zilberman (2024) setup by making the LTV ratio stochastic and assuming flat dividend taxes.

We also relate to the literature highlighting the prominent role of occasionally-binding credit frictions in explaining financial business cycles and corresponding asymmetries (Guerrieri and Iacoviello 2017; Jensen et al. 2020). For example, Jensen et al. (2020) demonstrate that the decrease in macroeconomic volatility largely hinges on the attributes of expansions, which diminish in magnitude due to the relaxation of collateral constraints. To our knowledge, we are the first to show how fluctuations and volatilities arising from estimated positive and negative LTV shocks are influenced by the prevailing dividend tax rate.

2 The Model

Households preferences are:

$$U(C_t, C_{t-1}) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - hC_{t-1})^{1-\varsigma}}{1-\varsigma},$$
(1)

where $\beta \in (0, 1)$ is the discount factor, ς is the curvature of the utility function, and h > 0 is the degree of habit formation.

Households own all initial corporate shares S_t , with the price per-stock given by p_t . Stock ownership entitles agents to earn an after-tax (net) dividend per share of $\bar{D}_t \equiv (1 - \tau^D) D_t$, with D_t the pre-tax dividend. Furthermore, intraperiod corporate bonds B_t pay a gross return $R_t = 1$. The budget constraint is thus:

$$C_t + p_t S_{t+1} + B_t \le \left[\left(1 - \tau^D \right) D_t + p_t \right] S_t + R_t B_t + W_t H_t + T_t,$$
(2)

where W_t denotes the wage rate, and T_t are government lump-sum transfers. Households allocate all their time endowment, normalized to 1, to productive work; $H_t = 1$.

Firms produce output Y_t by combining capital K_{t-1} and labor H_t :

$$F(K_{t-1}, H_t) = Y_t = K_{t-1}^{\alpha} H_t^{1-\alpha},$$
(3)

with $\alpha \in (0, 1)$. Capital evolves according to:

$$K_t = (1 - \delta) K_{t-1} + I_t, \tag{4}$$

where $\delta \in (0,1)$ is the depreciation rate. Denoting τ^{I} as a constant investment tax-subsidy, the pre-payout tax dividend is:

$$D_{t} = Y_{t} - W_{t}H_{t} - (1 + \tau^{I})I_{t} - \Phi\left(\frac{I_{t}}{K_{t-1}}\right) + B_{t} - R_{t}B_{t},$$
(5)

where $\Phi\left(\frac{I_t}{K_{t-1}}\right) = \frac{\gamma}{2} \left(\frac{I_t}{K_{t-1}} - \delta\right)^2 K_{t-1}$; $\gamma > 0$ denote capital adjustment costs. Introducing τ^I facilitates a more accurate steady-state calibration of both q and the financial constraint tightness, proxied by a credit spread measure, without any loss of generality (Ghilardi and Zilberman 2024).

Assuming $S_t = 1$ for all t, firms can use internal funds or issue more debt to finance investment. In the case of external borrowing and for $R_t = 1$ and $B_t \equiv I_t$, each firm faces the occasionallybinding borrowing constraint:

$$I_t \le \theta_t q_t K_{t-1},\tag{6}$$

where q_t is a market-based measure of Tobin's q (Ghilardi and Zilberman 2024). θ_t is the stochastic LTV ratio that follows:

$$\theta_t = (\theta)^{1-\rho_\theta} (\theta_{t-1})^{\rho_\theta} \exp(\varepsilon_{\theta,t}), \qquad (7)$$

with $\theta \in (0,1)$ the steady-state borrowing limit, $\rho_{\theta} \in (0,1)$ the degree of persistence, and $\varepsilon_{\theta,t} \sim i.i.d.\mathcal{N}(0,\sigma_{\theta}^2)$.

The government sets corporation taxes and rebates $T_t = \tau^D D_t + \tau^I I_t$ to households. Marketclearing requires:

$$Y_{t} = C_{t} + I_{t} + \frac{\gamma}{2} \left(\frac{I_{t}}{K_{t-1}} - \delta \right)^{2} K_{t-1}.$$
 (8)

3 Investment and Asset Prices with Financial Frictions and Dividend Taxes

The stock Euler equation determines the firm's value p_t , which is equal to the present discounted value of after-tax dividends:

$$p_{t} = \beta E_{t} \frac{\Lambda_{t+1}}{\Lambda_{t}} \left[p_{t+1} + (1 - \tau^{D}) D_{t+1} \right], \qquad (9)$$

where $\beta E_t \Lambda_{t+1} / \Lambda_t$ is the stochastic discount factor, and Λ_t is the forward-looking marginal utility of consumption:

$$\Lambda_t = (C_t - hC_{t-1})^{-\varsigma} - \beta E_t h (C_{t+1} - hC_t)^{-\varsigma}.$$
(10)

The firm maximizes the present discounted value of $\overline{D}_t \equiv (1 - \tau^D) D_t$. Denoting q_t and ϕ_t as the Lagrange multipliers on (4) and (6), respectively, and using the functional forms for $F(K_{t-1}, H_t)$ and $\Phi(I_t/K_{t-1})$, we derive the following proposition.

Proposition 1 The capital-investment Euler equation is:

$$(1 - \tau^{D}) \left[1 + \tau^{I} + \gamma \left(\frac{I_{t}}{K_{t-1}} - \delta \right) + \frac{\phi_{t}}{(1 - \tau^{D})} \right]$$

$$= \beta E_{t} \frac{\Lambda_{t+1}}{\Lambda_{t}} \left(1 - \tau^{D} \right) \left\{ \begin{array}{c} \alpha \frac{Y_{t+1}}{K_{t}} + \frac{\gamma}{2} \left[\left(\frac{I_{t+1}}{K_{t}} \right)^{2} - \delta^{2} \right] \\ + \left[1 + \tau^{I} + \gamma \left(\frac{I_{t+1}}{K_{t}} - \delta \right) + \frac{\phi_{t+1}}{(1 - \tau^{D})} \right] \left[(1 - \delta) + \theta_{t+1} \phi_{t+1} \right] \end{array} \right\}. (11)$$

Moreover, with $\phi_{t+j} > 0$ for j > 1, the recursively forward solution to q_t satisfies:

$$q_{t} = E_{t} \sum_{j=1}^{\infty} \left\{ \left[\prod_{i=0}^{j-1} \beta^{i+1} \frac{\Lambda_{t+i+1}}{\Lambda_{t+i}} \right] \left(1 - \tau^{D} \right) \left(1 + \tau^{I} - \delta + \theta_{t+j} \phi_{t+j} \right)^{j-1} u_{t+j} \right\},$$
(12)

where the user-cost-of-capital is defined as:

$$u_{t+j} = \alpha \frac{Y_{t+j}}{K_{t+j-1}} + \frac{\gamma}{2} \left[\left(\frac{I_{t+j+1}}{K_{t+j}} \right)^2 - \delta^2 \right].$$
 (13)

Finally, the optimal investment rate around the binding steady-state equilibrium is derived from the slackness condition:

$$\phi_t \left(\theta_t q_t K_{t-1} - I_t \right) = 0; \quad \phi_t \ge 0.$$
 (14)

Unlike Santoro and Wei (2011), proportional dividend taxes have an *asymmetric* impact on the marginal cost (left-hand side of (11)) and benefit of investment (right-hand side of (11)), implying that the 'old' view prevails when $\phi > 0$. Intuitively, ϕ drives a wedge between the frictionless external capital valuation, $(1 - \tau^D)$, and the adjustment cost-adjusted q in the credit-constrained economy; $q_t = (1 - \tau^D) \left[1 + \tau^I + \gamma \left(\frac{I_t}{K_{t-1}} - \delta \right) \right] + \phi_t$. Lowering τ^D raises q, and enables additional borrowing and investment against the existing capital stock. However, part of this investment increase is mitigated because firms discount the future more heavily in a persistently relaxed credit environment, which incentivizes higher payouts instead. The discounted marginal value from relaxing the constraint is reflected by the interaction term $(1 - \tau^D) \theta_{t+j}\phi_{t+j}u_{t+j}$ for j > 1 in (12). Moreover, notice from (5), (9), and \bar{D}_t that the firm's value and net dividends rise by $(1 - \tau^D)$ following a tax cut on the one hand, but decrease by $\phi/(1 - \tau^D)$ through the impact of τ^D on I on the other.

Finally, by substituting $I_t/K_{t-1} = \theta_t q_t$ for $\phi_t > 0$ in (13), u_t itself is also altered by θ_t and q_t through the effect adjustment costs have on the user-cost-of-capital. Around the neighborhood of a credit-bound steady-state, θ_t modifies investment decisions and therefore results in a higher $\Phi(\cdot)$

regardless of whether the shock is expansionary or contractionary. Nevertheless, starting from a low τ^D associated with a higher market firm valuation, changes in $\theta_t q_t$ (i.e., the investment rate) are not necessarily met with a substantial rise in u_t . Consequently, following LTV shocks and under a reduced tax regime, investment fluctuations are mitigated also via a financially-augmented adjustment cost channel.

Proposition 2 For $\theta < \frac{\delta}{(1-\tau^D)(1+\tau^I)}$ there exists a unique credit-bound steady-state equilibrium satisfying:

$$\phi = \frac{\delta}{\theta} - \left(1 - \tau^D\right) \left(1 + \tau^I\right) > 0, \tag{15}$$

and:

$$K = \left\{ \frac{\alpha}{\left(1 + \tau^{I} + \frac{\phi}{(1 - \tau^{D})}\right) \left(\beta^{-1} - 1\right) + (1 + \tau^{I}) \delta} \right\}^{\frac{1}{1 - \alpha}},$$
(16)

$$\frac{I}{Y} = \delta K^{1-\alpha},\tag{17}$$

$$p = \frac{\beta \left(1 - \tau^{D}\right) D}{\left(1 - \beta\right)} = \frac{\delta}{\theta} K,$$
(18)

$$q = \frac{\delta}{\theta} = \left(1 - \tau^D\right) \left(1 + \tau^I\right) + \phi, \tag{19}$$

$$\bar{D} = \left(1 - \tau^D\right) \left(\alpha K^\alpha - \delta K\right).$$
(20)

For a low θ corresponding with $\phi > 0$, cutting τ^D reduces ϕ , thereby raising K, I/Y, and p until the point where $\phi = 0$. When $\phi = 0$, (16) to (20) collapse to Santoro and Wei's (2011) steady-state conditions.

4 Quantitative Analysis

4.1 Calibration and Model Fit

We calibrate the model at a quarterly frequency for the U.S., setting typically employed parameters in the business cycle literature: $\beta = 0.99$, $\delta = 0.025$, $\alpha = 0.30$, h = 0.8, $\varsigma = 4$, and $\gamma = 1.7$. The borrowing limit and investment tax-subsidy are jointly set to $\theta = 0.0266$ and $\tau^I = 0.19$, ensuring that across all tax systems examined in the experiments below, the economy starts from an initially binding steady-state equilibrium with $\phi > 0$ and $q = 0.94 = q_{1960-2019}$ (see Proposition 2 – particularly (15) and (19)). In our benchmark case, we set $\tau^D = 0.24$, matching the average marginal effective dividend tax rate calculated for the 1960-2019 period (McGrattan 2023). Using (15) and the specified values for δ , θ , τ^{I} , and τ^{D} , we then calculate $\phi = 0.035$, which aligns with the 1960-2019 mean spread between Baa corporate yields and 3-month treasury bill rates.

We estimate the persistence parameter and standard deviation of the financial shock to reproduce the standard deviation of the percentage change in nonfinancial corporate investment-to-GDP ratio during 1960:Q1-2019:Q4.¹ The estimated occasionally-binding model implies choosing $\rho_{\theta} = 0.788$ and $\sigma_{\theta} = 0.045$. Table 1 reports the statistics of key variables and compares them with their respective data equivalents. We also showcase the model-implied moments when $\tau^{D} = 0.30$, and $\tau^{D} = 0.215$ – the minimum tax rate ensuring asymptotic convergence given our steady-state calibration with $\tau^{I} = 0.19$ and $\phi > 0$.

Table 1 - Business Cycle Moments with Different Dividend Tax Rates				
	$E\left(\frac{I}{Y}\right)$	$E\left(\frac{\bar{D}}{Y}\right)$	$\sigma\left(\ln\left(\frac{I_t}{Y_t}\right)\right)\%$	$\sigma\left(\ln\left(\frac{\bar{D}_t}{Y_t}\right)\right)\%$
$\tau^D = 30\%$				
Model	0.173	0.089	1.94	2.64
$\tau^D = 24\%$ (benchmark)				
Data	0.174	0.071	1.63	4.24
Model	0.177	0.093	1.63	2.36
$\tau^D = 21.5\%$				
Model	0.179	0.095	1.27	1.89

Notes: i) Model standard deviations $\sigma(\cdot)$ are computed from 10,000 simulations keeping θ_t stochastic.

ii) $E(\cdot)$ correspond with the steady-state averages; E(X) = X.

iii) Data statistics (detrended) are extracted from FRED.

Overall, the model fits the data fairly well for the benchmark $\tau^D = 0.24$, despite $\ln(\bar{D}_t/Y_t)$ exhibiting lower volatility in the model. This is expected since we only estimate one shock to match $\sigma(\ln(I_t/Y_t))$, and our abstraction from fiscal depreciation allowances and other forms of time-varying wedges that may influence equity prices and net payouts. Finally, as a result of permanent tax reductions, $\sigma(\ln(I_t/Y_t))$ and $\sigma(\ln(\bar{D}_t/Y_t))$ decline, while the average I/Y and \bar{D}/Y increase.

4.2 Simulations

Positive LTV Shocks.– Figure 1 displays the impulse responses to a positive $4 \times \sigma_{\theta}$ shock. We compare the model dynamics under a relatively high tax environment, $\tau^{D,H} = 0.30$, with the

¹Following Atesagaoglu (2012), the term "GDP" refers to "nonfinancial corporate GDP".

fluctuations that arise in a lower tax regime, $\tau^{D,L} = 0.24$. The 6-percentage-point difference between the two tax systems approximately reflects the observed changes in effective marginal payout tax rates following the tax reforms of 1981, 1986, and 2003 (McGrattan 2023).

Under both tax regimes, the collateral constraint is initially binding in the steady-state. However, for $\tau^{D,L} = 0.24$, the long-run value of ϕ^L is 0.035, compared to $\phi^H = 0.107$ when $\tau^{D,H} = 0.30$, keeping all other parameters constant. Thus, in the lower tax world, the constraint becomes slack more frequently for the same given shock size, resulting in a strong attenuation impact on all key variables.²

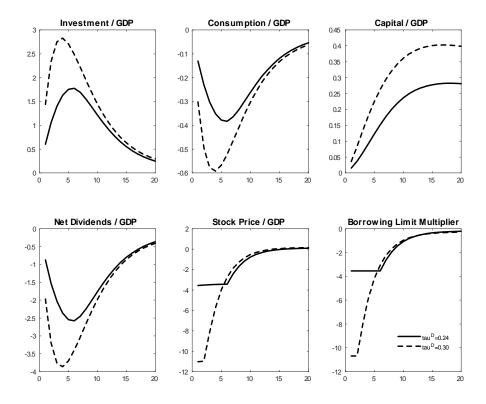


Figure 1: Apart from the borrowing limit multiplier, which is measured in percentage-point deviations, all other variables are measured in percentage deviations. Deviations are with respect to the *different* steady-states corresponding with the low and high tax rates.

The general equilibrium effects of the financial shock can be explained as follows. As θ_t rises, ϕ_t falls. Asset prices decline as capital becomes less valuable as a collateralized asset used for the purpose of obtaining investment loans. Simultaneously, the laxer lending constraint allows firms to take on more debt and consequently to raise investment, producing a cutback in dividend payouts.

²Mitigated fluctuations and volatilities in investment and asset prices under a lower τ^{D} also feature following productivity shocks, but to a smaller extent.

Furthermore, consumption slightly drops upon the shock impact, given that agents substitute away from spending and into further investment.³

The looser credit regime prevailing under $\tau^{D,L} = 0.24$ supports a higher steady-state stock price and a larger investment-to-GDP ratio (see Proposition 2 and Table 1). Because p and I play crucial roles in smoothing consumption, the higher values obtained for these variables when ϕ is lower imply significantly muted responses to a given credit shock. Importantly, ϕ remains at zero for 6 periods in the low tax environment, while staying at zero for only 2 period in the high tax regime. With the constraint slack for a longer duration and asset prices remaining relatively higher, firms pay out dividends from retained earnings and limit investment. Particularly, dividends fall by a smaller margin, while the investment-to-GDP response exhibits lumpiness and moderated fluctuations compared to the dynamics with $\tau^{D,H} = 0.30$. Moreover, in the slack environment, the constraint affects firms only through expectations. The longer is the duration of the slack regime, the more firms discount the constraint as implied from Proposition 1. Accordingly, the responses of key variables under $\tau^{D,L} = 0.24$ are not as dramatic as the reactions observed under $\tau^{D,H} = 0.30$. The upshot is that a permanently relaxed tax system makes the economy less prone to aggregate fluctuations instigated by positive financial disturbances. Considered more broadly, potential excessive debt levels associated with surging investment in the short-run can be contained by a permanently reduced τ^D that brings the economy closer to a slack credit regime.

Negative LTV Shocks.- Figure 2 displays the model dynamics following an adverse $4 \times \sigma_{\theta}$ credit shock under the same $\tau^{D,H}$ and $\tau^{D,L}$. A lower τ^{D} marginally mitigates investment-to-GDP fluctuations following negative shocks that tighten the borrowing constraint and that do not result in temporary credit regime switching. The very small attenuation mechanism in I/Y in this case is largely driven by the financially-augmented adjustment cost channel highlighted in Section 3.

³The negative comovement between I/Y, C/Y, and asset prices following financial shocks is shared with other notable works in the literature (e.g., Kiyotaki and Moore 2019).

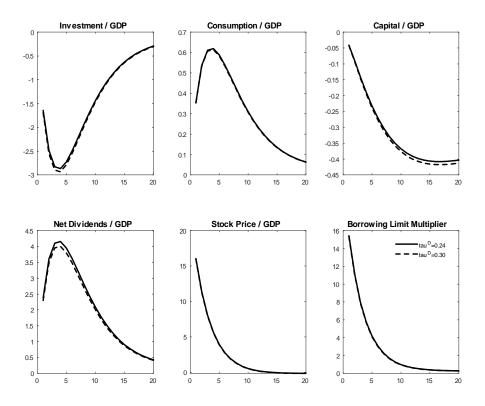


Figure 2: see note below Figure 1.

Asymmetric Effects of Financial Shocks.– Our final counterfactual exercise concerns the asymmetric dynamic responses of key variables to positive and negative LTV shocks of the same $4 \times \sigma_{\theta}$ magnitude. We set $\tau^{D,L} = 0.24$ to highlight the intuition behind Figure 3. After a favorable shock, the initially binding constraint turns slack and attenuates the otherwise more sizeable fluctuations arising in the credit-constrained model (see also solid lines in Figure 1). An adverse shock, on the other hand, raises ϕ and reduces the collateral capacity of the liquidity-constrained firm. The firm accordingly distributes more dividends and cuts back on investment. Quantitatively speaking, at its peak (trough), investment-to-GDP rises (falls) by +1.78% (-2.87%) following same-sized shocks moving in opposite directions. In the case of higher dividend taxes associated with less frequent regime switching, the model produces more symmetric macroeconomic reactions. Put differently, a permanently lower τ^D that pushes the economy closer to a slack credit environment can explain substantial macroeconomic asymmetries.

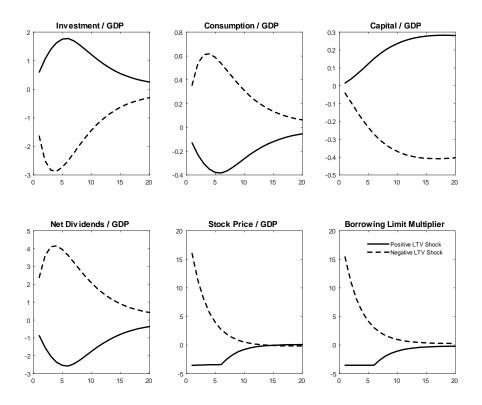


Figure 3: Apart from the borrowing limit multiplier, which is measured in percentage-point deviations, all other variables are measured in percentage deviations. Deviations are with respect to the *same* credit-bound steady-state with $\tau^{D,L} = 0.24$ and $\phi = 0.035$.

5 Conclusion

This note has clarified the potential role of different dividend tax systems in explaining the transmission channels of financial shocks. We contend that reductions in dividend taxation over the past five decades could have partly accounted for the vastly different and asymmetrical investment responses to credit disturbances.

Our analysis unveils two key policy implications. Firstly, a lower dividend tax regime may serve as a long-term macroprudential tool, countering the upward spirals of corporate investment-debt levels typically associated with positive credit cycles. Simultaneously, if the LTV shock is instead modeled as a state-contingent macroprudential instrument (e.g., Jensen et al. 2018; Rubio and Yao 2020), then permanently lower taxes could impede the efficacy of dynamic macroprudential policies aimed at stabilizing financial business cycles.

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