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Higher Education Finance**

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# **THE EVALUATION OF WELFARE UNDER ALTERNATIVE MODELS OF HIGHER EDUCATION FINANCE**

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## ABSTRACT

A model is developed which allows us to examine the welfare effects of alternative methods of financing access to higher education. Under an extreme specification of the social welfare function, it is shown that it does not matter whether higher education is financed privately or through the exchequer. Under a more general specification of the social welfare function, conditions may be derived under which (a) private finance is more welfare enhancing than public finance and (b) public finance is more welfare enhancing than private finance. Empirical estimates of the social welfare function are used to draw policy conclusions.

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# **THE EVALUATION OF WELFARE UNDER ALTERNATIVE MODELS OF HIGHER EDUCATION FINANCE**

## ***Introduction***

There has been a considerable recent literature on the appropriate means for funding higher education. Recent examples include the Greenaway and Haynes (2000) report, Chapman (1997) and Barr and Crawford (1998). These have generally been supportive of reforms that shift the burden of paying for higher education away from the general taxpayer and toward students. The argument is, essentially, that the main beneficiary should bear the main burden of the cost of tuition. This follows from the user pays principle, and - to mainstream economists, at least - has not been regarded as particularly contentious. This is so much so, that economists have not hitherto challenged themselves with the question of how to come up with estimates of the benefits of the user pays principle in this context.

This paper represents a first attempt to tackle this question. In order to do so, I shall develop a model which is quite general, and which is capable of accommodating conditions under which private funding of higher education is more efficient, equally efficient, or less efficient than the publicly funded alternative. The model will allow us to appreciate why the funding of higher education has become a politically contentious issue in many countries, in spite of the near consensus amongst economists in favour of the user pays principle. Finally, the model allows back-of-the-envelope calculations to be made concerning the welfare effects of various funding mechanisms.

The paper proceeds as follows. The theoretical framework is set out in the next section. This is followed by an empirical section which presents the back-of-the-envelope calculations. The paper ends with a conclusion that sets out a number of caveats that must attach to this analysis, along with suggestions for future research.

### *The model*

Consider the following, extremely simple, model of education finance. Individual  $i$  receives disposable income of  $Y_i$  where

$$Y_i = (Y_0 + s_i\theta_i)(1-\tau)$$

where  $Y_0$  is a constant,  $s_i$  is a binary variable that indicates whether or not  $i$  has attended tertiary schooling,  $\theta_i$  is a stochastic variable which is assumed to follow a uniform distribution and which varies between some strictly negative number and some strictly positive number, and  $\tau$  is the (proportional) rate of income tax. Tax in this model is paid solely for the purposes of financing education. It would be possible for the tax rate to equal zero if all education were paid for privately. Alternatively, if the tax rate exceeds zero there is some element of subsidy for education. We shall suppose that there is a single period, and that incomes and expenditures are incurred at the end of this period.

Education is assumed to take place instantaneously. This means that educated and uneducated individuals alike have the opportunity to earn an income. Direct costs of

education are covered by the cost term,  $c_0$ . This cost may be paid entirely by individuals, for example by securing a loan - in which case the tax rate is zero and the loan must be repaid at the end of the period. Or the cost may be borne entirely through the tax system. In this case the government pays for each educated worker's education, and the rate of proportional tax is set so as to offset this cost exactly.

For each worker,  $i$ , net income is defined as  $Y_i - c_i$  where  $c_i$  is the private cost of education to  $i$  in the  $t$ th period. Where education is paid for privately, this will either be  $c_0$ , if the individual is educated, or zero if she is not. In this case  $\tau=0$ . Where education is paid for entirely through the tax system, however,  $c_i$  will equal zero for all individuals - but in this case  $\tau>0$ .

Suppose individual  $i$  undertakes schooling iff

$$(Y_0 + s_i\theta_i)(1-\tau) - c_i \geq Y_0(1-\tau)$$

that is, iff the discounted value of post-schooling income (net of costs) is at least as great as the discounted value of income if no schooling is undertaken.

In solving the central problem of this paper, it will be necessary to work from a definition of social welfare. To provide a general definition, let social welfare,  $W$ , be defined as a weighted sum of the net incomes of all workers. Unit weight is attached to the incomes of uneducated workers, while a weight of  $\sigma$  is attached to the incomes of educated workers. This specification of the welfare function is quite general:

setting  $\sigma=1$  implies a utilitarian welfare function, while setting lower values of  $\sigma$  implies the attachment of a higher weight to the net income of the less well educated workers. As we shall see later, the most interesting values of  $\sigma$  lie in the range  $0 < \sigma < 1$ .

Assume that the government chooses  $\tau$  to maximise  $W$ . The distribution of  $\theta$  has supports  $\theta_{\min}$  and  $\theta_{\max}$ . Denote by  $n$  the population size.

Consider first the case where all education is privately financed, so that  $\tau=0$ . Those individuals who can augment their income by more than  $c_0$  by undertaking education, and paying for it, will choose to do so. The proportion of the population that does not undertake education is  $(c_0 - \theta_{\min}) / (\theta_{\max} - \theta_{\min})$ , and each of these individuals receives an income of  $Y_0$ . The remainder of the population receives an income which is distributed between  $Y_0 + c_0$  and  $Y_0 + \theta_{\max}$ . Hence total social welfare is given by

$$W = n \left\{ Y_0 (c_0 - \theta_{\min}) + \sigma \left[ Y_0 + (\theta_{\max} + c_0) / 2 - c_0 \right] (\theta_{\max} - c_0) \right\} / (\theta_{\max} - \theta_{\min})$$

Turn now to consider the case in which all education is funded through the tax system. This is a little more complicated, because it entails solving the model for the optimal rate of taxation. Suppose, as before, that those undertaking education are those at the top end of the  $\theta$  distribution, and suppose also that the proportion who undertake education is given by  $\lambda$ .

Total tax revenue is given by

$$\tau n \{ Y_0 + \lambda [\theta_{\max} - \lambda(\theta_{\max} - \theta_{\min})/2] \}$$

since the term in curly brackets is the pre-tax income of the typical individual. The total cost of education is  $c_0 \lambda n$ . Setting tax revenue equal to government expenditure allows us to solve for  $\lambda$ , which must of course lie within the unit interval. Hence

$$\lambda = \{ \tau \theta_{\max} - c_0 \pm \sqrt{(\tau \theta_{\max} - c_0)^2 + 2\tau^2 Y_0 (\theta_{\max} - \theta_{\min})} \} / \tau (\theta_{\max} - \theta_{\min})$$

Social welfare is given by the weighted sum of all disposable incomes, and hence

$$W = n(1-\tau) \{ (1-\lambda)Y_0 + \sigma \lambda Y_0 + \sigma \lambda [\theta_{\max} - \lambda(\theta_{\max} - \theta_{\min})/2] \}$$

Substituting  $\lambda$  out of this expression, we can investigate the effect that varying  $\tau$  would have on welfare. Substituting the welfare maximising value of  $\tau$  back into the social welfare function then allows the calculation of social welfare.

Some interesting observations are worth making at this stage. First, under a utilitarian regime (where  $\sigma=1$ ), the choice of funding system has no impact on welfare. Since social welfare is an unweighted sum of net incomes, it does not matter who pays for the education - so long as the socially optimal number of workers get educated (and here that is assured by choice of the tax rate), the level of economic welfare will be the same regardless of funding mechanism. Who pays differs according to regime, but who pays is not interesting given the nature of the social welfare function. This means that, if we adopt a utilitarian social welfare function, the question of whether education is paid for through the tax system or privately is a red herring.

Interestingly, a Rawlsian social welfare function also has the property that it does not matter whether education is publicly or privately financed. If it is privately financed, those who can benefit from it will invest in it, leaving the net income of the remainder unaffected. Meanwhile, if it were publicly financed, those who do not undertake education would be made worse off by tax payments; so in the Rawlsian model with public finance, nobody would receive education, and the utility of the poorest member of society would be the same as would be the case under private funding of education.

The same is not true, however, for more general cases in which  $\sigma$  is less than one. This is most easily appreciated by consideration of some empirical examples, and these form the basis of the next section of the paper.

### ***Empirical analysis***

Consider the following values for the key parameters of the model. Let  $\theta_{\max}=200000$ ,  $\theta_{\min}=-700000$ ,  $Y_0=300000$ ,  $c_0=25000$  and  $n=30$  million. The value of  $Y_0$  may be interpreted as the discounted value of lifetime earnings for the group of uneducated workers. If money values are measured in pounds, then the figures we have here may be thought of as a rough approximation to the United Kingdom context. Other assumed values of these parameters could clearly be used in order to reflect the position in other countries.

If  $\sigma=1$  (the utilitarian case),  $W=9.5104 \times 10^{12}$  in both the public finance and private finance scenarios. When financed through taxes, the optimal value of  $\tau$  in this case is 0.015, yielding  $\lambda=0.19$ .

Consider now some cases in which  $\sigma < 1$ . To begin, consider what would happen if  $\sigma=0.975$ . Under private finance,  $W=9.4539 \times 10^{12}$ , while under public finance,  $W=9.4518 \times 10^{12}$  with  $\tau=0.014$  and  $\lambda=0.1800$ . Clearly in this case private finance offers a higher level of welfare than does public finance. Notice that, under public finance, the level of education has fallen below the level that obtains under private finance.

Now consider the case where  $\sigma=0.75$ . This yields a value for  $W$  of  $8.9453 \times 10^{12}$  in the private finance scenario, but a value of  $9.0571 \times 10^{12}$  under public finance. The optimal value of  $\tau$  under public finance is 0.006, giving a value of  $\lambda$  of 0.0750. The level of education under public finance has fallen further. So while public finance offers greater welfare in this example than does private finance, it also provides a solution that yields a lower average level of education in the workforce and lower average earnings.

Why does this reversal take place from a situation where private finance is best for welfare to one in which public finance is better? Under private finance, those who can benefit from education do so. Under public finance, education is offered only to the extent that it raises social welfare. If the weight attached by society to the welfare of the richer (more highly educated) group is sufficiently low, then so will be society's

investment in education. More people will then belong to the less highly educated group, and the greater weight that these extra people have in the social welfare function more than offsets the loss of income to the highly educated group.

To clarify further, consider the impact on social welfare of public and private finance respectively, where education levels are kept constant across the two funding regimes. This means that  $\lambda$  is constrained to be 0.19 and  $\tau$  is constrained to be 0.015 under public finance. In this case, with  $\sigma=0.975$ , welfare under public finance is  $9.4515 \times 10^{12}$ . This compares with the  $9.4539 \times 10^{12}$  reported earlier for the private finance case. Likewise, where  $\sigma=0.75$ , welfare under public finance is now  $8.9211 \times 10^{12}$ , compared with  $8.9453 \times 10^{12}$  under private finance. These figures make clear that, given society's investment in education, private finance is more efficient than public finance. Public finance becomes more efficient at lower levels of  $\sigma$  only because it allows a welfare enhancing (albeit not libertarian) cap to be put on the extent of educational investment. Where prospective students are able to migrate to buy education privately in other countries, the imposition of such a cap may not in any event be feasible.

Using the figures provided for the example above, and once more allowing tax to settle to its welfare maximising level, it is possible to establish the critical value of  $\sigma$  at which welfare is the same under both public and private finance. This value is 0.94.

From the above discussion, it is clearly important to have some idea of the true value of  $\sigma$ . Fortunately, we have recently been given a clue. Recent work by Alesina *et al.* (2001) involves the construction of an empirical happiness function in which

macroeconomic variables play a part. In particular, these authors study the impact of the mean income level and the distribution of income upon happiness. Using their results as a guide,<sup>1</sup> we shall assume in the sequel a value of  $\sigma=0.3114$ . It is worth noting in passing that this low value is in accord with other work conducted using happiness measures by Easterlin (1995). The evidence provided by numerous studies is that the time trend of happiness has not mirrored the upward trend in incomes. This is not to say that income does not matter - at any one point in time, higher income individuals tend to be happier than lower income individuals. But in the aggregate, the level of income does not seem to be a particularly strong determinant of happiness, though the distribution of income is.

Since 0.3114 is below the critical value (of  $\sigma=0.94$ ), the above results can be interpreted as a case (in welfare terms, not in terms of income maximisation) for public funding of higher education in the United Kingdom. If, however,  $\lambda$  and  $\tau$  are chosen by government to replicate the free market levels of education, rather than to maximise social welfare, public finance entails a welfare loss. With  $\sigma=0.3114$ , welfare would then be  $7.9538 \times 10^{12}$  under private finance and  $7.8872 \times 10^{12}$  under public finance. The welfare loss would therefore amount to a little under one per cent.

### ***Conclusion***

Numerous caveats ought to be attached to the above analysis. First,  $\theta$  may more realistically have a non-uniform distribution. Secondly, the model as it stands

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<sup>1</sup> See appendix.

comprises a single period; a multiperiod model in which education takes time and where loans are repaid with interest would clearly match more closely what we observe in the world. It would allow foregone earnings to be included in the model. It would also allow consideration of situations in which  $\lambda$  is not stable over time. Thirdly, the estimate of  $\sigma$  given above is likely to be imprecise; it is obtained from the results of Alesina *et al.* (2001) for the United States which themselves are not estimated with precision. Further work should be aimed at providing better estimates of the empirical social welfare function, and in particular on the role played by the level and distribution of income. Fourthly, the model as it stands does not allow for externalities due to education. Fifthly, the model as it stands does not accommodate the incentive effects of taxation, and in particular does not allow taxation to influence work effort. Finally, the types of funding mechanisms considered here are extremes - full private finance and full public finance only.

The figures reported above must therefore be treated with some considerable caution. All of these caveats could be removed by further development of the model, and this in itself sets an agenda for further research. In particular, we need to know more precisely what the determinants of social welfare are - and that knowledge needs to inform economic policy, not only in the sphere of educational finance, but widely.

The central aim of this paper has been modest - to establish whether or not it is worth having an argument about how higher education is financed. The conclusion that we can draw from the above analysis is unambiguous, however - it is worth having that debate.

## Appendix

Suppose that the income distribution may be characterised, extremely simply, by a group of educated workers who earn on average  $Y_1$ , and a group of uneducated workers who earn  $Y_2$ . The former group comprise  $\xi$  of the whole. Note that, in a model of this type, the Gini coefficient is given by

$$\frac{1}{2} - [\xi^2 Y_1 + (1-\xi)(1-\xi)Y_2] / 2[\xi Y_1 + (1-\xi)Y_2]$$

Linearising around  $\xi=0.1$ ,  $Y_1=25000$  and  $Y_2=15000$  yields

$$\frac{1}{2} - (0.4951 - 0.2324\xi - 2.637 \times 10^{-6} Y_1 + 4.395 \times 10^{-6} Y_2)$$

Alesina *et al.* (2001, Table 3.4 column 1)<sup>2</sup> show that, on average, the marginal effect on happiness (H) of a one standard deviation change in mean income is 0.003, while the corresponding figure for a change in the Gini coefficient is -0.001. These figures imply that the marginal effect of a unit change in mean income is  $1.446 \times 10^{-6}$ , while that of a unit change in the inequality measure is -0.0357.

Hence the happiness function may be expressed as

$$H = 1.446 \times 10^{-6} [\xi Y_1 + (1-\xi)Y_2] - 0.0357 [\frac{1}{2} - (0.4951 - 0.2324\xi - 2.637 \times 10^{-6} Y_1 + 4.395 \times 10^{-6} Y_2)] + \text{constant}$$

Imposing  $\xi=0.1$ , this may be rewritten as:

$$H = b_1 \xi Y_1 + b_2 (1-\xi) Y_2 + \text{constant}$$

where

$b_1=5.0459 \times 10^{-7}$  and  $b_2=1.6203 \times 10^{-6}$ . This confirms that, in the social welfare function, more weight is attached to the income earned by the lower income group than to that of the higher income group. Using these values and grossing up, I can infer that  $\sigma$  equals  $b_1/b_2$ , that is 0.3114.

It should be noted at this stage that there remains some controversy about whether it is legitimate to represent the happiness function as a social welfare function. As Arrow (1951) demonstrated, the aggregation of individual preferences into a social welfare function may not be possible under reasonable conditions. But if, as here, it is supposed that interpersonal comparisons of utility can be made - I am using a cardinal measure of happiness - then Arrow's theorem does not hold (Sen, 1970). The controversy therefore boils down to the reliability of the happiness data. Much work

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<sup>2</sup> This is based on a model for the United States. Alesina *et al.* do not report marginal effects for the corresponding model using European data, although much of the work in their paper is based on data from the Eurobarometer data set.

in psychology (see, for example, Argyle, 1989), and an increasing body of work in economics (Oswald, 1997; di Tella *et al.*, 2001) suggests that the data are dependable.

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